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DIFFERENTIAL GEOMETRY

Course Content

July 17 - August 11, 2023 – SMI Summer School Perugia

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The course will be self-contained as much as possible. Only standard notions of multivariable calculus, group theory, and ordinary differential equations will be assumed.
Knowledge about smooth manifolds, fundamental group and coverings is useful but not mandatory. I will recall the notions needed for the course.

Suggested reading: any introduction to smooth manifolds is fine. A classical reference is
J.M. Lee, *Introduction to smooth manifolds.*
Another good reference is

B. Martelli, *An introductions to geoometric topology,*
 https://people.dm.unipi.it/martelli/Geometric_topology.pdf

In any case I will recall the main notions we need.

References
W. Ziller, Lie groups Representation Theory and Symmetric Spaces, lecture notes, https://www2.math.upenn.edu/~wziller/math650/LieGroupsReps.pdf

F. Paulin, Groupes et géométries, lecture notes, https://www.imo.universite-paris-saclay.fr/~frederic.paulin/notescours/cours_georiem.pdf

J.J.Duistermaat, J.A.C. Kolk, Lie groups, Universitext, Springer-Verlag Berlin Heidelberg 2000

J.F. Adams, Lecture on Lie groups, University of Chicago Press

Programme:
FUNDAMENTALS ON LIE GROUPS
- preliminaries on manifolds, smooth maps and vector fields;
- Definition of Lie groups and homomorphisms of Lie groups;
- Basic examples of classical groups;

- Exponential map;

- Lie subgroups;

- The Lie algebra of a Lie group.

- The adjoint representation;

- Lie's third Theorem;

STRUCTURE OF LIE GROUPS AND LIE ALGEBRAS

- Killing form

- Nilpotent and Solvable algebras

- Semisimple algebras

- Compact Lie groups

- Maximal tori and rank of a compact Lie group

- semi simple complex Lie groups

SYMMETRIC SPACES

- actions of Lie groups and quotients.

- preliminaries of Riemannian geometry.

- Symmetric spaces

- from a symmetric space to a Lie group

- from a Lie group to a symmetric space, Cartan involution

- symmetric space of non compact type.