Versatile weighting strategies for a citation-based research evaluation model

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joint work with Dario A. Binii

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1. The Problem
2. Common metrics
3. The model
4. Weighting strategies
The Problem

- Number of scientific journals and papers is increasing at an almost exponential rate
- What to read? What to cite? Which journals subscribe? How to evaluate research?
- This burden affects researchers, funding agencies, university administrators, reviewers

Difficult to give an in-depth evaluation of the research
Use indirect indicators of quality
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## The Problem

Most of “automatic” methods rely on citation analysis

The evaluation of research using citation analysis has weaknesses...

- Is a citation always a trusting vote?
- Data source and coverage
- How do authors choose the papers to cite?

... but also some pros

- Peer review is not always practicable
- There are plausible assumptions underlying the use of citation analysis as a heuristic
- Simple and objective

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weighting strategies for a citation-based model
Common metrics:

Different metrics for different purposes

- Ranking journals - Libraries, scholars for deciding where to publish, ...
- Ranking papers - What to read, what to cite, ...
- Ranking authors - distribution of grants, hiring people, ...
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## Ranking of Journals

**Citation Statistics:** Impact Factor, AMS MR, Citeseer,...

PageRank-like techniques: Eigenfactor, SCImago, RedJasper, ...
Ranking of Journals

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Ranking of Papers

- Relevance of the journal where the paper is published
  - Not all the papers in a journal have the same quality
- Number of citations received
  - Citation-gathering can be a very slow process.
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Ranking of Authors

- Top author if she publishes in “top” journals
- More accurate measures $h$-index, $m$-index, $g$-index, $g_1$-index.
- Count the number of distinct authors who are citing me.
Ranking of Authors

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Our Proposal

In the classical approach the ranking of journals is based on citations.

The ranking of papers and authors follows from the rank of the journals where the research is published.

We proposed an integrated ranking of authors, journals, papers, areas, and institutions.

Mutual reinforcement between papers, journals, authors.

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[ETNA 08, JCAM 09]

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Mutual reinforcement between papers, journals, authors
We have seen there are different aims of ranking

- Research evaluation by funding agencies
- Hiring in University or in an Industrial context
- Choosing individuals for a research team
- Many others...

We try to design a tunable method to capture the different needs.
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We try to design a **tunable** method to capture the different needs
General principles

- A paper is important if published in an important journal but also if cited by important papers and authored by important authors.
- An author is important if she has important co-authors and has written important papers published in important journals.
- A journal is important if collects citations from important journals, publishes important papers by important authors.

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Each paper can be described by means of

- List of authors
- Name of the journal
- List of references

Let \( n_P \) be the number of papers, \( n_A \) the number of authors and \( n_J \) the number of journals.
The three class model

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The three class model

\[ C \] is a \( n_p \times n_p \) binary matrix describing the citation process

\[ C(p_i, p_j) = 1 \text{ iff } p_i \text{ cites } p_j \]

\[ K \] is a \( n_A \times n_p \) binary matrix describing authorship

\[ K(a, p) = 1 \text{ if author } a \text{ has written paper } p \]

\[ F \] binary matrix \( n_j \times n_p \)

\[ F(j, p) = 1 \text{ if journal } j \text{ publishes paper } p \]
The system is represented by the matrix

\[ S = \begin{bmatrix}
  \mathbf{FCF}^T & \mathbf{FK}^T & \mathbf{F} \\
  \mathbf{KF}^T & \mathbf{KK}^T & \mathbf{K} \\
  \mathbf{F}^T & \mathbf{K}^T & \mathbf{C}
\end{bmatrix} \]
The system is represented by the matrix

\[
S = \begin{bmatrix}
F C F^T & F K^T & F \\
K F^T & K K^T & K \\
F^T & K^T & C
\end{bmatrix}
\]

The matrices in the first column contribute to the ranking of journals.
The system is represented by the matrix

\[
S = \begin{bmatrix}
FCF^T & FK^T & F \\
KF^T & KK^T & K \\
F^T & K^T & C
\end{bmatrix}
\]

The matrices in the second column contribute to the ranking of authors.
The system is represented by the matrix

\[ S = \begin{bmatrix}
FCF^T & FK^T & F \\
KF^T & KK^T & K \\
F^T & K^T & C
\end{bmatrix} \]

The matrices in the third column contribute to the ranking of papers.
The three class model

Let $P$ be a stochastic matrix “obtained” from $S$.

Working with a stochastic matrix guarantees that the amount of importance in the system is neither created nor destroyed.

Let

$$\pi = [\pi_J, \pi_A, \pi_P]$$

be the Perron vector of $P$, that is

$$\pi^T = \pi^T P,$$

The rank value of each subject is the value of the corresponding entry in $\pi$. 

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weighting strategies for a citation-based model
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weighting strategies for a citation-based model
The three class model

Existence and uniqueness of the Perron vector is forced into $P$.

We make the nine blocks row-stochastic.

We combine them into a larger stochastic matrix adding weights.
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The three class model

\[ P = \begin{bmatrix} \gamma_{1,1} A_{1,1} & \gamma_{1,2} A_{1,2} & \gamma_{1,3} A_{1,3} \\ \gamma_{2,1} A_{2,1} & \gamma_{2,2} A_{2,2} & \gamma_{2,3} A_{2,3} \\ \gamma_{3,1} A_{3,1} & \gamma_{3,2} A_{3,2} & \gamma_{3,3} A_{3,3} \end{bmatrix} \]

\[
\pi_J = \gamma_{1,1} \pi_J A_{1,1} + \gamma_{2,1} \pi_A A_{2,1} + \gamma_{3,1} \pi_P A_{3,1} \\
\pi_A = \gamma_{1,2} \pi_J A_{1,2} + \gamma_{2,2} \pi_A A_{2,2} + \gamma_{3,2} \pi_P A_{3,2} \\
\pi_P = \gamma_{1,3} \pi_J A_{1,3} + \gamma_{2,3} \pi_A A_{2,3} + \gamma_{3,3} \pi_P A_{3,3}
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\[ \pi_J = \gamma_{1,1}\pi_J A_{1,1} + \gamma_{2,1}\pi_J A_{2,1} + \gamma_{3,1}\pi_P A_{3,1} \]

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\[
\pi_A = \gamma_{1,2} \pi_J A_{1,2} + \gamma_{2,2} \pi_A A_{2,2} + \gamma_{3,2} \pi_P A_{3,2}
\]

\[
\pi_P = \gamma_{1,3} \pi_J A_{1,3} + \gamma_{2,3} \pi_A A_{2,3} + \gamma_{3,3} \pi_P A_{3,3}
\]
Weighting strategies

The weights $\gamma_{ij}$ can be used to tune how much of their importance each player transfers to the subject.

Let

$$\mu_J = \sum_{i=1}^{n_J} \pi_i, \quad \mu_A = \sum_{i=n_J+1}^{n_J+n_A} \pi_i, \quad \mu_P = \sum_{i=n_J+n_A+1}^{n_J+n_A+n_P} \pi_i,$$

be the “energy” of each class.
From the Coupling Theorem $\mu = [\mu_J, \mu_A, \mu_P]$, is such that

$$\mu^T = \mu^T \Gamma.$$ 

How to choose $\Gamma$?

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weighting strategies for a citation-based model
Weighting strategies

From the Coupling Theorem $\mu = [\mu_J, \mu_A, \mu_P]$, is such that

$$\mu^T = \mu^T \Gamma.$$

How to choose $\Gamma$?
Uniform weights

Uniform weights, i.e.

$$\Gamma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

we get $$\mu = 1/3 \, [1, 1, 1].$$
Uniform weights

If $\mu = 1/3 [1, 1, 1]$, the mean value of a generic subject in a class is

$$\mathcal{M}_C = 1/3 \frac{1}{n_C}, \quad C \in J, A, P$$

In practical situation $n_J << n_A << n_P$, hence

$$\mathcal{M}_J >> \mathcal{M}_A >> \mathcal{M}_P.$$

Journals are much more important than papers and authors!
Uniform weights

If \( \mu = 1/3 \) \([1, 1, 1]\), the mean value of a generic subject in a class is

\[
M_C = \frac{1}{n_C}, \quad C \in J, A, P
\]

In practical situation \( n_J \ll n_A \ll n_P \), hence

\[
M_J \gg M_A \gg M_P.
\]

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Uniform weights

If $\mu = 1/3 [1, 1, 1]$, the mean value of a generic subject in a class is

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In practical situation $n_J << n_A << n_P$, hence

$$M_J >> M_A >> M_P.$$ 

Journals are much more important than papers and authors!
Balancing weights

We want to find a possible \( \Gamma \) such that

\[
M_J = M_A = M_P.
\]

It is easy to prove that

\[
\Gamma = \frac{1}{N} \begin{bmatrix}
  n_J & n_A & n_P \\
  n_J & n_A & n_P \\
  n_J & n_A & n_P
\end{bmatrix}
\]

is such that

\[
\mu = 1/N [n_J, n_A, n_P],
\]

and hence \( M_J = M_A = M_P \).
Balancing weights

We want to find a possible $\Gamma$ such that

$$\mathcal{M}_J = \mathcal{M}_A = \mathcal{M}_P.$$ 

It is easy to prove that

$$\Gamma = \frac{1}{N} \begin{bmatrix} n_J & n_A & n_P \\ n_J & n_A & n_P \\ n_J & n_A & n_P \end{bmatrix}$$

is such that

$$\mu = 1/N [n_J, n_A, n_P],$$

and hence $\mathcal{M}_J = \mathcal{M}_A = \mathcal{M}_P.$
Note that although it is possible to know in advance the average value of a particular class by looking at $\Gamma$, we cannot predict or influence the outcome of the algorithm.

The rank value of each subject is obtained combining too many ingredients!