Differential Equations in Mathematical Physics

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Objective (goals, type of students for whom useful, outcome etc): We will model various physical problems to obtain some well-known differential equations. We will focuses on the rigorous study of three primary partial differential equations(PDEs) arising from nature: Laplace's equation, heat equation, and wave equation, representing second order elliptic, parabolic, and hyperbolic PDEs, respectively. Each of these leads to its own branch of mathematics that one could study for a lifetime. As well as being an engaging mathematical topic in its own right, this theory is essential for many topics in analysis, geometry, probability theory, mathematical physics, engineering maths etc. Many of the differential equations that we study, especially nonlinear pdes do not in general possess a smooth solution, therefore it is essential to devise some sort of proper notion of generalized or weak solution and therefore it is important to learn Sobolev space which in some sense has a proper setting to study many linear and nonlinear pde via energy methods. The last part is introduction to calculus of variation, where we will set forth a careful derivation of direct method for deducing the existence of minimizers, and also discuss a variety of variational systems and constarined problems.

Syllabus:

1. From physical models to differential equations

2. Brief discussion on transport equations

2. Laplace Equations (properties of harmonic functions, fundamental solution, solving bvp using Green function)

3. Heat Equations (fundamental solution, solving Cauchy problem, various properties of heat equations)

4. Wave equations in 1D, 2D, 3D, uniqueness, domain of dependence for wave equation,

5. Sobolev spaces,

6. Brief introduction to Calculus of variation

Prerequisites: Real Analysis, basic measure theory and integration, ordinary differential equations (optional)

References: L.C. Evans, Partial Differential Equations, AMS, 1998

Y. Pinchover and J. Rubinstein, An Introduction to Partial Differential Equations, Cambridge, 2005.

Fritz John: Partial Differential Equations 4th edition, Springer, 1981.

H. Brezis, Functional Analysis, Sobolev spaces and partial differential equations, Springer, 2011.