

SYLLABUS FOR ALGEBRA

SACHIN GAUTAM

OVERVIEW

The aim of this course is to provide an introduction to algebraic techniques featuring prominently in the representation theory of groups and Lie algebras. We will assume some familiarity with basic linear algebra. The list of topics given below is somewhat tentative, and could be modified depending on how the course is going.

1. TOPICS AND REFERENCES

1.1. Week 1. Group theory. Definitions: groups, subgroups, normal subgroups, group homomorphisms. Isomorphism theorem. Examples. Group actions on sets - counting results. Groups given by generators and relations. Semidirect products. Sylow theorems. Solvable, nilpotent and simple groups. Structure theorem for finite abelian groups.

References.

1. N. Bourbaki, *Algebra I*, Springer-Verlag.
2. S. Gautam, *Notes from Math 6111, Fall 2017* - available at <http://people.math.osu.edu/gautam.42/F17/notes.html>.
3. S. Lang, *Algebra*, Graduate Texts in Mathematics, Springer.

1.2. Week 2. Representations of finite groups. Review of linear algebraic constructions. Definitions: group representations, intertwiners, direct sum and tensor product of representations. Maschke's theorem. Schur's lemma. Induced representations and Frobenius reciprocity. Group algebra and the fundamental theorem. Characters and orthonormality.

References.

1. P. Etingof et al. *Introduction to representation theory*, AMS.
2. J. P. Serre, *Linear representations of finite groups*, Graduate Texts in Mathematics, Springer.

1.3. Week 3. Lie algebras. Definitions: Lie algebras, ideals, representations. Examples. Jordan decomposition. Solvable, nilpotent and simple Lie algebras. Killing

form and Cartan's criterion. Enveloping algebra of a Lie algebra and Poincaré–Birkhoff–Witt theorem. Representation theory of \mathfrak{sl}_2 . Simple Lie algebras - structure theory: Cartan subalgebras, root systems and Weyl group.

1.4. Week 4. Representation theory of Lie algebras. Representations of simple Lie algebras. Semisimplicity. Highest weight theory. Classification of irreducible, finite-dimensional representations. Example of \mathfrak{sl}_n . Weyl character formula.

References for Weeks 3, 4.

1. N. Bourbaki, *Lie groups and Lie algebras: Chapter 1-3; 4-6*, Springer.
2. J. Humphreys, *Introduction to Lie algebras and representation theory*, Graduate Texts in Mathematics, Springer.
3. J. P. Serre, *Complex, semisimple Lie algebras*, Springer.