DISUGUAGLIANZE

$$x^2 \geq 0 \quad \forall x \in \mathbb{R}$$

$$(x-y)^2 \geq 0 \quad \forall x,y \in \mathbb{R}$$

$P(x) = x^2+2x+2 \qquad \mathrm{MIN}(P(x)) \qquad x \in \mathbb{R}$

$= x^2+2x+1+1 = (x+1)^2+1$

SE $x=-1 \to 0+1$

$P(x) = x^2+\underbrace{8x}_{2\cdot x\cdot 4}+90 = \underbrace{(x+4)^2}_{x^2+8x+16}+74 \qquad \mathrm{MIN}=74$

DISUGUAGLIANZE TRA MEDIE $(M=2)$

$AM = \frac{a+b}{2} \qquad GM = \sqrt{a\cdot b} \qquad HM = \frac{2}{\frac{1}{a}+\frac{1}{b}} \qquad QM = \sqrt{\frac{a^2+b^2}{2}}$

$a,b>0$

$\min(a,b) \leq HM \leq GM \leq AM \leq QM \leq \max(a,b)$

$(a-b)^2 \geq 0 \iff a^2-2ab+b^2 \geq 0 \qquad \cancel{2}ab \leq \frac{a^2+b^2}{2}$

$\sqrt{ab} \leq \sqrt{\frac{a^2+b^2}{2}}$ QUASI....

V

$\underbrace{(\sqrt{a}-\sqrt{b})^2 \geq 0} \iff \underbrace{a+b-2\sqrt{ab} \geq 0} \iff \underbrace{\sqrt{ab} \leq \frac{a+b}{2}}_{GM \leq AM}$ V

V

$\underbrace{\frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}}} \iff \frac{a^2+b^2+2ab}{4} \leq \frac{a^2+b^2}{2} \iff$

$\iff a^2+b^2+2ab \leq 2a^2+2b^2 \iff a^2-2ab+b^2 \geq 0$

$(a-b)^2 \geq 0$ V

V

$\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \sqrt{ab} \iff \frac{2ab}{a+b} \leq \sqrt{ab} \iff$

$\iff \frac{ab}{\sqrt{ab}} \leq \frac{a+b}{2} \iff \sqrt{ab} \leq \frac{a+b}{2} \quad GM \leq AM$

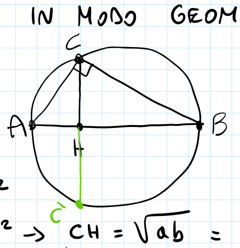
V

$GM \leq AM$ DIMOSTRATA IN MODO GEOMETRICO

$AH = a > 0$

$HB = b > 0$

$a+b = 2\cdot R = 2\cdot AM$



2° EUCLIDE: $AH \cdot HB = CH^2$

$a \cdot b = CH^2 \to \underline{CH = \sqrt{ab} = GM(a,b)}$

$CC' = \underline{2\cdot CH} \qquad CC' \leq AB$

$\cancel{2}\cdot GM \leq \cancel{2}\cdot AM$

$\mathrm{MIN}\left(45x + \frac{125}{x}\right) \qquad x>0$

$GM \leq AM$

$\sqrt{ab} \leq \frac{a+b}{2}$

$a = 45x \qquad b = \frac{125}{x}$

$\sqrt{45\cancel{x}\cdot\frac{125}{\cancel{x}}} \leq \frac{a+b}{2}$

$\frac{a+b}{2} \geq \sqrt{45\cdot 125}$

"MINIMO" è QUANDO c'è $=$

$$\frac{a+b}{2} \geq \sqrt{45\cdot 125}$$ MINIMO è QUANDO C'È =

$$\frac{a+b}{2} = \sqrt{3^2\cdot 5\cdot 5^3} = 3\cdot 25 = 75$$

$$a+b = 150$$

"=" SI VERIFICA QUANDO $a=b$

$$45X = \frac{125}{X} \qquad X^2 = \frac{25}{9}$$

$$X = \frac{5}{3}$$

$$45\cdot\frac{5}{3} + \frac{125}{\frac{5}{3}}$$

$$75 + 75 = 150$$

$$3a+5b+10c = 9 \qquad \text{MAX}\,(a\cdot b\cdot c) = ?$$

$$GM \leq AM \qquad \sqrt[3]{X\cdot Y\cdot Z} \leq \frac{X+Y+Z}{3}$$

$X = 3a$
$Y = 5b$
$Z = 10c$

$$\sqrt[3]{3a\cdot 5b\cdot 10c} \leq \frac{3a+5b+10c}{3} = 3$$

$$3a\cdot 5b\cdot 10c = 3^3$$

$$abc = \frac{3^{3}}{3\cdot 5\cdot 10} = \frac{9}{50}$$

$$X, Y > 0 \qquad X+Y = 1 \qquad \text{MAX}\,(X^2\cdot Y) = ?$$

$$GM \leq AM$$

$$m=2 \quad \sqrt{XY} \leq \frac{X+Y}{2} = \frac{1}{2}$$

NON CI FACCIO NULLA

$$m=3 \quad \sqrt[3]{X\cdot X\cdot Y} \leq \frac{X+X+Y}{3}$$

$$m=3 \quad a = \frac{X}{2} \quad b = \frac{X}{2} \quad c = Y$$

$$\sqrt[3]{\frac{X}{2}\cdot\frac{X}{2}\cdot Y} \leq \frac{\frac{X}{2}+\frac{X}{2}+Y}{3} = \frac{X+Y}{3} = \frac{1}{3}$$

$$\frac{X^2Y}{4} = \frac{1}{3^3} \rightarrow X^2Y = \frac{4}{27}$$

$$\frac{X}{2} = \frac{X}{2} = Y \rightarrow \begin{cases} X = 2Y \\ X+Y = 1 \end{cases} \qquad Y = \frac{1}{3} \quad X = \frac{2}{3} \qquad X^2\cdot Y = \frac{4}{9}\cdot\frac{1}{3} = \frac{4}{27}$$

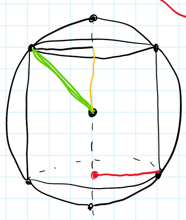
10. Cilindro circolare retto C inscritto nella sfera S con raggio 10. Trova C con max volume. Calcola $\sqrt{10800\cdot p}$, con p proba che un punto a caso di S appartenga a C.

$$P = \frac{V_{CIL}}{V_{SFE}}$$

r

$\frac{h}{2}$

$R = 10$



$$R^2 = r^2 + \left(\frac{h}{2}\right)^2$$

$$4r^2 + h^2 = 400$$

$$\pi\cdot \text{MAX}\,(r^2\cdot h) = ?$$

$$GM \leq QM$$

$$m=2 \quad \sqrt{a\cdot b} \leq \sqrt{\frac{a^2+b^2}{2}} \qquad a = r^2 \quad b = h$$

$$\sqrt{r^2 h} \leq \sqrt{\frac{r^4+h^2}{2}} \quad \text{No!}$$

$$m=3 \quad \sqrt[3]{a\cdot b\cdot c} \leq \sqrt{\frac{a^2+b^2+c^2}{3}} \qquad a = r \quad b = r \quad c = h$$

$$\sqrt[3]{r^2 h} \leq \sqrt{\frac{r^2+r^2+h^2}{3}}$$ QUASI...

$n=3 \quad \sqrt[3]{a\cdot b\cdot c} \le \sqrt{\frac{a^2+b^2+c^2}{3}}$ $\quad a=r \quad b=r \quad c=h$

$\sqrt[3]{r^2 h} \le \sqrt{\frac{r^2+r^2+h^2}{3}}$ QUASI....

$a=\sqrt{2}r \quad b=\sqrt{2}r \quad c=h$

$\sqrt[3]{\sqrt{2}r\cdot\sqrt{2}r\cdot h} \le \sqrt{\frac{2r^2+2r^2+h^2}{3}} = \sqrt{\frac{400}{3}} = \frac{20}{\sqrt{3}}$

$2r^2h = \frac{8000}{3\sqrt{3}} \to r^2h = \frac{4000}{3\sqrt{3}}$

$\sqrt{10800}\cdot p = 60\cancel{\sqrt{3}}\cdot \frac{\cancel{\pi}\cdot \frac{4000}{\cancel{3\sqrt{3}}}}{\frac{1}{3}\cancel{\pi}\, 4000} = 60$ ☺

DISUGUAGLIANZA DI RIARRANGIAMENTO

10€ 20€ 50€ 3 5 8

MIN = ? $3\cdot 50 + 5\cdot 20 + 8\cdot 10$

MAX = ? $3\cdot 10 + 5\cdot 20 + 8\cdot 50$

$x_1 \le x_2 \le \dots \le x_n$ e $y_1 \le y_2 \le \dots \le y_n$

MIN $= x_1 y_n + x_2 y_{n-1} + \dots + x_n y_1$

MAX $= x_1 y_1 + x_2 y_2 + \dots\ x_n y_n$

DIM CON $n=2$

$x_1 \le x_2 \qquad y_1 \le y_2$

VOGLIO DIM $\boxed{x_1y_1 + x_2y_2 \ge x_1y_2 + x_2y_1}$ ✓

$\iff x_1y_1 + x_2y_2 - x_1y_2 - x_2y_1 \ge 0$

$\iff x_2(y_2-y_1) - x_1(y_2-y_1) \ge 0$

$\iff \underbrace{(x_2-x_1)}_{\ge 0}\underbrace{(y_2-y_1)}_{\ge 0} \ge 0$ ✓

CAUCHY - SCHWARZ

$x_1, x_2, y_1, y_2 \in \mathbb{R}$

$n=2 \qquad (x_1^2+x_2^2)\cdot(y_1^2+y_2^2) \ge (x_1\cdot y_1 + x_2\cdot y_2)^2$

$V_1 = (x_1 ; x_2) \qquad |\vec{V_1}|^2 \ |\vec{V_2}|^2 \ge (\vec{V_1}\cdot\vec{V_2})^2$

PRODOTTO SCALARE

$V_2 = (y_1 ; y_2)$

$|\vec{V_1}|\cdot|\vec{V_2}| \ge \vec{V_1}\cdot\vec{V_2} = |\vec{V_1}|\cdot|\vec{V_2}|\cdot\cos(\alpha)$

$$(x_1^2 + \dots x_n^2)(y_1^2 + \dots + y_n^2) \ge (x_1\cdot y_1 + \dots + x_n\cdot y_n)^2$$

4) $x, y, z \in \mathbb{R}^+$ MIN $\underbrace{\left(\frac{1}{x} + \frac{1}{y} + \frac{4}{z}\right)}_{H}$ $\quad \underbrace{x+y+z = \frac{2}{25}}_{A}$

HM $\le$ AM $\qquad$ SÌ

$n=3 \quad \frac{3}{\frac{1}{x}+\frac{1}{y}+\frac{4}{z}} \le \frac{x+y+\frac{z}{4}}{3}$ $\quad$ NON MI DÀ NULLA

$a=x \quad b=y \quad c=\frac{z}{4}$

$\frac{1}{x} + \frac{1}{y} + \frac{2}{z} + \frac{2}{z} \qquad x+y+\frac{z}{2}+\frac{z}{2} = \frac{2}{25}$

$n=4 \quad a=x \quad b=y \quad c=\frac{z}{2} \quad d=\frac{z}{2}$

$\frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}} \le \frac{a+b+c+d}{4} =$

$$\frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}} \le \frac{a+b+c+d}{4} =$$

$$\frac{4}{\frac{1}{x}+\frac{1}{y}+\frac{2}{z}+\frac{2}{z}} \le \frac{x+y+\frac{z}{2}+\frac{z}{2}}{4} = \frac{\frac{2}{25}}{4} = \frac{2}{25}\cdot\frac{1}{4} = \frac{1}{50}$$

$$\frac{1}{x}+\frac{1}{y}+\frac{2}{z}+\frac{2}{z} \ge 4\cdot 50 = 200$$

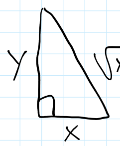
MIN $= 200$

8)

$$P(x,y) = \underbrace{25x^2}+\underbrace{16y^2}-\underbrace{40x}_{2\cdot 5x\cdot ?}-\underbrace{40y}_{2\cdot 4y\cdot ?}+\underbrace{50}$$

$$\underbrace{(5x-4)^2}_{\substack{MIN\,=\,0\\ x=\frac{4}{5}}}+\underbrace{(4y-5)^2}_{\substack{MIN\,=\,0\\ y=\frac{5}{4}}}+\underbrace{50-16-25}_{9}$$

10)



IPOTENUSA $= 1$

$\sqrt{x^2+y^2} = 1 \leftarrow$ QM

MAX (AREA)

$A = \frac{x\cdot y}{2}$ ← GM

$$GM \le QM$$

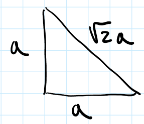
$$\sqrt{x\cdot y} \le \sqrt{\frac{x^2+y^2}{2}} = \sqrt{\frac{1}{2}}$$

$$xy \le \frac{1}{2} \qquad MAX: = \frac{1}{2}$$

$$A = \frac{xy}{2} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$$

10' IN MODO FURBO USANDO IL 10

L'IPOTENUSA MINIMA (A PERIMETRO FISSATO) C'È QUANDO IL TRIANGOLO RETTANGOLO È LA METÀ DI UN QUADRATO



$$a+a+\sqrt{2}a = 16$$
$$a(2+\sqrt{2}) = 16$$
$$a = \frac{16}{2+\sqrt{2}}\cdot\frac{(2-\sqrt{2})}{(2-\sqrt{2})} = 8(2-\sqrt{2})$$

$$\text{IPOT MIN} = 16-2a = 16-2\cdot 8\cdot(2-\sqrt{2})$$
$$= 16-32+16\sqrt{2} = 16\sqrt{2}-16$$

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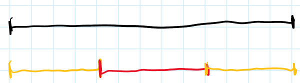
14)



13 m

$$4\cdot 5\cdot 4 = 80$$

$$3\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2 = 96$$

$$3\ \ 3\ \ 3\ \ 2\ \ 2 = \boxed{108}$$

15) $\forall\ a,b,c > 0$

$$\frac{a+b+c}{abc} \le \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

15) $\forall\, a,b,c > 0$

$$\frac{a+b+c}{abc} \le \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

RIARRANGIAMENTO

$$\frac{1}{bc}+\frac{1}{ac}+\frac{1}{ab} \le \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

$$\frac{1}{b}\cdot\frac{1}{c}+\frac{1}{a}\cdot\frac{1}{c}+\frac{1}{a}\cdot\frac{1}{b} \le \boxed{\frac{1}{a}\cdot\frac{1}{a}+\frac{1}{b}\cdot\frac{1}{b}+\frac{1}{c}\cdot\frac{1}{c}}$$

MAX

WLOG $a \le b \le c$ $\quad \frac{1}{a} \ge \frac{1}{b} \ge \frac{1}{c}$

PER RIARRANGIAMENTO

16) $x_1^2 + 2\cdot x_2^2 + 3\cdot x_3^2 + \dots + 49\cdot x_{49}^2 = 1$

$\text{MAX}\left(x_1 + 2\cdot x_2 + 3\cdot x_3 + \dots + 49\cdot x_{49}\right) = ?$

$a_1, \dots, a_n \quad b_1, \dots, b_n \in \mathbb{R}$

C-S: $(a_1^2 + a_2^2 + \dots + a_{49}^2)\cdot(b_1^2 + b_2^2 + \dots + b_{49}^2) \ge (a_1\cdot b_1 + \dots + a_{49}\cdot b_{49})^2$

FACCIO IN MODO CHE

$$\begin{cases} a_1\cdot b_1 = x_1 \to \\ a_2\cdot b_2 = 2x_2 \to \\ a_3\cdot b_3 = 3x_3 \\ \dots \\ a_{49}\cdot b_{49} = 49\cdot x_{49} \end{cases} \qquad \begin{cases} a_1^2 = x_1^2 \to a_1 = x_1 \\ a_2^2 = 2x_2^2 \to a_2 = \sqrt{2}\, x_2 \\ \dots \\ a_{49}^2 = 49\, x_{49}^2 \to a_{49} = \sqrt{49}\cdot x_{49} \end{cases}$$

$a_n = \sqrt{n}\cdot x_n$

$a_2\cdot b_2 = 2\cdot x_2$

$$b_2 = \frac{2\cdot x_2}{a_2} = \frac{2\cdot x_2}{\sqrt{2}\cdot x_2} = \sqrt{2}$$

$b_n = \sqrt{n}$

$a_2\cdot b_2 = \sqrt{2}\, x_2\cdot\sqrt{2}$

$a_n = \sqrt{n}\cdot x_n$

$b_n = \sqrt{n}$

$$\underbrace{\overbrace{(1\cdot x_1^2 + 2\cdot x_2^2 + 3\cdot x_3^2 + \dots 49\cdot x_{49}^2)}^{1}}_{1}\cdot\underbrace{(1+2+3+\dots+49)}_{\frac{49\cdot 50}{2} = 7^2\cdot 5^2} \ge \underbrace{(1x_1 + 2x_2 + \dots 49x_{49})^2}$$

$$\text{MAX} = \sqrt{1\cdot 7^2\cdot 5^2} = 7\cdot 5 = 35$$