JAMES SERRIN: FROM HIS LEGACY TO THE NEW FRONTIERS

Perugia, January 30 – February 3, 2017
Sala della Partecipazione
Palazzo Cesaroni – Piazza Italia

Abstracts

Harnack, Hölder, Gauss and Widder: Serrin's Parabolic Legacy

Don G. Aronson
School of Mathematics, University of Minnesota

Wednesday, 1st February – 16:40-17:20

James Serrin's fundamental contributions to the theory of quasilinear elliptic equations are well-known and widely appreciated. He also made less well-known contributions to the theory of quasilinear parabolic equations which we discuss in this note. Jügen Moser gave greatly simplified proofs of the De Giorgi-Nash regularity results for linear divergence structure elliptic and parabolic differential equations using an original iterative technique. Serrin extended Moser's techniques and applied them to the study of divergence structure quasilinear elliptic and, in collaboration with Aronson, to divergence structure quasilinear parabolic equations. Specifically, among other results, they proved a maximum principle, Hölder continuity of generalized solutions and derived a Harnack principle for a very broad class of quasilinear parabolic equations. In subsequent work, Aronson applied these results to study non-negative solutions to divergence structure linear equations without regularity assumptions on the coefficients. The results include a two-sided Gaussian estimate for the fundamental solution and a generalization of the Widder Representation Theorem.

A priori estimates and ground states of solutions of an Emden–Fowler equation with gradient term

Marie Françoise Bidaut–Véron
Faculté des Sciences et Techniques, Université Francois–Rabelais de Tours

Thursday, 2nd February – 12:10-12:50

Here we consider the nonnegative solutions of equation

$$-\Delta u = u^p |\nabla u|^q$$

in a punctured ball $B(0,R)\setminus\{0\} \subset \mathbb{R}^N$, where $p, q \in \mathbb{R}$, $p+q > 1$ (which reduces to the Emden-Fowler equation when $q = 0$). We look for a priori estimates on the solutions and their gradient, in particular beyond the first critical case $(N-2)p + (N-1)q = N$. We use Bernstein technique and Osserman's or Gidas-Spruck's type results. In particular, such estimates hold when $p + q < 1 + 4/N$, or when $q \geq 2$. The most interesting case is $0 \leq q < 1$, where we also give Liouville type results. (Joint work with *M. Garcia–Huidobro* and *L. Véron.*)

A family of degenerate operators: maximum principle and some unusual phenomena

Isabeau Birindelli
Dipartimento di Matematica *Guido Castelnuovo*, Sapienza Università di Roma
Monday, 30th January – 16:40-17:20

We shall discuss the validity and the consequences of the maximum principle for degenerate elliptic operators whose higher order term is the sum of k eigenvalues of the Hessian. In particular we will show some very unusual phenomena due to the degeneracy of the operator. We prove moreover Lipschitz regularity results and boundary estimates for problems in convex domains which are intersections of ball of same radius called *hula hoop domains*.

J. Serrin and G. Stampacchia methods as starting blocks for some new results on the Dirichlet problem

Lucio Boccardo
Dipartimento di Matematica *Guido Castelnuovo*, Sapienza Università di Roma
Monday, 30th January – 11:50-12:30

We assume that Ω is a bounded, open subset of $\mathbb{R}^N$, $p > 1$, $f \in L^m(\Omega)m$, $m \geq 1$, and we consider the **model case** of a nonlinear boundary value problem

$$\begin{cases} -\mathrm{div}(a(x)|\nabla u|^{p-2}\nabla u) = f(x), & \text{in } \Omega; \\ u = 0, & \text{on } \partial\Omega. \end{cases}$$

If $f \in L^m(\Omega)$, $m \geq (p^*)'$, the summability of u (depending on m) was proved by G. Stampacchia and B-Giachetti (see also [B-Croce, book]).

However, the summability of ∇u is not a strictly increasing function of m, even in the linear case (see [Boccardo 2014]): $p = 2$, $m > N/2$.

In the case $f \in L^m(\Omega)$, $1 \leq m < (p^*)'$, is proved in [B-Gallouet, 1989], [B-Gallouet, 1992], the existence of a distributional solution $u \in W_0^{1,[(p-1)m]^*}(\Omega)$ if $\sup(1, \frac{N}{N(p-1)+1}) < m < (p^*)'$. If $1 \leq m < \frac{N}{N(p-1)+1}$, the definition of entropy solution is needed ([paper-6]).

If we assume $f \in L^m(\Omega)$, $m = \frac{N}{N(p-1)+1}$, $1 < p < 2 - \frac{1}{N}$, then there exists a distributional solution $\boxed{u \in W_0^{1,1}(\Omega)}$ [B-Gallouet 2012].

The presence of lower order term depending on u [Cirmi 1995] or depending on ∇u [B-Gallouet 1993] produces finite energy solutions even if $m < (p^*)'$.

In [B-Cirmi, 2016] some borderline cases of m which give solutions in $\boxed{W_0^{1,1}(\Omega)}$ are studied.

A few recent results on the weak maximum principle

Italo Capuzzo Dolcetta
Dipartimento di Matematica *Guido Castelnuovo*, Sapienza Università di Roma
Monday, 30th January – 10:00-10:40

I will discuss a few topics concerning the weak maximum principle for second order operators with nonnegative characteristic form and, more generally, for fully nonlinear degenerate elliptic ones. The first part of the talk will be focused on a characterization of the validity the weak maximum principle in the spirit of previous results of Donsker-Varadhan and Berestycki-Nirenberg-Varadhan. In the second part, I will present some recent research, steaming from a series of papers by Caffarelli-Li-Nirenberg, about the validity of the weak maximum principle for one-directional elliptic operators on some specific classes of unbounded domains, possibly of infinite measure.

Vortex dynamics in Euler flows and the Liouville equation

Manuel del Pino
Departamento de Ingenieria Matemática, Universidad de Chile
Monday, 30th January – 11:00-11:40

We consider the two-dimensional Euler flow for an incompressible fluid confined to a smooth domain. We construct regular solutions with concentrated vorticities around k points which evolve according to the Hamiltonian system for the Kirchoff-Routh energy. We capture the core of the desingularization of each vortex as a scaled finite mass solution of Liouville's equation plus small, more regular terms.

This is joint work with *Juan Dávila*, *Monica Musso* and *Juncheng Wei*.

Multivector fields, k-measures and their boundaries

Antonio DiCarlo
CECAM–IT–SIMUL Node, Roma
Friday, 3rd February – 12:10-12:50

Boundary-related issues were occasionally considered by Jim Serrin: think of his *interstitial work flux* (Dunn & Serrin 1985). I view boundaries as interfaces across which some relevant physical quantity exhibits a more or less abrupt contrast. Seen this way, whether a boundary appears sharp or blunt depends on the scale of observation. It is therefore desirable to have a unique formalism covering the whole range of blurriness.

With this in mind, I introduce the boundary of (compactly supported) smooth k-vector fields as primary, and define the exterior derivative of smooth differential k-forms via an integral duality. This is nicely consistent with the way in which boundary and coboundary are introduced in algebraic topology and discrete exterior calculus.

It emerges naturally that (locally summable) k-vector fields do not matter *per se*, but as densities of k-measures absolutely continuous with respect to the Lebesgue measure. This allows extending the notion of boundary to general k-measures. Then, identifying each k-dimensional submanifold with boundary with the corresponding characteristic k-measure reconciles the different notions of boundary and justifies my terminology. In conclusion, both diffuse and clear-cut boundaries are uniformly covered.

Gagliardo-Nirenberg inequalities for differential forms in Heisenberg groups

Bruno Franchi
Alma Mater Studiorum – Università di Bologna
Friday, 3rd February – 10:20-11:00

The L^1-Sobolev inequality states that for compactly supported functions u on the Euclidean n-space, the $L^{n/(n-1)}$-norm of a compactly supported function is controlled by the L^1-norm of its gradient. The generalization to differential forms (due to Lanzani & Stein and Bourgain & Brezis) is recent, and states that a the $L^{n/(n-1)}$-norm of a compactly supported differential h-form is controlled by the L^1-norm of its exterior differential du and its exterior codifferential δu (in special cases the L^1-norm must be replaced by the $\mathcal{H}^1$-Hardy norm). In a joint paper with *A. Baldi* and *P. Pansu*, we extend this result to Heisenberg groups in the framework of Rumin's complex of differential forms.

Existence of energy minimizing solutions to the Kirchhoff–Plateau problem

Eliot Fried
Okinawa Institute of Science and Technology Graduate University
Wednesday, 1st February – 12:10-12:50

The Kirchhoff–Plateau problem concerns the equilibrium shapes of a flexible filament in the form of a closed loop is spanned by a soap film, with the filament being modeled as a Kirchhoff rod and the action of the spanning surface being solely due to surface tension. We establish the existence of an equilibrium shape that minimizes the total potential energy of the system under the physical constraint of non-interpenetration of matter, but allowing for self-contact of the filament. In our treatment, the bounding filament is of finite thickness and, thus, has nonvanishing volume, while the soap film is represented by a set with finite two-dimensional Hausdorff measure. Moreover, the region of contact between the soap film and the surface of the filament is not prescribed a priori. This is joint work with *Giulio Giusteri* and *Luca Lussardi*.

Structure and regularity of the singular set in the obstacle problem for the fractional Laplacian

Nicola Garofalo
ICEA, Università di Padova
Friday, 3rd February – 11:20-12:00

I will present some new results on the structure and regularity of the singular free boundary in the obstacle problem for the fractional Laplacian. One essential ingredient is a new one-parameter family of Monneau type monotonicity formulas. This is joint work with *Xavier Ros-Oton*.

Stability, solitons and stiffness in suspension bridges

Filippo Gazzola
Dipartimento di Matematica, Politecnico di Milano
Wednesday, 1st February – 15:50-16:30

We model the deck of a suspension bridge as a degenerate plate, that is, a central beam with cross sections. This model displays two degrees of freedom: the vertical displacement of the beam and the torsional angles of the cross sections. We investigate the stability of the vertical movements and we prove the existence of solitons (stable solitary waves). Under certain stiffness assumptions, we show that the solitons have a nontrivial torsional component: this appears relevant for the security since several suspension bridges collapsed due to torsional oscillations. Our conclusion is that more stiffness may lead to collapses. We confirm this conclusion by analyzing more reliable and sophisticated models.

Based on joint works with *V. Benci* and *D. Fortunato*, with *E. Berchio*, with *G. Arioli*.

Minimising a relaxed Willmore functional for graphs
subject to Dirichlet boundary conditions

Hans–Christoph Grunau
Institut für Analysis und Numerik, Universität Magdeburg
Wednesday, 1st February – 15:00-15:40

One of James Serrin's most celebrated results appeared 1968 in Crelle's journal in the joint work "The Dirichlet problem for the minimal surface equation in higher dimensions" with Howard Jenkins. We quote from [2]:

> Let D be a bounded domain in E^n whose boundary is of class C^2. Then the Dirichlet problem for the minimal surface equation in D is well posed for C^2 boundary data if and only if the mean curvature of Γ is non-negative everywhere.

The minimal surface equation has been one of the main driving forces in the investigation of quasilinear *second* order elliptic equations. Currently the theory of quasilinear *fourth* order elliptic equations is by far less well developed. The lack of maximum principles and Stampacchia-De Giorgi-Nash-Moser-like techniques is an important obstruction. But perhaps, in 50 years the Willmore equation will have played a comparable role in this field as the minimal surface equation in second order equations. The goal of the present talk is to outline a very first step towards a possible future theory for the Dirichlet problem for the Willmore equation. The hope is that at some time in future somebody may be able to prove a Jenkins-Serrin-type result also in this fourth order problem.

For a bounded smooth domain $\Omega \subset \mathbb{R}^2$ and a smooth boundary datum $\varphi : \overline{\Omega} \to \mathbb{R}$ we consider the minimisation of the Willmore functional

$$W(u) := \frac{1}{4} \int_\Omega H^2 \sqrt{1+|\nabla u|^2}\, dx$$

for graphs $u : \overline{\Omega} \to \mathbb{R}$ with mean curvature $H := \operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right)$ subject to Dirichlet boundary conditions, i.e. in the class

$$\mathcal{M} := \{u \in H^2(\Omega) : (u - \varphi) \in H_0^2(\Omega)\}.$$

Making use of a celebrated result by L. Simon [4, Lemma 1.2] we first show that *in this class*, bounds for the Willmore energy imply area and diameter bounds. Examples show that stronger bounds in terms of the Willmore energy are not available. This means that $L^\infty \cap BV(\Omega)$ is the natural solution class where, however, the original Willmore functional is not defined. So, we need to consider its L^1-lower semicontinuous relaxation. Our main result states that this relaxation coincides on $\mathcal{M}$ with the original Willmore functional so that the relaxed functional is indeed its largest possible L^1-lower semicontinuous extension to $BV(\Omega)$. Moreover, finiteness of the relaxed energy encodes attainment of the Dirichlet boundary conditions in a suitable sense. Finally, we obtain the existence of a minimiser in $L^\infty \cap BV(\Omega)$ for the relaxed/extended energy.

The major benefit of our non-parametric approach is the validity of a-priori diameter and area bounds, which are not available in the general setting of R. Schätzle's work [3]. On the other hand we need to leave open most of the regularity issues.

This talk is based on the joint work [1] with *Klaus Deckelnick* and *Matthias Röger*.

References

[1] Klaus Deckelnick, Hans-Christoph Grunau, Matthias Röger, Minimising a relaxed Willmore functional for graphs subject to boundary conditions, Preprint 2015, arxiv:1503.01275.

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An Inverse Problem in Potential Theory for Picone Elliptic–Parabolic PDEs

Ermanno Lanconelli

Alma Mater Studiorum – Università di Bologna,

Monday, 30th January – 15:50-16:30

Let Ω be a domain in $\mathbb{R}^N$. A density with the mean value property for non-negative harmonic functions in Ω is a positive l.s.c. function w such that, for a suitable $x_0 \in \Omega$,

$$u(x_0) = \frac{1}{w(\Omega)} \int_\Omega u(y)w(y)dy$$

for every non-negative harmonic function u in Ω. In this case we say that (Ω, w, x_0) is a Δ-triple. Existence of Δ-triples on every sufficiently smooth domain has been proved in 1994-1995, by Hansen and Netuka, and by Aikawa.

Very recently, we have given positive answers to the following inverse problem:

"Let (Ω, w, x_0) and (D, w', x_0) be Δ-triples such that $\frac{w}{w(\Omega)} = \frac{w'}{w'(D)}$ in $D \cap \Omega$. Then is it true that $\Omega = D$?"

Our result contains, as particular cases, several classical potential theoretical characterizations of the Euclidean balls. Densities with the mean value property for solutions to wide classes of Picone's elliptic-parabolic PDEs have appeared in literature since the 1954 pioneering work by B.Pini on the mean value property for caloric functions. In this talk we present an abstract inverse problem Theorem allowing to extend the previously recalled result on the Δ-triples to elliptic, parabolic and sub-elliptic PDEs. The results have been obtained in collaboration with *G. Cupini* (Univ. Bologna).

Some recent results in singularly perturbation problems

Giovanni Leoni

Department of Mathematical Sciences, Carnegie Mellon University

Tuesday, 31st January – 9:30-10:10

In this talk we will present a survey of some singular perturbation models and their applications to the study of phase transitions problems.

Almost–critical points in geometric variational problems

Francesco Maggi
International Centre for Theoretical Physics (ICTP), Trieste

Tuesday, 31st January – 15:00-15:40

We review some recent works where the quantitative geometric description of almost–critical points in a given variational problem is investigated. The study of boundaries with almost–constant mean curvature is addressed for studying critical points in capillarity theory in joint papers with *Giulio Ciraolo* (Univ. Palermo) and *Brian Krummel* (Univ. Texas, Austin). The study of conformally flat metric with positive almost–constant scalar curvature is addressed for obtaining new estimates for convergence to equilibrium in fast diffusion equations in a joint paper with *Giulio Ciraolo* and *Alessio Figalli* (ETH Zurich).

On singular Liouville equations

Andrea Malchiodi
Scuola Normale Superiore di Pisa

Thursday, 2nd February – 16:40-17:20

Liouville equations play a crucial role both in Geometry and in Mathematical Physics. We consider singular versions of the equation, describing either conical singularities or vortex points. We will discuss both existence results, obtained via improved functional inequalities of Moser–Trudinger type combined with topological arguments, as well as non existence results, via generalized Pohozaev type identities.

There are joint works with *D. Bartolucci*, *A. Carlotto*, *F. De Marchis* and *D. Ruiz*.

Some remarks in the Calculus of Variations inspired by James Serrin's work

Paolo Marcellini
Dipartimento di Matematica e Informatica *U. Dini* – Università di Firenze

Thursday, 2nd February – 9:30-10:10

I will discuss about some new and old problems of the Calculus of the Variations having in mind some James Serrin's ideas, mainly about lower semicontinuity and about elliptic and parabolic partial differential equations.

Schrödinger equations and applications to semilinear problems

Moshe Marcus
Department of Mathematics, Technion – Israel Institute of Technology, Haifa

Tuesday, 31st January – 15:50-16:30

We consider Schrödinger equations of the form $\Delta u + \gamma V u = 0$ in a bounded Lipschitz domain $D \subset \mathbb{R}^{\mathbb{N}}$. We assume that the potential $V \in C(D)$ is positive and satisfies the condition $V(x) \leq a\mathrm{dist}(x, \partial D)^{-2}$, $\forall x \in D$, where a is a constant. The coefficient γ is a constant such that $\gamma < C_H(D; V)$ (= the Hardy constant with weight V in D).

We discuss properties of the linear equation, including sharp two-sided estimates of Green functions. In continuation we discuss applications to the study of semilinear problems of the form $-(\Delta u + \gamma V u) + g(u) = 0$, where $g \in C(\mathbb{R})$ is monotone increasing and superlinear on $\mathbb{R}_+$.

Prescribed mean curvature graphs with Neumann boundary conditions in some Friedman-Lemaître-Robertson-Walker spacetimes

Jean Mawhin
Ecole de Mathématique, Université Catholique de Louvain
Friday, 3rd February – 9:30-10:10

The general purpose of the lecture is the study of the prescribed mean curvature problem with Neumann condition in a certain family of Friedmann-Lemaître-Robertson-Walker (FLRW) spacetimes. The mathematical problem under consideration is described as follows.

Let $I \subseteq \mathbb{R}$ be an open interval with $0 \in I$, and let $f \in C^1(I)$ a positive function. For a given continuous and radially symmetric function $H : \mathbb{R} \times [0, +\infty) \to \mathbb{R}$, we look for radially symmetric solutions of the problem

$$(P) \qquad \begin{cases} \operatorname{div}\left(\dfrac{\nabla u}{f(u)\sqrt{f(u)^2 - |\nabla u|^2}}\right) + \dfrac{f'(u)}{\sqrt{f(u)^2 - |\nabla u|^2}}\left(n + \dfrac{|\nabla u|^2}{f(u)^2}\right) = nH(u, |x|) \\ |\nabla u| < f(u) \;\text{ in } B(R), \\ \dfrac{\partial u}{\partial \nu} = 0 \quad \text{in} \quad \partial B(R), \end{cases}$$

where $\dfrac{\partial u}{\partial \nu}$ denotes the outward normal derivative of u.

Our purpose is to identify a family of warping functions f such that for any prescribed curvature function H, problem (P) possesses at least one solution for a given range of values of the radius R, independent of the curvature function. This contrasts with the results obtained for Neumann problems in Lorentz-Minkowski case, corresponding to $f(t) \equiv 1$, which require some sign condition upon H but do not put limitations upon R.

Such family contains some examples of interest in cosmology. This is a joint work with *Pedro J. Torres* from the University of Granada.

Semilinear biharmonic coercive equations and inequalities on $\mathbb{R}^N$

Enzo Mitidieri
Dipartimento di Matematica e Geoscienze, Università di Trieste
Wednesday, 1st February – 11:20-12:00

During a private conversation in Perugia in July 2008, Professor James Serrin asked whether is it possible to extend Liouville theorems for second order quasilinear equations and inequalities and for changing sign solutions to higher order versions of similar problems. Complete and sharp answers to this problem for second order semilinear and quasilinear coercive elliptic inequalities have been obtained among others by *Alberto Farina* and *James Serrin* in 2011-2012. In this talk we present some results for semilinear biharmonic equations/inequalities of the type

$$(*) \qquad \Delta^2 u + f(u) = 0 \quad \text{on } \mathbb{R}^N,$$

where $N \geq 1$ and $f : \mathbb{R} \to \mathbb{R}$ is a superlinear function. Our analysis is motivated by the following result: *Let* $u \in L^q_{loc}(\mathbb{R}^N)$*,* $q > 1$*, be a nontrivial distributional solution of*

$$\Delta^2 u + |u|^{q-1} u = 0 \quad \textit{on } \mathbb{R}^N,$$

then u *must change sign.*

We prove several Liouville theorems for $(*)$, revealing unexpected phenomena related to the dimension N.

This is a joint work with *Lorenzo D'Ambrosio* from the University of Bari.

Existence of solutions of some fully nonlinear elliptic problems

Filomena Pacella
Dipartimento di Matematica *Guido Castelnuovo*, Sapienza Università di Roma
Thursday, 2nd February – 11:20-12:00

I will discuss the existence or nonexistence of solutions of fully nonlinear elliptic equations of the type

$$-F(x, D^2u) = |u|^{p-1}u$$

with homogeneous boundary conditions in bounded domains. Similarities and differences with respect to the semilinear case will be emphasized.

Towards a deterministic KPZ equation with fractional diffusion: The stationary problem

Ireneo A. Peral
Department of Mathematics, Universidad Autónoma de Madrid
Thursday, 2nd February – 10:20-11:00

The classical KPZ model involves a nonlinearity depending on the modulus of the gradient, but the diffusion is typically the Laplacian.

The goal of the talk is to explore a direction in which the diffusion is a fractional Laplacian.
(Work supported by Project MTM2013-40846-P, MINECO, Spain.)

On entropy production and energy loss

Paolo Podio–Guidugli
Accademia Nazionale dei Lincei, Roma
Tuesday, 31st January – 12:10-12:50

In [1], Jim Serrin summarized and completed his views about the format of the basic laws of continuum mechanics. Starting from suitable weak forms of the postulates about cyclic processes of classic thermodynamics of homogeneous processes, he showed how the two state functions of importance, internal energy and entropy, could be consistently constructed for a fairly general and representative constitutive class of materials; moreover, he proved that the standard dissipation inequality that filters unphysical constitutive prescriptions *à la* Coleman–Noll could be arrived at also by replacing the usual internal-energy equation by an imbalance law under form of an inequality. In so doing, he implied that not only, as is customary, the internal source of entropy should be taken positive, but also that the non-constitutively specified part of the internal source of internal energy should be taken negative: in other words, that entropy production should be accompanied by energy loss.

In my talk, building partly on [2], I shall show how a careful hybridization of statistical and continuum mechanics leads to identifying those internal sources as purely kinetic macroscopic consequences of microscopic motion randomness, consequences that continuum mechanics alone could not predict.

References

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Two classical results with lack of monotonicity

Vicenţiu D. Rădulescu
Institute of Mathematics of the Romanian Academy, Bucharest
Tuesday, 31st January – 16:40-17:20

We establish that the monotonicity assumption can be removed in the framework of two classical results in nonlinear analysis. We are first concerned with the general version of the maximum principle, which is due to P. Pucci and J. Serrin [1, 2, 3]. Next, we consider the Keller-Osserman condition (cf. [4, 5]), which characterizes the existence of singular solutions with boundary blow-up. We expect that related arguments can be used in order to show that monotonicity hypotheses are not necessary in the case of several models described by nonlinear partial differential equations.

References

[1] P. Pucci and J. Serrin, A note on the strong maximum principle for elliptic differential inequalities, *J. Math. Pures Appl.* **79** (2000), 57-71.
[2] P. Pucci and J. Serrin, The strong maximum principle revisited, *J. Differential Equations* **196** (2004), 1-66.
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On the interplay among weak and strong maximum principles, compact support principles and Keller-Osserman conditions on complete manifolds

Marco Rigoli
Dipartimento di Matematica "Federigo Enriques", Università degli Studi di Milano
Tuesday, 31st January – 11:20-12:00

We study the influence of geometry on some classical problems of analysis, well studied on Euclidean space, in the case of complete manifolds. In order to do so we need to introduce new techniques to deal with global phenomena of the ambient space due to the presence of curvature, the existence of a non empty cut locus and in general that of a non trivial topology. In particular, we consider the strong and weak maximum principles, the Liouville property and the compact support principle also in the presence of gradient terms obtaining new results even for the flat space.

On the moving plane method for semilinear and quasilinear elliptic problems

Berardino Sciunzi
Dipartimento di Matematica e Informatica, Università della Calabria
Tuesday, 31st January – 10:20-11:00

Starting from the seminal paper of J. Serrin, driven by the celebrated work of B. Gidas, W. M. Ni and L. Nirenberg, the PDE community started studying symmetry and monotonicity properties of solutions to elliptic PDE via the *Moving Plane Technique*. Bounded or unbounded domains can be considered and each case exhibits its peculiarities. We will consider the leading examples given by the case of bounded domains, the case of the whole space and the case of the half space. I will recall the main ingredients of the technique and then I will discuss some generalizations in the case of semilinear or quasilinear elliptic problems. Some of the results that I will present have been obtained in collaboration with *L. Damascelli*, *A. Farina* and *L. Montoro*.

Solutions to overdetermined elliptic problems in nontrivial exterior domains

Pieralberto Sicbaldi

Aix–Marseille Université, CNRS - École Centrale Marseille

Thursday, 2nd February – 15:50-16:30

Overdetermined elliptic systems of the form

$$\begin{cases} \Delta u + f(u) = 0 & \text{in} \quad \Omega \subset \mathbb{R}^n, n \geq 2 \\ u = 0 & \text{on} \quad \partial\Omega \\ \frac{\partial u}{\partial \vec{n}} = \text{constant} & \text{on} \quad \partial\Omega \end{cases}$$

appear in many problems in Physics and Applied Mathematics. A surprising parallelism between such problems and constant mean curvature surfaces has been observed in the last years, and this allowed to obtain some strong classification results, and some nontrivial and unexpected solutions.

In this talk, I will present the construction of new solutions in the case where the PDE is the Nonlinear Schrödinger Equation and the domain Ω is a nontrivial exterior domain, i.e. the complement of a compact region that is not the ball. These new solutions are the first examples that, up to our knowledge, have no clear counterpart in the theory of constant mean curvature surfaces. They provide also an answer, with a counterexample for all dimension $n \geq 2$, to a conjecture by Berestycki, Caffarelli and Nirenberg, which was still open in dimension 2.

This is a joint work with *A. Ros* and *D. Ruiz*.

Boundary conditions and corners in elliptic problems

Guido Sweers

Mathematisches Institut, Universität zu Köln

Wednesday, 1st February – 9:30-10:10

Homogeneous Dirichlet boundary conditions have a stable dependence on the domain. That is, an approximation of the domain by for example polygons usually leads to the solution on the limit domain. It is known that such a result does not hold for second order elliptic problems under Neumann boundary conditions. For higher order elliptic problems the number of physically relevant boundary condition increases and for most of them an approximation through polygonal domains becomes quite subtle. Many authors have contributed to these questions and I would like to present some of their results.

Entire solutions and spiralling asymptotic profiles of competition–diffusion systems

Susanna Terracini
Dipartimento di Matematica "Giuseppe Peano", Università di Torino

Monday, 30th January – 15:00-15:40

In this talk we consider solutions of the competitive elliptic system

$$\begin{cases} -\Delta u_i = -\beta \sum_{j\neq i} a_{ij} u_i u_j & \text{in } \Omega \subset \mathbf{R}^N \\ u_i = g & \text{in } \partial\Omega \end{cases} \qquad i = 1, \dots, k,$$

and their asymptotic profiles when $\beta \to +\infty$. We shall focus our attention on the asymmetric case: $a_{ij} \neq a_{ji}$. This is a joint result with *A. Salort, G. Verzini, A. Zilio.*

Gradient estimates in quasilinear Riccati equations and applications to singularities and growth problems

Laurent Véron
Faculté des Sciences et Techniques, Université Francois–Rabelais de Tours

Thursday, 2nd February – 15:00-15:40

We study local and global properties of solutions of

$$(P) \qquad -\Delta_p u + |\nabla u|^q = 0,$$

in a domain in the superhomogeneous range $0 < p - 1 < q$. In the flat case $\Omega \subset \mathbb{R}^N$, we use a combination of Bernstein and Keller-Osserman methods to obtain that any C^1 solution verifies the basic estimate

$$|\nabla u(x)| \leq c_{N,p,q} \left(\operatorname{dist}(x, \partial\Omega)\right)^{-\frac{1}{q+1-p}} \qquad \forall x \in \Omega.$$

The method extends to a geometric framework where (M^d, g) is a complete Riemannian manifold with metric g, Ricci and sectional curvatures $Ricc_g$ and $Sect_g$ bounded from below. As an application we prove: *If $q > p - 1$ and $Ricc_g \geq (1 - d)B^2$ for some $B \geq 0$ and if the convexity radius is infinite if $1 < p \leq 2$ or*

$$\lim_{\operatorname{dist}(x,a)\to\infty} \frac{|Sect_g(x)|}{\operatorname{dist}(x, a)} = 0,$$

for some $a \in M$, if $p \geq 2$, then any solution of (P) in M satisfies

$$|\nabla_g u(x)| \leq c_{N,p,q} B^{\frac{1}{q+1-p}} \qquad \forall x \in M.$$

We also prove: *Let (M^d, g) be an asymptotically flat manifold with $\tau \geq 1$ i.e.*

$$g_{ij}(x) \sim \delta_{ij} \text{ and } \left(\operatorname{dist}(x, a)\right)^{\tau} |\nabla_g g_{ij}(x)| + \left(\operatorname{dist}(x, a)\right)^{\tau+1} |\nabla_g^2 g_{ij}(x)| \leq A,$$

for some $A > 0$ and $a \in M$. Then any nonnegative p-harmonic function in M admits a limit (finite or infinite) at infinity.

Cauchy–Liouville Theorems for the m–Laplacian Lane–Emden Equation and Their Applications

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Wednesday, 1st February – 10:20-11:00

We study the quasi-linear elliptic equations

$$\text{(I)} \qquad \begin{aligned} \operatorname{div}(|\nabla u|^{m-2}\nabla u) + B(z,u,\nabla u) &= 0 \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \partial\Omega, \end{aligned}$$

where $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ is a connected smooth domain, and the exponent $m \in (1,n)$ is a positive number. The homogeneous boundary conditions $u = 0$ is imposed only when $\partial\Omega \neq \emptyset$. We are concerned with the questions of a priori estimates, existence and non-existence of positive solutions of (I). A canonical prototype of (I) is the so called m-Laplacian Lane-Emden equation where $B = |u|^{p-1}u$, $p > m-1$, in which the results are generically optimum.

Part of this work is joint with *J. Serrin*.