Booklet of abstracts

Two Nonlinear Days in Perugia
on the occasion of Patrizia Pucci's 65th birthday

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PARTICIPANTS & INDEX

Monica Marras
Cagliari (Italy)
→ Page 9

Enzo Mitidieri
Trieste (Italy)
→ Page 10

Giovanni Molica Bisci
Reggio Calabria (Italy)
→ Page 11

Stella Piro Vernier
Cagliari (Italy)
→ Page 12

Vicenţiu D. Rădulescu
Craiova and Romanian Academy (Romania)
→ Page 13

Bentahar Rajae
Tetouan (Morocco)

Paola Rubbioni
Perugia (Italy)

Xu Runzhang
Harbin (P.R.C.)
→ Page 14

Sara Saldi
Florence (Italy)
→ Page 15

Simone Secchi
Milano Bicocca (Italy)

Raffaella Servadei
Urbino (Italy)
→ Page 16

Kamel Thari
Tlemcen (Algeria)

Enzo Vitillaro
Perugia (Italy)
→ Page 17

Vardan Voskanyan
King Abdullah University (Saudi Arabia)

Youssef Zguir
(Morocco)

Multiple solutions for some supercritical problems

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In the last years we have investigated the existence of one or more weak solutions of the quasilinear elliptic model problem

$$\begin{cases} -\mathrm{div}(A(x,u)|\nabla u|^{p-2}\nabla u) + \frac{1}{p}A_t(x,u)|\nabla u|^p = g(x,u) + h(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, $N \geq 2$, the real terms $A(x,t)$, $A_t(x,t) = \frac{\partial A}{\partial t}(x,t)$ and $g(x,t)$ are Carathéodory functions on $\Omega \times \mathbb{R}$, $h : \Omega \to \mathbb{R}$ is a given measurable map.

We note that coefficient $A(x,t)$ makes the variational approach more difficult, as we have to look for critical points of the functional

$$\mathcal{J}(u) = \frac{1}{p}\int_\Omega A(x,u)|\nabla u|^p dx - \int_\Omega G(x,u)dx - \int_\Omega h(x)u(x)dx$$

in the Banach space $X = W_0^{1,p}(\Omega) \cap L^\infty(\Omega)$ (here, $G(x,t) = \int_0^t g(x,s)ds$).

Anyway, if it satisfies "good" growth assumptions, then at least one (bounded) critical point exists also when the nonlinear term $G(x,t)$ has a suitable supercritical growth but using variants of classic theorems which are based on a weak version of the Cerami–Palais–Smale condition.

More precisely, if $h \equiv 0$, we use a variant of the Ambrosetti–Rabinowitz theorems and prove both an existence result and, if $A(x,\cdot)$ and $G(x,\cdot)$ are even, also a multiplicity one.

On the other hand, if $h \not\equiv 0$ but $p = 2$, if $A(x,\cdot)$ and $G(x,\cdot)$ are even infinitely many solutions still exist in spite of the lack of symmetry and the proof is based on a variant of the variational perturbation techniques introduced by Rabinowitz.

These are joint works with Giuliana Palmieri and Addolorata Salvatore.

Hyperbolic systems in domains with growing cracks

Maicol Caponi
SISSA

In this talk I consider the hyperbolic system $\ddot{u} - \operatorname{div}(\mathbb{A}\nabla u) = f$ in the time varying cracked domain $\Omega \setminus \Gamma_t$. Here u is an $\mathbb{R}^d$–valued vector function, $\mathbb{A}$ satisfies the usual assumptions in linear elasticity, and the cracks Γ_t are closed subsets of $\overline{\Omega}$ increasing with respect to inclusion and contained in a prescribed C^2 manifold. I prove existence and uniqueness of weak solutions, provided that there is a regular change of variables which reduces $\Omega \setminus \Gamma_t$ to the fixed domain $\Omega \setminus \Gamma_0$. Moreover, I show an energy equality and, as a consequence, a continuous dependence result on the cracks for the solutions of this system.

Analysis of the nonlocal elastic rod by means of the Fractional Laplacian operator

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The rod analyzed in the paper is an example of nonlocal composite material. The nonlocal behavior is modeled through the Fractional Laplacian operator. The effect of the variation of the parameters of the model is investigated, with particular attention devoted to the order of the FL. The response is evaluated by means of a numerical procedure where the fractional Laplacian operator is estimated using a finite-difference quadrature technique. This is a joint work with G. Autuori, V. Gusella and P. Pucci.

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Radial solutions to p-Laplacian Neumann problems

Francesca Colasuonno
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In this talk, I will present some results on p-Laplacian problems set in a ball or an annulus of $\mathbb{R}^N$ with homogeneous Neumann boundary conditions. The considered problems admit at least one positive constant solution and involve a nonlinearity which can be taken supercritical in the sense of Sobolev embeddings. The results concern existence, multiplicity, oscillating behavior around the constant solution, and *a priori* estimates of radial positive solutions. I will show that the situation changes drastically depending on whether $1 < p < 2$, $p = 2$, or $p > 2$, and I will try to give a unified description of these three different cases. The main tools used are variational techniques and the shooting method for ODEs.

This is a joint work with A. Boscaggin (Torino) and B. Noris (Amiens).

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Nonexistence results for parabolic inequalities

Roberta Filippucci
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In this talk some Fujita type results for strongly p-coercive quasi linear parabolic differential inequalities will be discussed. The problem investigated involves both a diffusion term and a dissipative term, furthermore singular or degenerate weights are included in the nonlinearities. The protype we consider extends a wellknown model containing a reaction term u^q, $u \geq 0$, and a term of Hamilton-Jacobi type $b|\nabla u|^s$ given by

$$u_t - \Delta u = u^q - b|\nabla u|^s, \quad b > 0, \quad q \geq 1,$$

in $\Omega \times \mathbb{R}^+$, where Ω is a generic domain of $\mathbb{R}^N$. This equation was first introduced in [1] by Chipot and Weissler and then proposed in [2] by Souplet as a model in population dynamics.

The results discussed are contained in a joint work with S. Lombardi.

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Recent results and open questions on fractional Kirchhoff problems involving singular terms

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In this talk we discuss about Kirchhoff type problems driven by nonlocal fractional operators, involving critical Sobolev nonlinearities and different singular terms. In particular we focus our attention on singular terms of type $u^{-\gamma}$, with $\gamma > 0$, and on fractional Hardy potentials $u^2/|x|^{2s}$. Our problems present some difficulties due to the bi–nonlocal nature of the elliptic part, the lack of compactness at critical level and the presence of singularities. For this, in order to state multiplicity results, we introduce different proof techniques based on variational methods, approximation arguments and topological tools. Finally, we discuss on some interesting open questions, concerning the study of our problems.
This talk is mainly based on some joint works with V. Ambrosio, T. Isernia and P.K. Mishra.

Elliptic and parabolic equations under general and p, q growth conditions

Paolo Marcellini
University of Florence

We give some recent *existence* and *interior regularity results* - partly obtained in collaboration with *Giovanni Cupini, Michela Eleuteri* and *Elvira Mascolo* - for elliptic partial differential *equations* in divergence form, or elliptic *systems* of m partial differential equations in divergence form of the type

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_i} a_\alpha^i \left(x, u\left(x\right), Du\left(x\right)\right) = b_\alpha \left(x, u\left(x\right), Du\left(x\right)\right), \quad \alpha = 1, 2, \ldots, m,$$

for maps $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$. Here the vector field $\left(a_\alpha^i \left(x, s, \xi\right)\right)$ assumes values in the set of $m \times n$ matrices and it satisfies some *general growth conditions* with respect to the gradient variable $\xi \in \mathbb{R}^{m \times n}$, sometine p, q *growth conditions.*

As a part of a joint research-project started in 2013 with *Verena Bögelein and Frank Duzaar*, we consider the evolution problem associated with a convex integrand $f \colon \mathbb{R}^{m \times n} \to [0, \infty)$ satisfying - for instance - some p, q-growth assumption. To establish the existence of solutions we introduce the concept of *variational solutions.* In contrast to weak solutions, i.e. mappings $u \colon \Omega_T \subset \mathbb{R}^{n+1} \to \mathbb{R}^m$ which solve

$$\partial_t u - \mathrm{div} Df(Du) = 0$$

weakly in Ω_T, variational solutions in general exist under a much weaker assumption on the gap $q - p$.

In particular, if $2 \leq p \leq q < p + \min\left\{1, \frac{4}{n}\right\}$, we obtain the existence of variational solutions and we also show that they are actually - in this case - weak solutions. This means that any solution u automatically admits the necessary higher integrability of the spatial derivative Du to satisfy the parabolic system in the weak sense, i.e. we prove that

$$u \in L^q_{\mathrm{loc}} \left(0, T; W^{1,q}_{\mathrm{loc}}(\Omega, \mathbb{R}^m)\right).$$

Bernstein theorems for graphs with prescribed mean curvature

Luciano Mari
Scuola Normale Superiore

The core of this talk is the investigation of Bernstein-Liouville type theorems for entire graphs $u : M \to \mathbb{R}$ over a complete Riemannian manifold, under the condition that the mean curvature of the graph is a prescribed function. Emphasis will be put on minimal graphs, on solitons for the mean curvature flow and on capillary graphs. After a review of some classical results that stress the peculiar features of the mean curvature operator, we will focus on recent contributions based on joint works with B. Bianchini, M. Rigoli and P. Pucci. The geometry of the underlying manifold is taken into account just via the growth of the volume of its geodesic balls. The main Liouville type theorems are also applicable to more general quasilinear inequalities of the type

$$\operatorname{div}\big(A(|\nabla u|)\nabla u\big) \geq b(x)f(u)l(|\nabla u|).$$

Lifespan for solutions to 4-th order hyperbolic systems

Monica Marras
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Università di Cagliari

We discuss blow-up phenomena to solution of some classes of nonlinear hyperbolic coupled systems of fourth order under Dirichlet or Navier boundary conditions. In particular we consider Euler-Bernoulli type system for which we derive a safe interval of existence of the solution and a lower bound of the lifespan.
This is a joint work with S. Vernier-Piro.

Quasilinear elliptic system in divergence form associated to general nonlinearities

Enzo Mitidieri
University of Trieste

We present recent results on a-priori estimates of positive solutions of a class of quasilinear elliptic systems associated to general nonlinearities.

A group-theoretical approach for nonlinear Schrödinger equations

Giovanni Molica Bisci
University of Reggio Calabria

In this talk we present some recent results on the existence of weak solutions for some classes of Schrödinger equations defined on the Euclidean space $\mathbb{R}^d$ ($d \geq 3$). These equations have a variational structure and, thanks to this, if $d \neq 5$, we are able to find $(-1)^d + \left[\frac{d-3}{2}\right]$ pairs of sign-changing weak solutions for them by using the Palais principle of symmetric criticality and a group-theoretical approach on the orthogonal group $O(d)$ inspired by some seminal papers of Bartsch and Willem and recently used by Kristály and co-authors in different settings.

For instance, let

$$E_V := \left\{ u \in H^1(\mathbb{R}^d) : \int_{\mathbb{R}^d} (|\nabla u(x)|^2 + V(x)|u(x)|^2)dx < +\infty \right\}$$

be the Hilbert space endowed by the inner product

$$\langle u, v \rangle := \int_{\mathbb{R}^d} \nabla u(x) \cdot \nabla v(x)dx + \int_{\mathbb{R}^d} V(x)u(x)v(x)dx, \quad \forall\, u, v \in E_V$$

and induced norm

$$\|u\|_{E_V} := \left(\int_{\mathbb{R}^d} |\nabla u(x)|^2 dx + \int_{\mathbb{R}^d} V(x)|u(x)|^2 dx \right)^{1/2},$$

for every $u \in E_V$.

A simple prototype of the main results is the following.

Theorem 1 *Let $d > 3$ and let $1 < r < 2 < s < 2^*$, where $2^* := 2d/(d-2)$. Furthermore, let s' be the conjugate exponent of s and let $W \in L^\infty(\mathbb{R}^d) \cap L^{s'}(\mathbb{R}^d) \setminus \{0\}$ be a non-negative radially symmetric map. Finally, let V be a continuous and positive potential.*

Then, for λ sufficiently small, the following problem

$$\begin{cases} -\Delta u + V(x)u = \lambda W(x)(|u|^{r-2}u + |u|^{s-2}u) \ \text{ in } \ \mathbb{R}^d \\ u \in H^1(\mathbb{R}^d), \end{cases} \tag{C_λ}$$

admits at least

$$\zeta_S^{(d)} := 1 + (-1)^d + \left[\frac{d-3}{2}\right]$$

pairs of non-trivial weak solutions $\{\pm u_{\lambda,i}\}_{i \in J'} \subset E_V$, where $J' := \{1, ..., \zeta_S^{(d)}\}$, such that

$$\lim_{\lambda \to 0^+} \|u_{\lambda,i}\|_{E_V} = 0,$$

and $|u_{\lambda,i}(x)| \to 0$, as $|x| \to \infty$, for every $i \in J'$.

Moreover, if $d \neq 5$ problem (C_λ) admits at least

$$\tau_d := (-1)^d + \left[\frac{d-3}{2}\right]$$

pairs of sign-changing weak solutions $\{\pm z_{\lambda,i}\}_{i \in J} \subset E_V$.

A key ingredient of our proof is a compact embedding result due to Lions valid for some classes of H-invariant functions (where H is a subgroup of $O(d)$). We also emphasize that the method adopted here should be applied for a wider class of energies largely studied in the current literature also in non-Euclidean framework as, for instance, concave-convex nonlinearities on Cartan-Hadamard manifolds with poles.

Hölder regularity for bounded solutions to anisotropic equations

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In this talk we show the Hölder regularity for bounded solutions to the class of anisotropic elliptic equations

$$\sum_{i=1}^{N-1} \frac{\partial}{\partial x_i} A_{q,i}(x,u,Du) + \frac{\partial}{\partial x_N} A_p(x,u,Du) = 0 \quad \text{in } \Omega,$$

with Ω a regular domain in $\mathbb{R}^N$, with $p > q > 1$ (singular case).

The functions $A_{q,i}(x,u,Du)$ and $A_p(x,u,Du) : \Omega \times \mathbb{R}^{N+1} \to \mathbb{R}^N$ are assumed to be measurable and satisfying the structure conditions. This result is the dual of the one proved by Liskevich and Skrypnik [2] and Düzgüm, Marcellini, Vespri [1].

The results have been established in a joint research with Francesco Ragnedda (Cagliari) e Vincenzo Vespri (Firenze).

This talk is dedicated to prof. Patrizia Pucci in the occasion of her 65th birthday.

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Small and high perturbations of nonhomogeneous elliptic problems

Vicenţiu D. Rădulescu
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We report on several recent results concerning various perturbation effects for some classes of nonlinear elliptic equations with Dirichlet boundary condition. These problems are driven by two nonhomogeneous differential operators, recently introduced by A. Azzollini, resp. I.H. Kim & Y.H. Kim. These new differential operators extend the classical $p(x)$-Laplace operator, the generalized mean curvature operator, and the anisotropic capillarity operator.

We are concerned both with *small* and *high* perturbations and we establish existence and nonexistence results. This study is motivated by phenomena arising in the study of non-Newtonian fluids, image processing, and other applied fields. Problems of this type are in relationship with the contributions of P. Marcellini, R. Mingione *et al.* in the study of non-autonomous problems with integrands satisfying unbalanced growth conditions.

This is a joint work with S. Baraket, S. Chebbi and N. Chorfi.

This talk is dedicated with gratitude to Professor Patrizia Pucci on the occasion of her 65th birthday.

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The initial-boundary value problems for sixth order nonlinear wave equation

Xu Runzhang
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In this talk we consider the initial boundary value problem of solutions for a class of sixth order 1-D nonlinear wave equations. We discuss the probabilities of the existence and nonexistence of global solutions and give some sufficient conditions for the global and non-global existence of solutions at three different initial energy levels, i.e., sub-critical level, critical level and sup-critical level.
This is a joint work with Mingyou Zhang, Shaohua Chen, Yanbing Yang and Jihong Shen.

Asymptotic stability for nonlinear damped Kirchhoff systems involving the fractional p–Laplacian operator

Sara Saldi
University of Florence

In this talk we present some results concerning the question of global and local asymptotic stability for nonlinear damped Kirchhoff systems, with homogeneous Dirichlet boundary conditions, under fairly natural assumptions on the external force, the distributed damping, the perturbation term and the dissipative term, when the initial data are in a special region. Particular attention is devoted to the asymptotic behavior of the solutions in the linear case. This talk is based on a joint work with P. Pucci, where we extend in several directions recent theorems and cover also the so–called degenerate case, that is when the Kirchhoff function M is zero at zero.

References

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Topological methods for fractional critical equations

Raffaella Servadei
University of Urbino "Carlo Bo"

Motivated by the interest shown in the literature for nonlocal operators of elliptic type, in recent years we considered various problems in this setting.

In this talk we will present a recent multiplicity result for nonlocal fractional equations in presence of critical nonlinearity. These results, which extend the validity of some theorems known in the classical case of the Laplacian to the nonlocal framework, were obtained via topological methods in a joint paper with Giovany Figueiredo and Giovanni Molica Bisci.

On the the wave equation with hyperbolic dynamical boundary conditions, interior and boundary damping and sources

Enzo Vitillaro
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The talk will deals with local and global existence, uniqueness and well–posedness results, together with blow–up results, for the wave equation with hyperbolic dynamical boundary conditions, interior and boundary damping and sources. The typical problem studied is

$$\begin{cases} u_{tt} - \Delta u + P(x,u_t) = f(x,u) & \text{in } (0,\infty)\times\Omega, \\ u = 0 & \text{on } (0,\infty)\times\Gamma_0, \\ u_{tt} + \partial_\nu u - \Delta_\Gamma u + Q(x,u_t) = g(x,u) & \text{on } (0,\infty)\times\Gamma_1, \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x) & \text{in } \overline{\Omega}, \end{cases}$$

where Ω is a bounded open C^1 subset of $\mathbb{R}^N$ ($N \geq 2$), $\Gamma = \partial\Omega$, (Γ_0, Γ_1) is a measurable partition of Γ, Δ_Γ denotes the Laplace–Beltrami operator on Γ, ν is the outward normal to Ω, and the terms P and Q represent nonlinear damping terms, while f and g are nonlinear perturbations.