

Title: Solvable Lie algebras in the null cone of $O(p, q)$ -actions

Abstract: Given a finite-dimensional, real vector space V equipped with a scalar product of signature (p, q) , we consider an action of the Lie group $O(p, q)$ on the space $V \otimes \wedge^2 V^*$ that preserves the subset $\mathcal{B} \subset V \otimes \wedge^2 V^*$ of Lie brackets, and such that all Lie brackets in the same orbit define Lie algebras on V that are isomorphic via an isometry of V . The null cone of this action is the set of Lie brackets in $\mathcal{B}$ whose orbit closure contains 0. If a Lie bracket μ is in the null cone, any Lie group having (V, μ) as its Lie algebra, equipped with the left-invariant metric induced from the scalar product on V , has vanishing polynomial scalar curvature invariants (it is VSI). Using a theorem of Richardson and Slodowy from real geometric invariant theory, we establish an equivalent condition for a Lie bracket to lie in the null cone. We say that a real, finite-dimensional Lie algebra is in the null cone of the $O(p, q)$ -action if it is isomorphic to (V, μ) for some μ in the null cone of this action. Hervik showed that up to dimension six, the only family of solvable Lie algebras not in the null cone of any $O(p, q)$ -action is $\mathfrak{s}_{3,3}$. We use the established condition to show that this statement holds for all finite dimensions.

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