Algebraic Geometry

Scuola Matematica Interuniversitaria (SMI), Perugia

July 22 – August 16, 2024

Instructor: Hirotachi Abo, University of Idaho

Prerequisite: A solid understanding of linear algebra and abstract algebra. More precisely, students are expected to be familiar with materials from Sections 7.1-13.6 of Abstract Algebra by David S. Dummit and Richard M. Foote.

Goals: At its most basic, algebraic geometry is the study of the common zero loci of systems of multivariate polynomials. Such systems arise very naturally throughout mathematics, theoretical physics, mathematical biology, statistics, computer science, and engineering. The purpose of this course is three-fold; namely, the first is to discuss topics in commutative algebra that are for use in algebraic geometry, the second is to introduce the basic theory of algebraic geometry, and the third is to explore computational aspects of algebraic geometry.

Textbook: This course will be based on the following textbook:

- *Introduction to Algebraic Geometry* by Brendan B. Hassett, Cambridge University Press, Cambridge, 2007, xii+252 pp.

Software: We use a computer algebra system called Macaulay2. You can install Macaulay2 on your computer. Alternatively, you can try out Macaulay2 in a web browser.

Topics: This course will cover the following topics:

- Preliminaries from commutative algebra, including Gröbner bases and primary decompositions
- Affine and projective varieties
- Morphisms and rational maps
- Intersections of varieties

Learning outcomes:

- The students will develop a grasp of the basic concepts in algebraic geometry and will be able to give examples and non-examples of such concepts.
- The students will understand fundamental theorems in algebraic geometry, such as Hilbert's Nullstellensatz and Bézout's Theorem.
- The students will understand the interplay between commutative algebra and algebraic geometry.
- The students will learn methods of manipulating varieties.
- The students will familiarize themselves with a computer algebra system.

Homework: Homework will be due approximately once a week.

Exams: There will be one exam at the end of the term. It will be in class and based on chapter material and homework problems. The exam is closed-book and closed-notes.