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Differential Geometry Lecture

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Differential geometry is ubiqitous in mathematics. Our aim in this lecture is to give an introduction to it with special emphasis on topological applications. Once the basic concepts like smooth manifolds and exterior forms are introduced, we use them to define cohomological invariants which eventually allow us to distinguish objects in the smooth category and prove several classical theorems. The main subjects of the lecture are listed below. They will be completed by exercice sessions where complementary topics will be discussed.

- Tensor algebra, exterior algebra
- Differential forms on R^n, exterior differential
- De Rham cohomology
- Smooth manifolds
- Tangent bundle, vector fields, differential forms
- Classical theorems (sphere theorem, Brower's theorem)
- Orientation, integration, Stokes' formula
- Poincaré duality, applications

Prerequisites:

General topology, Differential Calculus (inverse function theorem, integration in R^n)

References:

John Lee. *Introduction to Smooth Manifolds*. Springer-Verlag. ISBN 978-1-4419-9981-8. (2013).
Available online at the address:
https://math.berkeley.edu/~jchaidez/materials/reu/lee_smooth_manifolds.pdf