COMPLEX ANALYSIS
SMI – PERUGIA 2022
LOREDANA LANZANI, Universita' di Bologna & Syracuse University

Description. Complex Analysis is foundational to any Mathematics PhD program. Complex analysis lies at the crossroad of analysis and geometry: as such, it contains the ``germs'' of many among the fundamental features of its higher dimensional counterparts (e.g., several complex variables; pluripotential theory; harmonic analysis; PDE; complex geometry) yet it is unburdened by the technicalities of the higher dimensional world, which is to say: complex analysis is as close to ``mathemagics'' as it ever can be!

In this course we will go over the topics typically covered in the PhD programs at most US-based institutions, which roughly correspond to Ch. 1 through 11 in the main reference (Gamelin, below). To the extent possible, content will be adjusted to the students' background and mathematical taste. Some of the theory may be developed in the problem sessions.

- The complex plane & elementary functions
- Analytic functions
- Complex line integrals & Harmonic functions
- Complex integration: Cauchy Thm; Cauchy Formula, etc.
- Power series of analytic functions

- Laurent decomposition & isolated singularities
- Residue calculus
- Logarithmic integral: Rouche' thm
- Schwarz Lemma
- Harmonic Functions & the reflection principle
- Conformal mapping & the Riemann mapping Thm

Prerequisites. A good knowledge of calculus, in particular: parametrized curves; tangent vectors; arc-length; gradient; line integrals; Green's theorem.

References:

- T. D. Gamelin, *Complex Analysis,* Springer (Main Reference)
- S. Ponnusamy & H. Silverman, *Complex Variables with Applications*, Birkhauser