Differential Equations in Mathematical Physics and Applications

The course will present the basic theory of partial differential equations having in mind several applications. The first part of the course will focus on scalar conservation laws in one space variable. These are a class of partial differential equations ubiquitous in Mathematical Physics, which model the conservative transport of physical quantities. In more detail, the following topics will be addressed:

(i) derivation of conservation laws;
(ii) the method of characteristics;
(iii) discontinuous solutions and weak formulation of a conservation law;
(iv) Riemann problems;
(v) the case of strictly convex/concave flux: (v.a) shock waves and the Rankine-Hugoniot condition, (v.b) rarefaction waves, non-uniqueness of the solution and entropy criteria;
(vi) extension to the case of fluxes with variable concavity.

The second part of the course will focus on reaction-diffusion equations in one-dimension and more, having in mind several applications, mainly biomedical. In more details the following topics will be addressed:

(i) derivation of reaction-diffusion equations;
(ii) link between microscopic dynamics and macroscopic equations;
(iii) fundamental solutions;
(iv) deduction of boundary and interface conditions;
(v) stability problems;
(vi) backward heat equation;
(vii) eikonal equation.

Exercise sessions will be devoted to the concrete solution of prototypical models and to a physical interpretation of their solutions.

Pre-requisites

A good knowledge of the theory of ordinary differential equations is welcome, but not strictly necessary.

Bibliography

- R. J. LeVeque, Numerical Methods for Conservation Laws, Birkhäuser, 1992 (We will refer, in particular, to the first part of the book, which is devoted to the mathematical theory of conservation laws)
- Some lecture notes will be made available before the course, others during the course.