

Algebraic Geometry - Syllabus

The purpose of this course is to motivate algebraic geometry via classical linear and multi-linear algebra.

Open questions in elementary linear algebra give rise to deep problems in algebraic geometry, such as: How to determine if a space of $n \times n$ matrices is simultaneously diagonalizable? This turns out to be equivalent to problems regarding (a special case of) the Quot scheme defined by Grothendieck.

Week 1: Classical linear algebra revisited with a geometric perspective: fundamental theorem, determinants, Jordan normal form.

Weeks 2 and 3: Basic definitions and first examples from algebraic geometry: e.g., varieties, Zariski topology, regular and rational maps, incidence correspondences, Grassmannians, determinantal varieties, secant varieties.

Week 4: Examples of elementary questions in linear algebra that lead to deep and open questions in algebraic geometry.

Notes will be handed out for the material in weeks 1 and 4. Weeks 2 and 3 will cover chapters 4 and 5 from "Tensors, Geometry and Applications" and selected topics from chapters 1 and 2 of "Basic Algebraic Geometry I"

Textbooks:

Landsberg, J. M. "Tensors: geometry and applications". Graduate Studies in Mathematics, 128. American Mathematical Society, Providence, RI, 2012. xx+439 pp. ISBN: 978-0-8218-6907-9

Shafarevich, Igor R. "Basic algebraic geometry. 1. Varieties in projective space". Third edition. Translated from the 2007 third Russian edition. Springer, Heidelberg, 2013. xviii+310 pp. ISBN: 978-3-642-37955-0; 978-3-642-37956-7

Prerequisites: A solid background in linear algebra and basic notions from algebra (groups, rings, algebras)