THE CATEGORY OF AUTOMATA

The following definition, restricted to objects, is [Sipser, Definition 1.5].

Definition 0.1. An *automaton* consists of a 5-tuple $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ where Q and Σ are sets, $q_0 \in Q$ is an element of Q and $F \subset Q$ is a subset of Q, and where $\delta \colon \Sigma \times Q \to Q$ is a function. The elements of Q are called *states* and the elements of Σ are called *letters*.

Suppose that $\mathcal{A}' = (Q', \Sigma', \delta', q_0', F')$ is another automaton. A *morphism of automata*, denoted $(f, g) \colon \mathcal{A} \to \mathcal{A}'$, consists of a function $f \colon Q \to Q'$ and a function $g \colon \Sigma \to \Sigma'$ such that $f(q_0) = q_0'$, $f(F) \subset F'$ and the diagram

$$\begin{array}{ccc} Q \times \Sigma & \xrightarrow{\delta} & Q \\ \downarrow{\scriptstyle f \times g} & & \downarrow{\scriptstyle f} \\ Q' \times \Sigma' & \xrightarrow[\delta']{} & Q' \end{array}$$

commutes. We denote the category of automata by **Automata**.

Definition 0.2. Let $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ be an automaton. We call the monoid Σ^* of sequences in Σ the *action monoid* for $\mathcal{A}$, and note that δ gives Q the structure of a right Σ^*-set. For $a \in \Sigma^*$ and $q \in Q$, we write $q \cdot a$ to denote the result of acting a on q.

The *language* of $\mathcal{A}$, denoted $L(\mathcal{A})$ is the subset of elements $a \in \Sigma^*$ such that $q_0 \cdot a \subset F$. Note that a morphism $(f, g) \colon \mathcal{A} \to \mathcal{A}'$ induces a morphism of sets $L(\mathcal{A}) \to L(\mathcal{A}')$, so that $L \colon$ **Automata** $\to$ **Sets** is a functor.

I'm not sure what the point of the following definition is, but it is natural to define.

Definition 0.3. Let $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ be an automaton. An element $a \in \Sigma^*$ is said to *preserve acceptance* if, for all $q \in F$ one has $q \cdot a \in F$. Note that the set of letters that preserve acceptance is a submonoid of Σ^*.

References

[Sipser] Sipser, M. *Introduction to the Theory of Computation* Second edition.