Numerical Analysis: basics on spline functions

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Presentation

Spline functions are ubiquitous, although sometimes hidden, in numerical methods. They are probably the most popular approximation tool in several contexts. Besides their theoretical interest, they have application in several branches of the sciences including geometric modeling, signal processing, data analysis, visualization, and numerical treatment of PDEs, just to mention a few. The success of splines mainly relies in their special and renowned basis: the B-spline basis which possesses several properties that are important from both theoretical and computational point of view. The course aims to present an introduction to splines and to the B-spline basis. Some applications in approximation and modeling will be also discussed time permitting.

Program

Basic concepts of polynomial interpolation

1. Lagrange and Hermite polynomial interpolation (1H)
2. Error estimates and Lebesgue constants for Lagrange polynomial interpolation (1H)
3. Interpolation at Tchebycheff points (1H)

Bernstein Polynomials

1. Definition and main properties (4H)
2. Uniform polynomial approximation in C[a,b]: the Weierstrass theorem (1H)
3. Bernstein polynomials in geometric modeling: Bézier curves (2H)

B-splines

1. Polynomial splines (1H)
2. B-splines: definition and main properties (6H)
3. B-splines as a basis of polynomial splines (1H)
4. B-splines in geometric modeling (2H)

Applications (time permitting)

1. Interpolation in spline spaces (1 H)
2. Continuous and discrete least square approximation in spline spaces (1H)
3. B-splines and FEM (2 H)

Text books

T. Lyche, C. Manni, and H. Speleers. *Foundations of spline theory: B-splines, spline approximation, and hierarchical refinement*. In: T. Lyche, C. Manni, and H. Speleers (eds.) Splines and PDEs: From Approximation Theory to Numerical Linear Algebra, Lecture Notes in Mathematics 2219, Springer International Publishing, pp. 1–76, 2018**.**

C. Manni, H. Speleers*. Standard and non-standard CAGD tools for isogeometric analysis: A tutorial*. In: A. Buffa and G. Sangalli (eds.) IsoGeometric Analysis: A New Paradigm in the Numerical Approximation of PDEs, Lecture Notes in Mathematics 2161, Springer International Publishing, pp. 1–69, 2016.

Other readings

C. de Boor: *A Practical Guide to Splines*, Springer, 2001.

Prerequisites

Contents of the basic courses of Analysis, Geometry and Algebra of the undergraduate degree. Knowledge of basic notions of Numerical Analysis.