DISCRETE POTENTIAL THEORY AND SOME APPLICATIONS TO ANALYSIS IN THE CONTINUUM

The two courses of the school aim at providing an in-depth introduction to potential theory, linear and nonlinear, in the discrete context of trees, and to highlight some of its applications to analysis in the continuum. No previous exposure to potential theory is assumed. The prerequisites are real analysis, basic functional analysis and, for some topics, some exprosure to probability theory (martingales) and complex analysis. The courses will move fast from the basic theory to areas of current research, and a number of open problems and questions will be discussed.

The two courses are interlaced, and especially in the first few days, they will recall notation and results from each other. The course of N. Arcozzi will be more focused on trees, while the course of P. Mozolyako will proceed to explore the recently developed area of n-trees.

Nicola Arcozzi

i. The origins: Gauss' theory of electrostatics
ii. Axiomatic nonlinear potential theory
iii. Hausdorff measures; Ahlfors regular spaces
iv. Strong capacitary inequality according to Maz'ya and according to Adams
v. Maximal functions and Marcinkievic interpolation
vi. A crash course on reproducing kernel Hilbert spaces
vii. Applications: Carleson measures, multipliers, and boundary values for the holomorphic Dirichlet space
viii. Quotient structures, product structures, conformal invariance
ix. The recurrence relation for capacity on the tree
x. The characterization of equilibrium measures
xi. Applications: tiling rectangles by squares
xii. Applications to Diophantine approximation
xiii. Probabilistic interpretation and related topics

Pavel Mozolyako

i. Important examples: Riesz-Bessel kernels and dyadic potentials
ii. Sobolev spaces vs. potential spaces
iii. The tree and its boundary as a metric space
iv. Frostman lemma
v. Hardy (trace) inequality on trees: characterizations involving capacity and mass/energy (one and multiple boxes)
vi. The Muckenhoupt-Wheeden-Wolff inequality
vii. Applications: trace inequalities for Sobolev spaces
viii. The bitree: the two-weight inequality problem for the Hardy operator
ix. The bitree: Carleson's counterexample for the Hardy space (Tao's version)
x. The bitree and the failure of the maximum principle
xi. The Hardy inequality for the bitree: capacitary characterizations
xii. The Hardy inequality for the bitree; mass/energy
xiii. Open problems on the bitree

xiv. Applications: Carleson measures for the Dirichlet space on the bidisc

xv. Probabilistic interpretation and related topics

Bibliography:

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b) Nicola Arcozzi, Pavel Mozolyako, Karl-Mikael Perfekt, Giulia Sarfatti, Bi-parameter Potential theory and Carleson measures for the Dirichlet space on the bidisc, arXiv:1811.04990

c) N Arcozzi, R Rochberg, ET Sawyer, BD Wick, Potential theory on trees, graphs and Ahlfors-regular metric spaces, Potential Analysis 41 (2), 317-366

d) Aronszajn, Nachman "Theory of Reproducing Kernels". *Transactions of the American Mathematical Society*. (1950) **68** (3): 337–404

e) Kenneth Falconer, The *Geometry of Fractal Sets*, Cambridge University Press, 1985, 184 pp.

f) Russel Lyons, Yuval Peres, Probability on Trees and Networks, `Cambridge University Press, New York`, 2016, `xv+699`, `Available at \url{https://rdlyons.pages.iu.edu/`

g) Peter Moerters, Yuval Peres, Brownian Motion, Cambridge, 2010, xii+403 pp.

h) Pavel Mozolyako, Georgios Psaromiligkos, Alexander Volberg, Pavel Zorin-Kranich, Carleson embedding on tri-tree and on tri-disc, arXiv:2001.02373

i) P.M. Soardi, Potential Theory on Infinite Networks, LNM 1590, Springer, 1994

j) Terence Tao, Hausdorff dimension, https://terrytao.wordpress.com/2009/05/19/245c-notes-5-hausdorff-dimension-optional/

k) Brett Wick, Lectures on Multiparameter Harmonic Analysis, Internet Analysis Seminar, http://internetanalysisseminar.gatech.edu/lectures_mpha.html