

ALGEBRA

Course Content

July 17 - August 11, 2023 - Perugia, Italy

Instructor: Dr. Gene Abrams, University of Colorado, USA

In the first day or two we will review some of the basic ideas in (associative) ring theory. Then, the course will consist of investigation into these topics:

1. the structure of various algebras over fields (e.g. semigroup algebras, and specifically path algebras coming from directed graphs)
2. definition and example of modules over rings
3. properties of projective modules; the monoid $\mathcal{V}(R)$; the Grothendieck group $K_0(R)$
4. Invariant Basis Number, and module type
5. purely infinite simple rings
6. the definition of the Leavitt path algebra $L_K(E)$ of a directed graph E over a field K
7. properties of $L_K(E)$.

If there is time in the final week of the course we will consider Morita equivalence of rings, and show how that idea related to Leavitt path algebras.

It will be assumed that the student has seen a first year course in basic abstract algebra, and that the student is familiar with the notions of *group*, *ring*, and *field*. We will not assume that the student is familiar with the notions of *algebra over a field*, nor of *module*, nor of *directed graph*.

There is no official textbook for the course. However, there will be two documents that will form the foundation of the ideas:

1) “Lectures on Non-Commutative Rings” (notes from a course given by Professor Frank W. Anderson, University of Oregon, 2002)

available online: <https://pages.uoregon.edu/anderson/rings/COMPLETENOTES.PDF>

2) “Leavitt path algebras: the first decade” by G. Abrams, Bulletin of Mathematical Sciences 5(1), 2015, pp. 59-120.

available online: <https://academics.uccs.edu/gabrams/> (scroll down to “Publications”)