

FUNCTIONAL ANALYSIS

Course Content

July 17 - August 11, 2023 – SMI Summer School Perugia

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Abstract:

One of the aims of Functional Analysis is to develop general tools on linear and/or topological spaces to tackle practical applications in Analysis. This class will be devoted to a study of some selected topics in the field. In the last part, some applications to the variational formulation\index{Variational} and resolution of partial differential equations\index{Partial differential equation} will be given.

These notes are strongly influenced by the courses taught by the author at Université de Lorraine (France) and the books cited in the references.

Prerequisite: Topology (especially of metric spaces), Topology of linear normed spaces including projection on convex in Hilbert spaces, Differential Calculus, Integration. Some reminders will be given during the lectures.

References: Brezis « Functional Analysis», Rudin «Functional Analysis» and «Real and Complex Analysis»

Programme:

Part I: continuous functions

Chap 1: Picard's fixed point theorem

Chap 2: Stone-Weierstarss theorem and density

Chap 3: Relatively compact subsets and Ascoli's theorem

Part II: Baire spaces and convexity

Chap 5: Hahn-Banach's theorems and applications

Chapter 6: Intermission: ODEs with continuous coefficient via Zorn's lemma

Part III: continuous linear functions

Chap 7: The method of continuity

Chap 8: Hilbert spaces

Part IV: Sobolev spaces and applications to elliptic PDEs

Chap 9: Dimension 1 and main properties

Chap 10: Applications to 1D boundary problems

Part V: The Fourier transform

Chap 11: The L1-Fourier transform

Chap 12: The inversion theorem and applications top ODEs

Chap 13: The L2-Fourier transform

Part VI: a very short introduction to distributions.