Marco Radeschi: Singular Riemannian foliations, structure and applications

Lectures: 9-11am
Problem Sessions: 4-6pm

Abstract:

Singular Riemannian foliations are rigid structures on Riemannian manifolds that generalize isometric group actions, Riemannian submersions, isoparametric families in space forms, etc. In this course we will study the local structure of singular Riemannian foliations. The end goal will be to prove the Algebraicity Theorem which states that these foliations are locally cut by polynomials, in appropriate charts. If time permits, we will see some application of this theory, to Invariant theory.

Topics:

1) Local structure around a point: distinguished tubular neighbourhoods, local equidistance, infinitesimal foliation.
2) Local structure around a leaf: linearized maps, Slice theorem, foliated holonomy, stratification
3) Structure around a horizontal geodesic: holonomy and projectable Jacobi fields, equifocality.
4) Infinitesimal foliations: basic mean curvature, Algebraicity Theorem.
5) Invariant theory without groups.

Literature:

- P. Molino, Riemannian Foliations
- Personal Lecture Notes, available at https://www.marcoradeschi.com/

Prerequisites:

1) Basics of differential topology: Manifolds, tangent space, vector fields, flows (e.g. Do Carmo "Riemannian Geometry", Chapter 0)
2) Basics of Riemannian curvature: Riemannian metrics, geodesics, curvature, Jacobi fields, e.g. DoCarmo "Riemannian Geometry", Chapters 1-5.
3) Basics of submanifold geometry: normal bundle, tubular neighbourhood theorem, principal curvature, mean curvature vector field.

Wolfgang Ziller: Geometry of isometric group actions on Riemannian manifolds

Lectures: 2pm-4pm
Problem Sessions: 4pm-6pm

Abstract:
Lie group actions occur naturally in many different subjects in mathematics and in physics.
We will concentrate on proper Lie group actions. In this case there exists a Riemannian metric for which the Lie group acts by isometries. We will then use some elementary Riemannian geometry to give simpler proofs of the main properties of such actions with a detailed study of the geometry of the quotient.
We will finally discuss applications to various subjects. Students will apply this theory to a wealth of examples in order to understand the intricate structure and detailed properties of such actions and its orbit stratification..

Topics:

1) Slice theorem, stratification into orbit types and (Riemannian) geometry of the quotient.
2) Polar actions, symplectic reductions and cohomogeneity one actions.
3) Non-negative curvature, Einstein metrics and solitons.

Literature:

- Bredon, Introduction to compact transformation groups.
- Alexandrino - Bettiol, Lie groups and geometric aspects of isometric actions.

Prerequisites:

1) Basics of Riemannian curvature: Geometry of geodesics and curvature. e.g. DoCarmo, Riemannian geometry, Chapters 1-4.
2) Basics of Lie groups, Lie algebras, exponential map, and adjoint representation. A quick review is possible if desired) e.g. Ziller, Lie Groups, Representation theory and symmetric spaces, p. 1-17 is sufficient. (https://www2.math.upenn.edu/~wziller/math650/LieGroupsReps.pdf)
3) 3) Elementary representation theory.(A quick review is possible if desired). E.g. the above notes p. 90-96 is mostly sufficient.