FUNCTIONAL ANALYSIS

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Description. Functional Analysis treats functions and other objects as points of suitable spaces, and operations on such objects as maps between the corresponding spaces. At the heart of the field lies the linear theory, where spaces are topological vector spaces and maps are linear. A basic feature is that most vector spaces of interest are infinite dimensional, and new difficulties and phenomena arise. The interplay between the abstract theory and concrete instances is twofold: abstract results are applied to specific problems; special and concrete objects "represent" abstract classes of them.

The class will move from the basic definitions to some version of the spectral theorem. Below there is a somehow generic of topics, more or less corresponding to chapters I-IV, VI-VII of Reed and Simon's book. The content of the topics will also depend on the audience, its expectations, and background. Parts of the theory and of the applications will be developed in the exercise sessions.

- A brief review of Measure Theory and Complex Analysis
- Hilbert spaces and Reproducing Kernels
- Banach spaces
- Bounded linear operators and their topologies
- Compact operators
- Spectral theory

Prerequisites:

Topology (especially of metric spaces), Basic Measure Theory, Differential Calculus, Basic Complex Analysis (power series, Cauchy Theorem and Cauchy formula). Some reminding will be given during the lectures.

References:

Michael Reed, Barry Simon, Functional Analysis, Academic Press, 1981 (main reference)

Peter D. Lax, Functional Analysis, John Wiley & Sons, 2002

Walter Rudin, Real and Complex Analysis, McGraw Hill, 2001