Trace Complexity

Flavio Chierichetti
Sapienza University of Rome

Online social networks

The total number of
active user accounts
exceeds
two billions!

Advertisement

Online Social Networks

Sociology

Economics

Political Science

Computer Science

Online Social Networks

Sociology
Economics
Political Science

Computer Science

Models of human behavior

Models Improvements ← Models Validations

Online Social Networks

Sociology

Economics ⟷ Computer Science

Political Science

Models of human behavior

Models Improvements ← **Models Validations**

Empirical Analysis

- This sort of analysis is becoming more and more important in social networks research
- Exact user data is **very hard** to obtain for **privacy** related reasons

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- Empirical researchers have a hard time testing their hypotheses / validating their models

“Data Traces”

Researchers try to **guess**
the *missing data*,
using *existing observations* and a
reasonable *theoretical model*

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the *missing data*,
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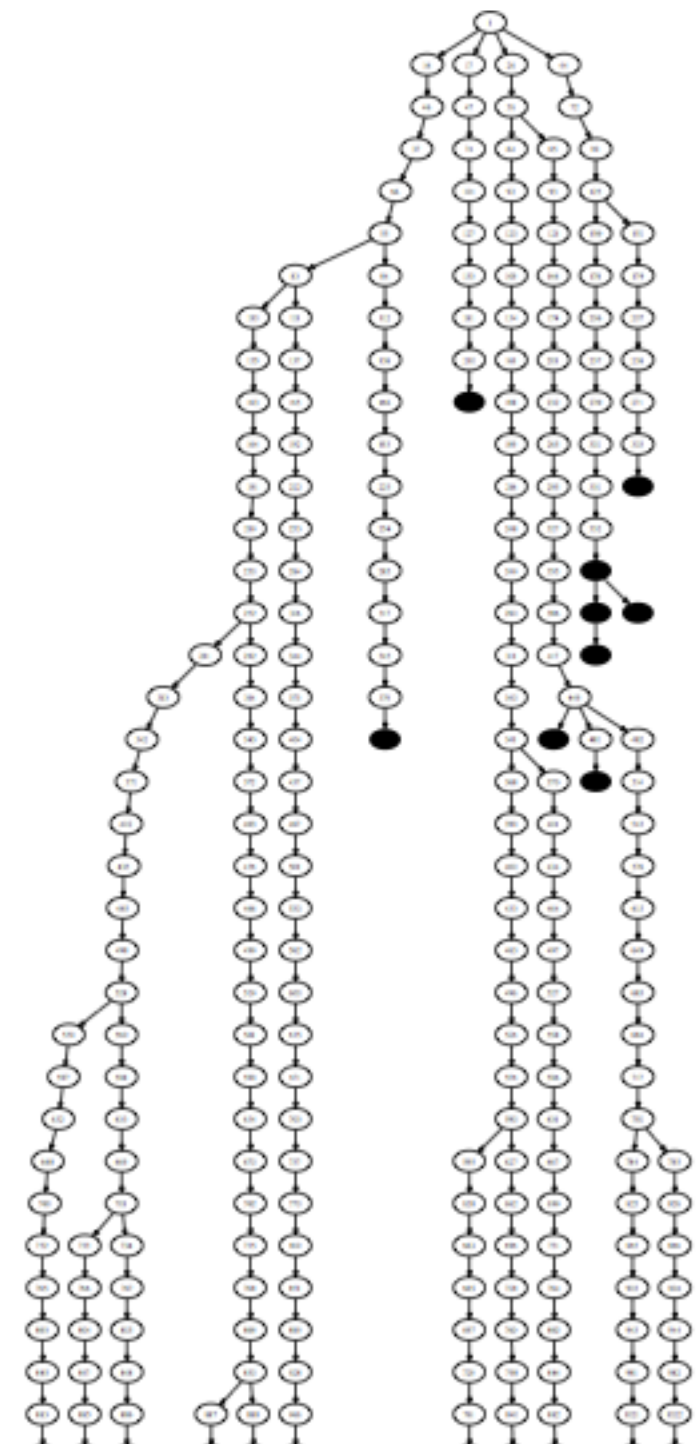
Theoretical researchers can help by
trying to understand whether
sound guessing algorithms *exist*

Online Guessing Tasks

- Which of these people are friends?

Online Guessing Tasks

- Which of these people are friends?
- How many people were infected by this meme?



Online Guessing Tasks

- We see little pieces of some online social process.
- We would like to make some **sensible** guesses on the process as a whole.

Network Reconstruction

Joint with

Bruno Abrahao, Robert Kleinberg, Alessandro Panconesi

Network Reconstruction

- Incomplete Traces of Information Flow.
- Wish to infer the hidden network.

Network Reconstruction

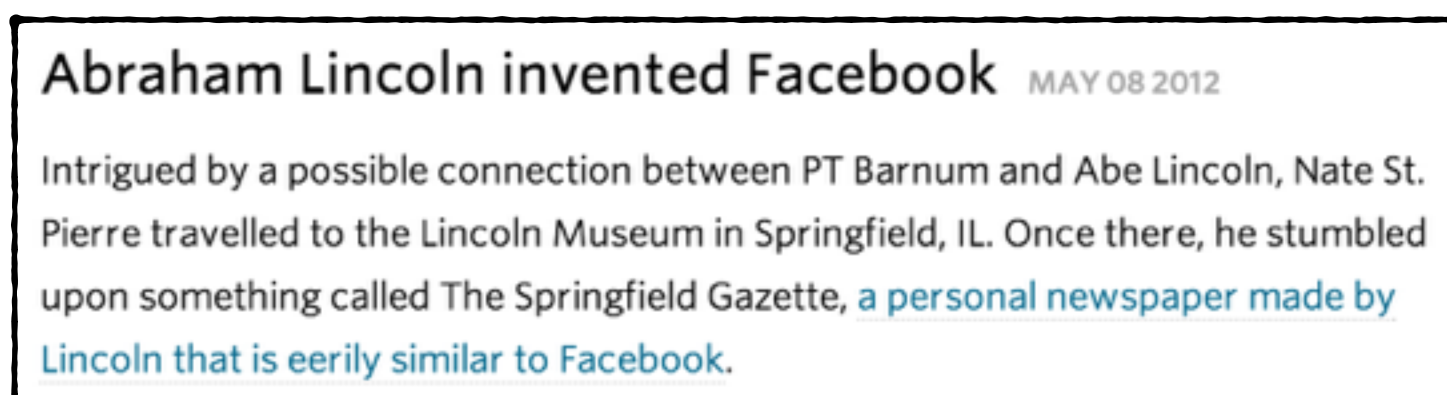
- Rich area of study, pioneered by [Adar, Adamic, ’05] in the context of social networks.
- MLE-based approaches:
  - [Gomez-Rodriguez, Leskovec, Krause ’10],
  - [Gomez-Rodriguez, Balduzzi, Scholkopf ’11],
  - [Myers, Leskovec ’11],
  - [Du et al. ’12]
- Information theoretic approaches
  - [Netrapalli, Sanghavi ’12],
  - [Grippon, Rabbat ’13]

“Inferring networks of diffusion and influence”

Gomez-Rodriguez, Leskovec, Krause *[KDD’10]*

“

- Gomez-Rodriguez et al studied a large collection of blogs and memes to guess the *blogger network* that allowed *memes* to *spread*.


Abraham Lincoln invented Facebook MAY 08 2012

Intrigued by a possible connection between PT Barnum and Abe Lincoln, Nate St. Pierre travelled to the Lincoln Museum in Springfield, IL. Once there, he stumbled upon something called The Springfield Gazette, a personal newspaper made by Lincoln that is eerily similar to Facebook.

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- They proposed a random meme-diffusion model, and used it for making the guess.

Flow of Memes

Alice's Blog

Bob's Blog - Live from Lewisville

c.
charlie's blog

The World According to Dave
My unsolicited ramblings, rants, musings & bloviations (totally free and worth every penny)

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Mar 4, 2014, 8:25am
Mrs. Hudson's cake shop will reopen!

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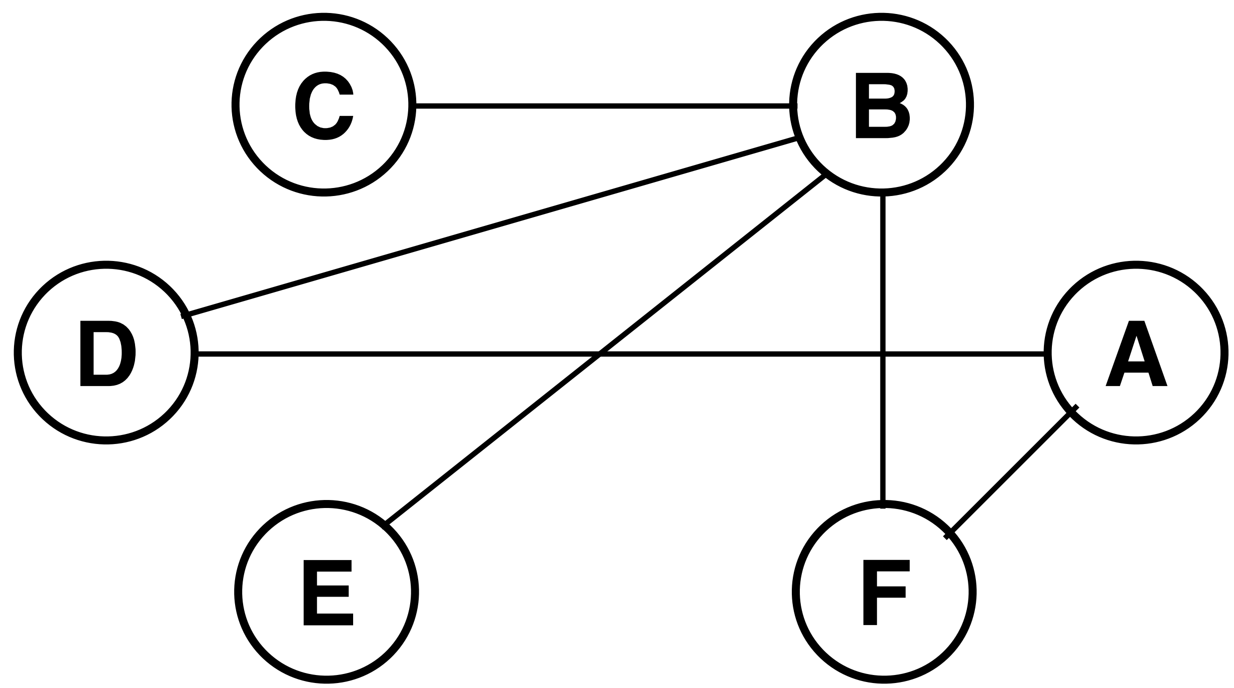
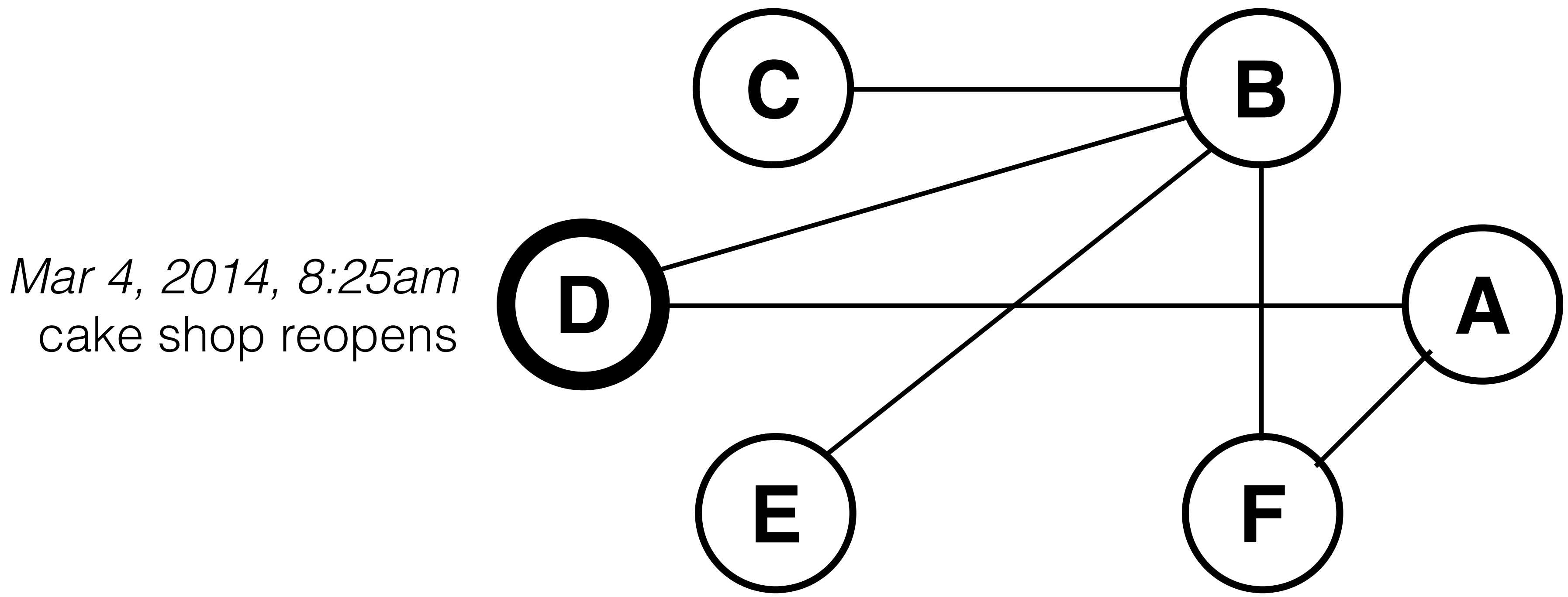
Can we infer which blogger follows which blog?

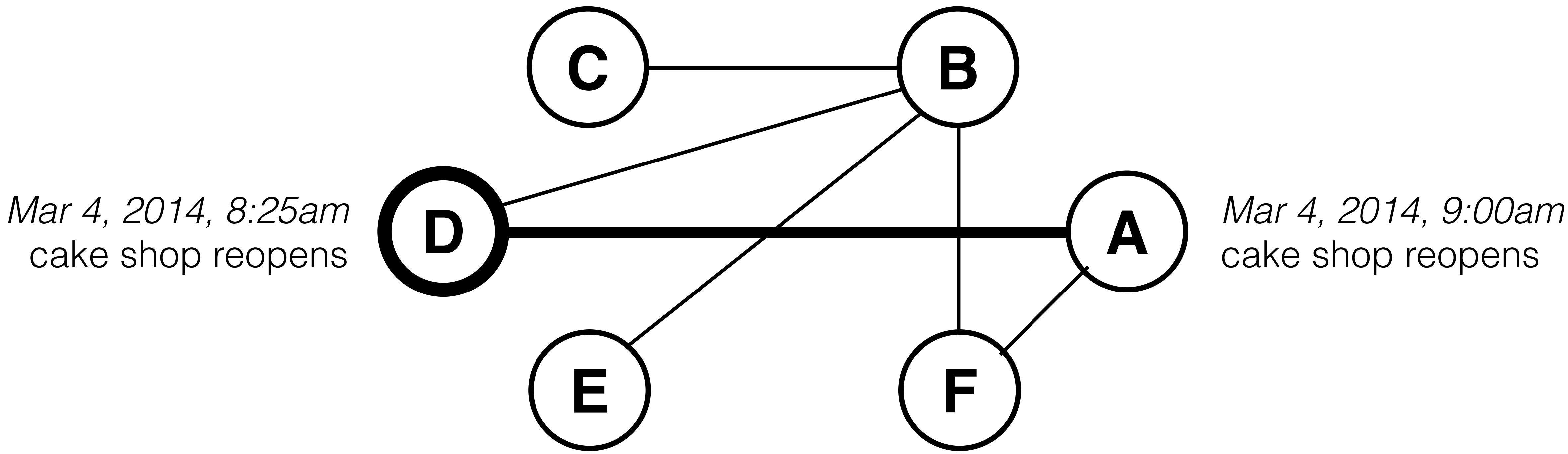
Meme Flow



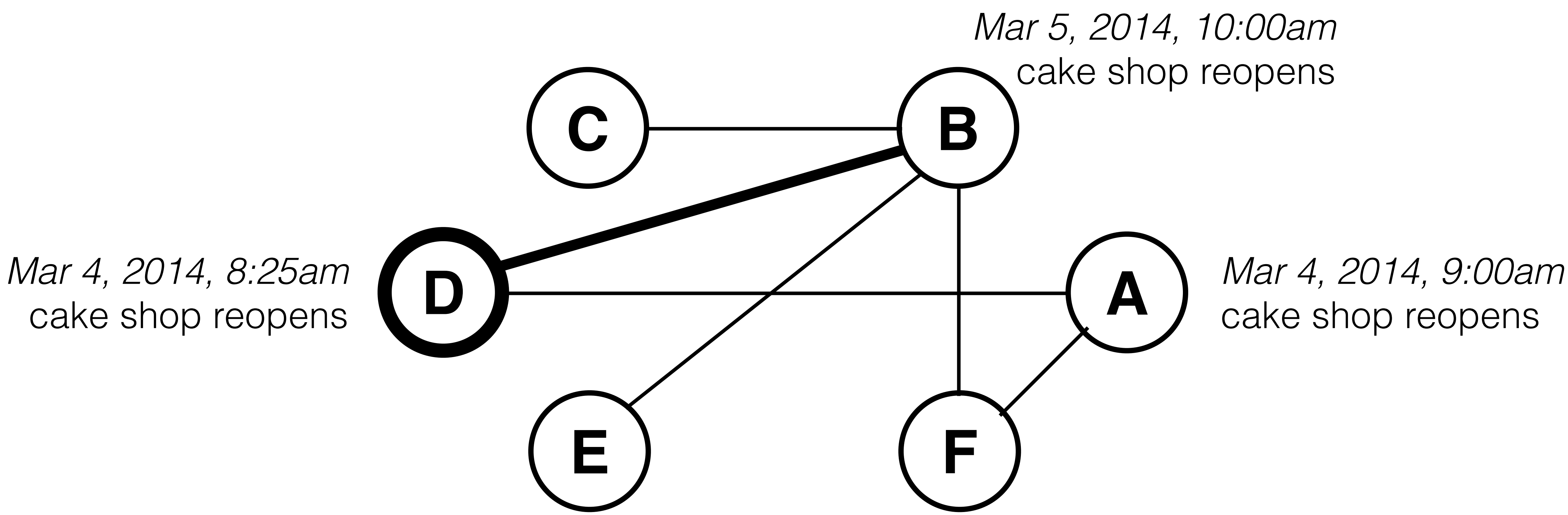
Meme Flow



Meme Flow


C
B
D
A
E
F
Mar 4, 2014, 8:25am
cake shop reopens
Mar 4, 2014, 9:00am
cake shop reopens

Meme Flow


Mar 5, 2014, 10:00am
cake shop reopens
C
B
Mar 4, 2014, 8:25am
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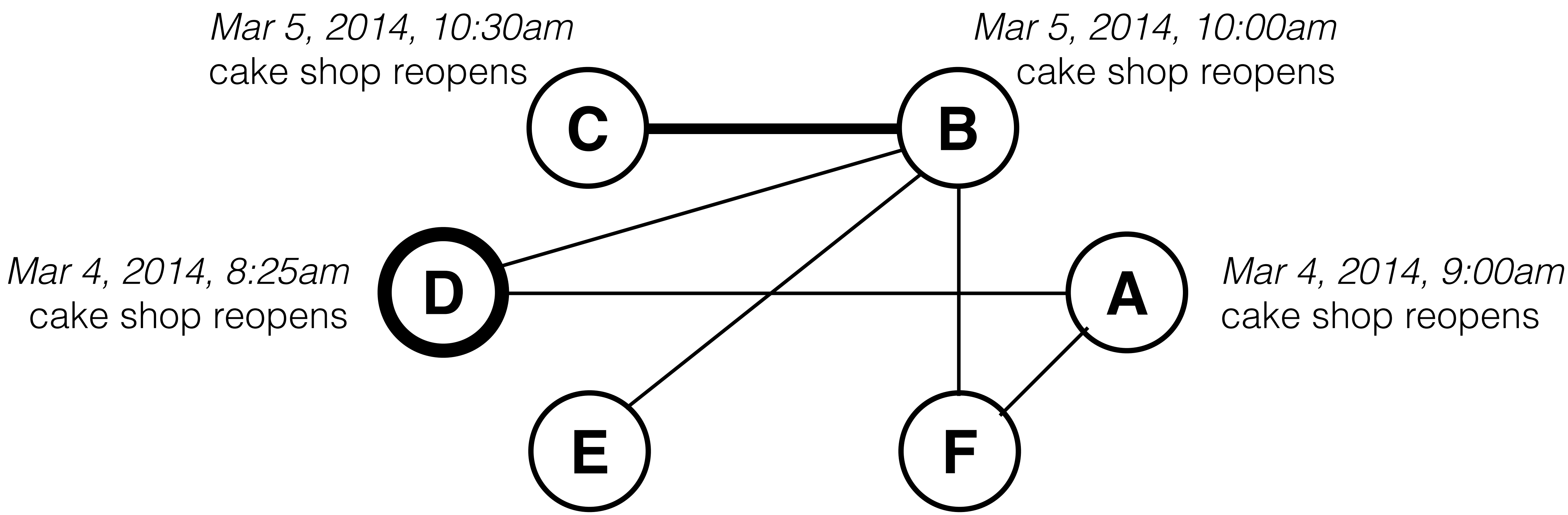
D
A
Mar 4, 2014, 9:00am
cake shop reopens
E
F

Meme Flow



Mar 5, 2014, 10:30am
cake shop reopens
C
B
Mar 5, 2014, 10:00am
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Mar 4, 2014, 8:25am
cake shop reopens
D
A
Mar 4, 2014, 9:00am
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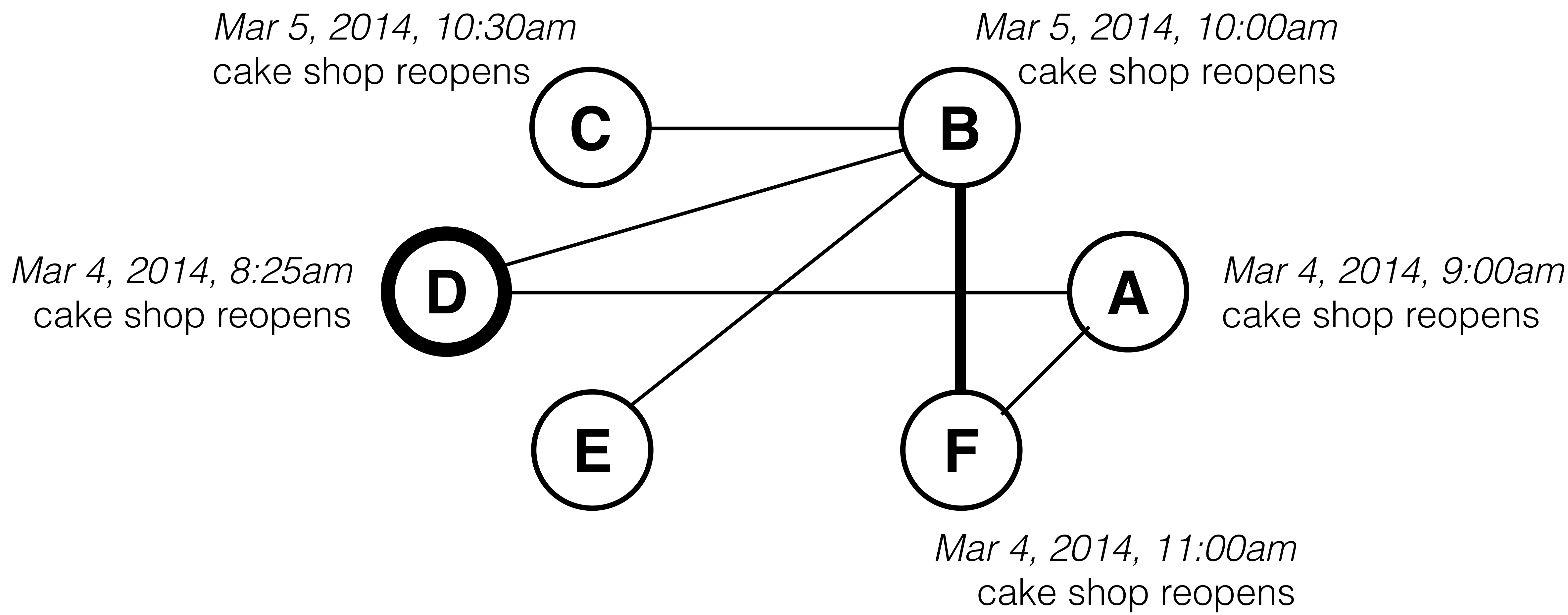
E
F

Meme Flow



Mar 5, 2014, 10:30am
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cake shop reopens
B
Mar 4, 2014, 8:25am
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D
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Mar 4, 2014, 9:00am
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E
F
Mar 4, 2014, 11:00am
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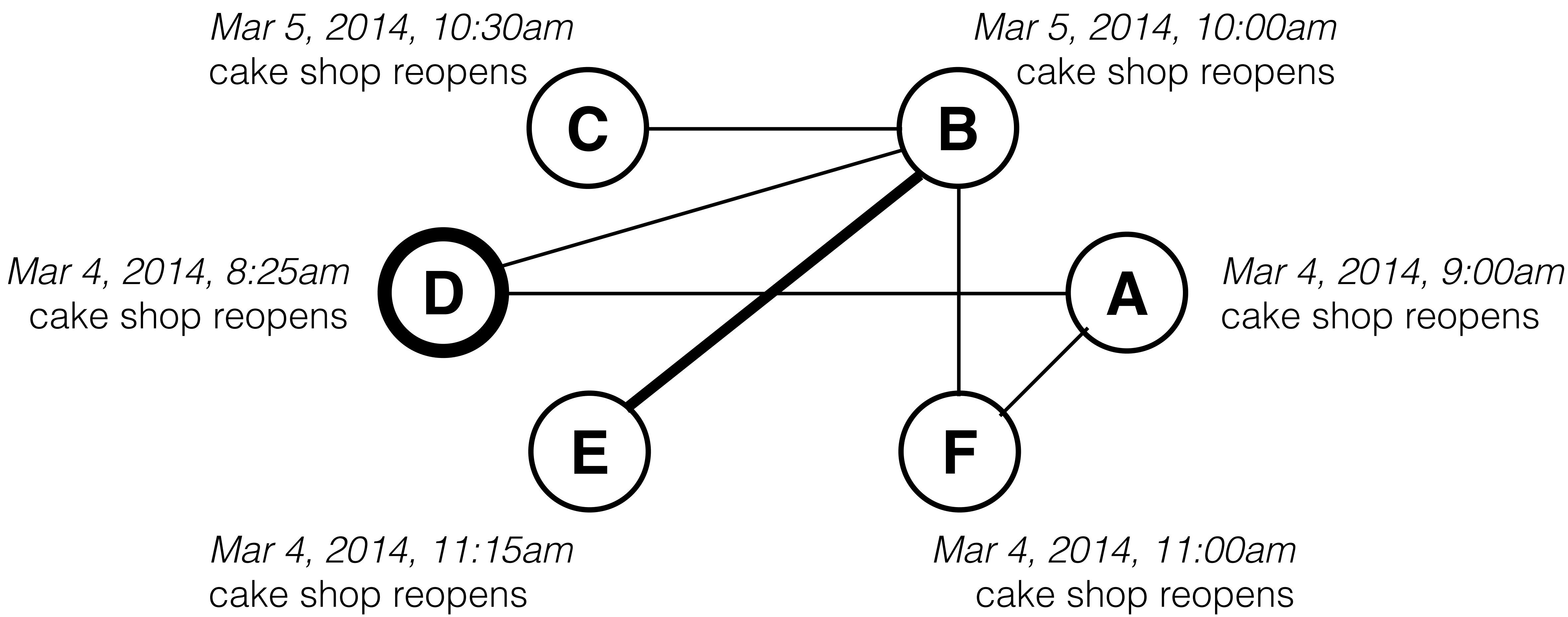
Meme Flow


Mar 5, 2014, 10:30am
cake shop reopens
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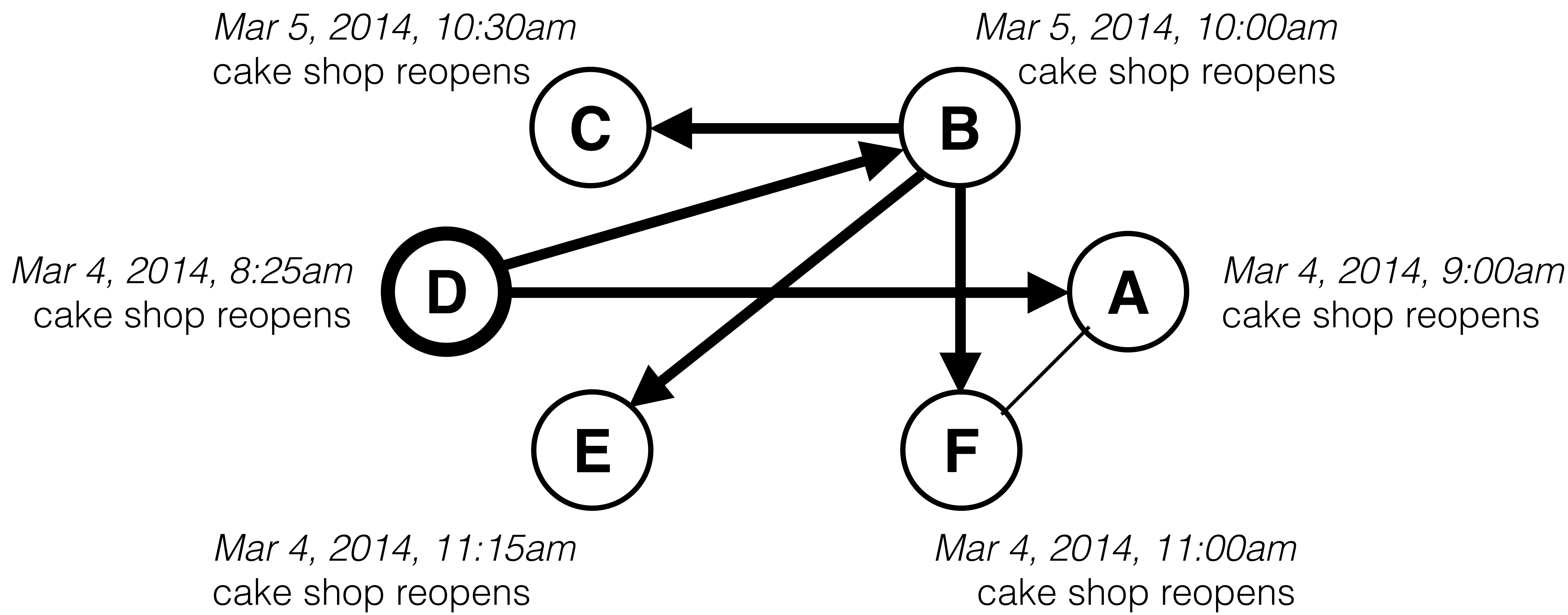
C
B
Mar 4, 2014, 8:25am
cake shop reopens
D
A
Mar 4, 2014, 9:00am
cake shop reopens
E
F
Mar 4, 2014, 11:15am
cake shop reopens
Mar 4, 2014, 11:00am
cake shop reopens

Meme Flow


Mar 5, 2014, 10:30am
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E
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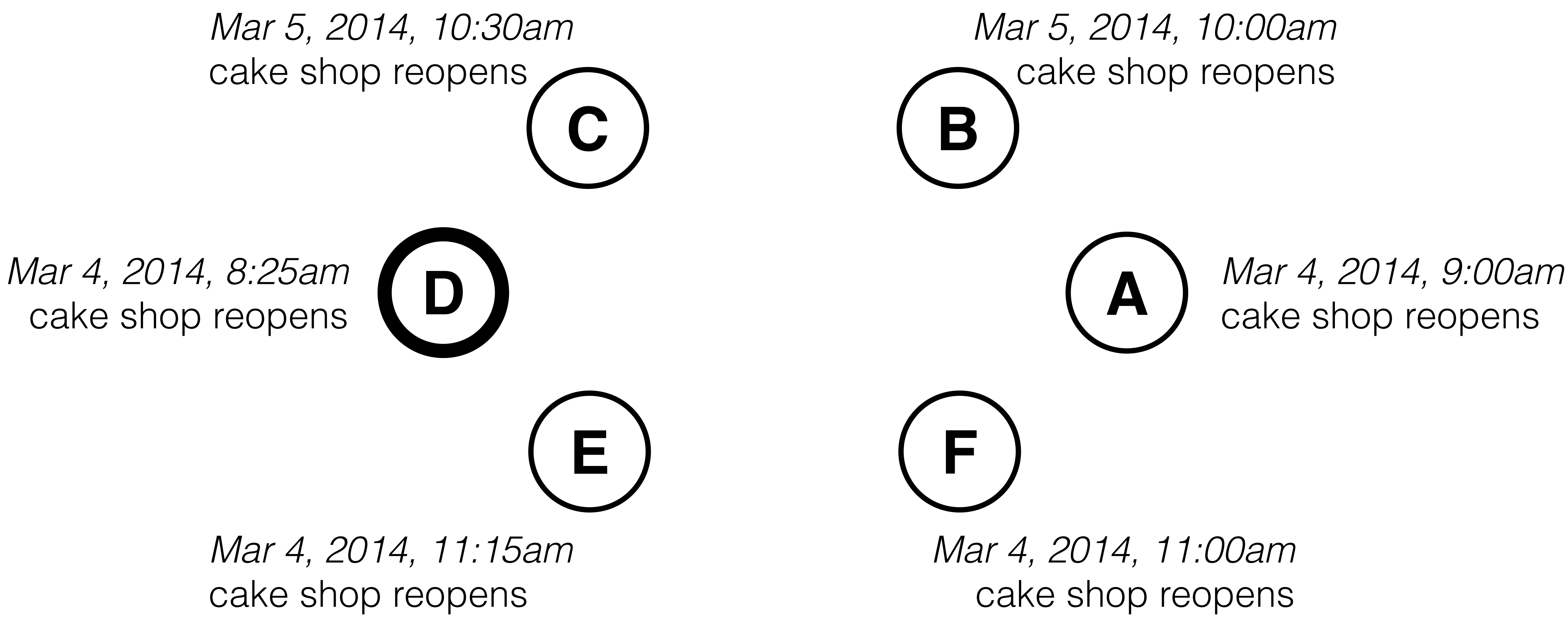
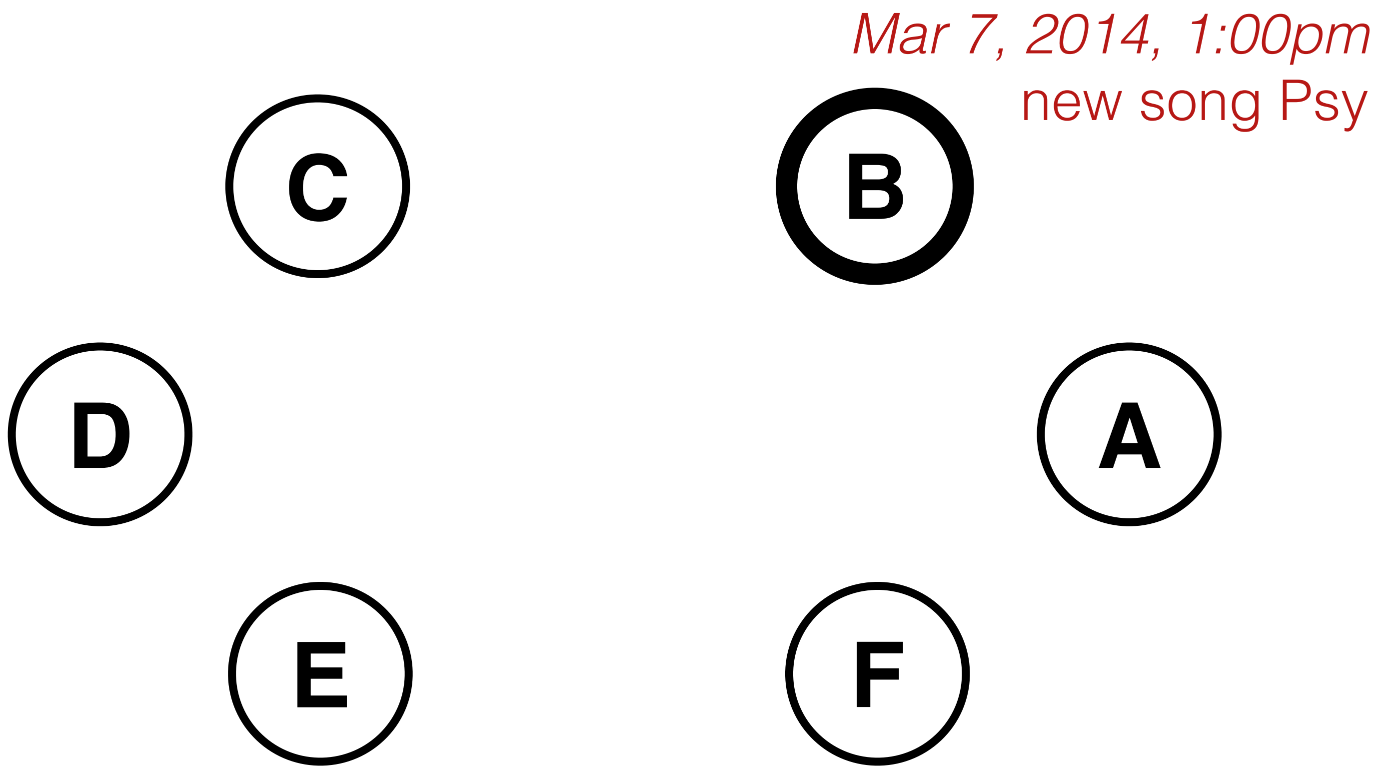
Traces



D *35’* A *60’* B *30’* C *30’* F *15’* E

Traces



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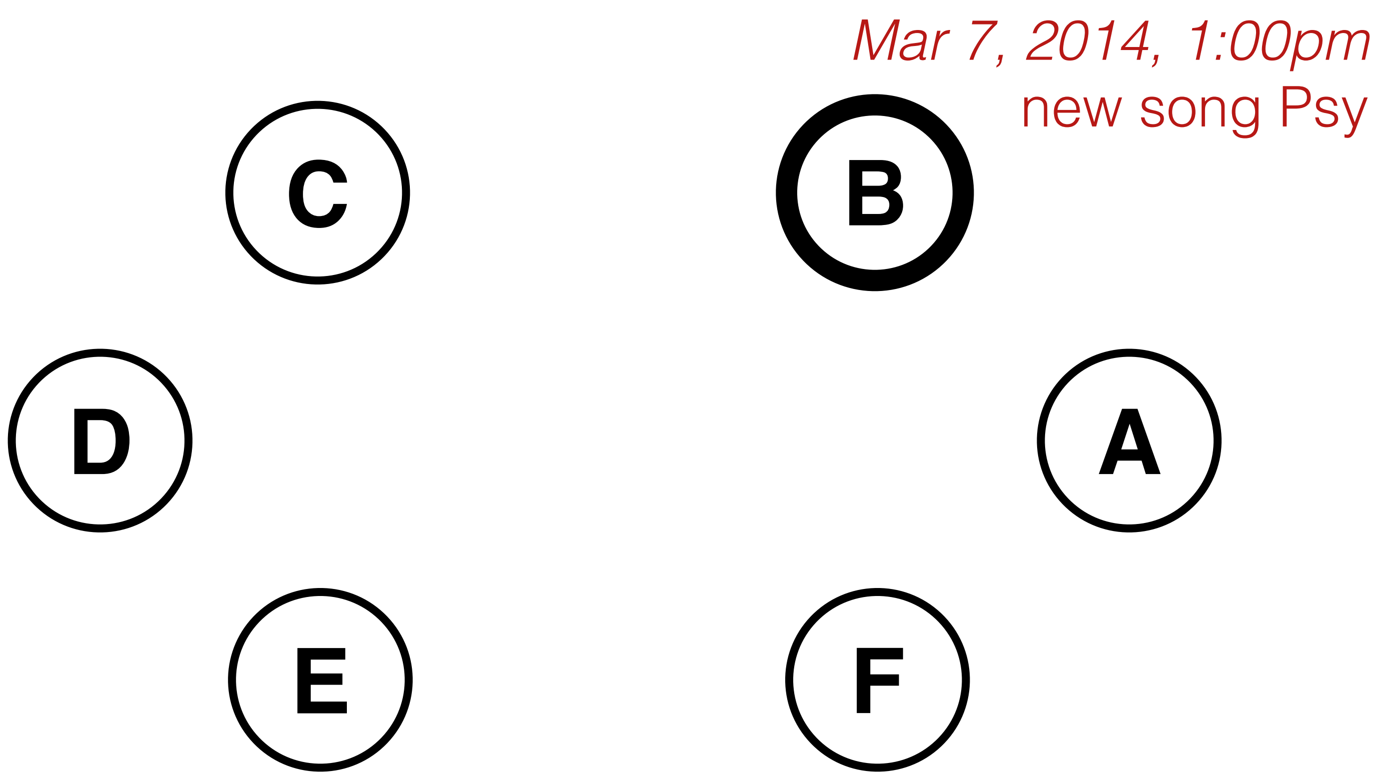
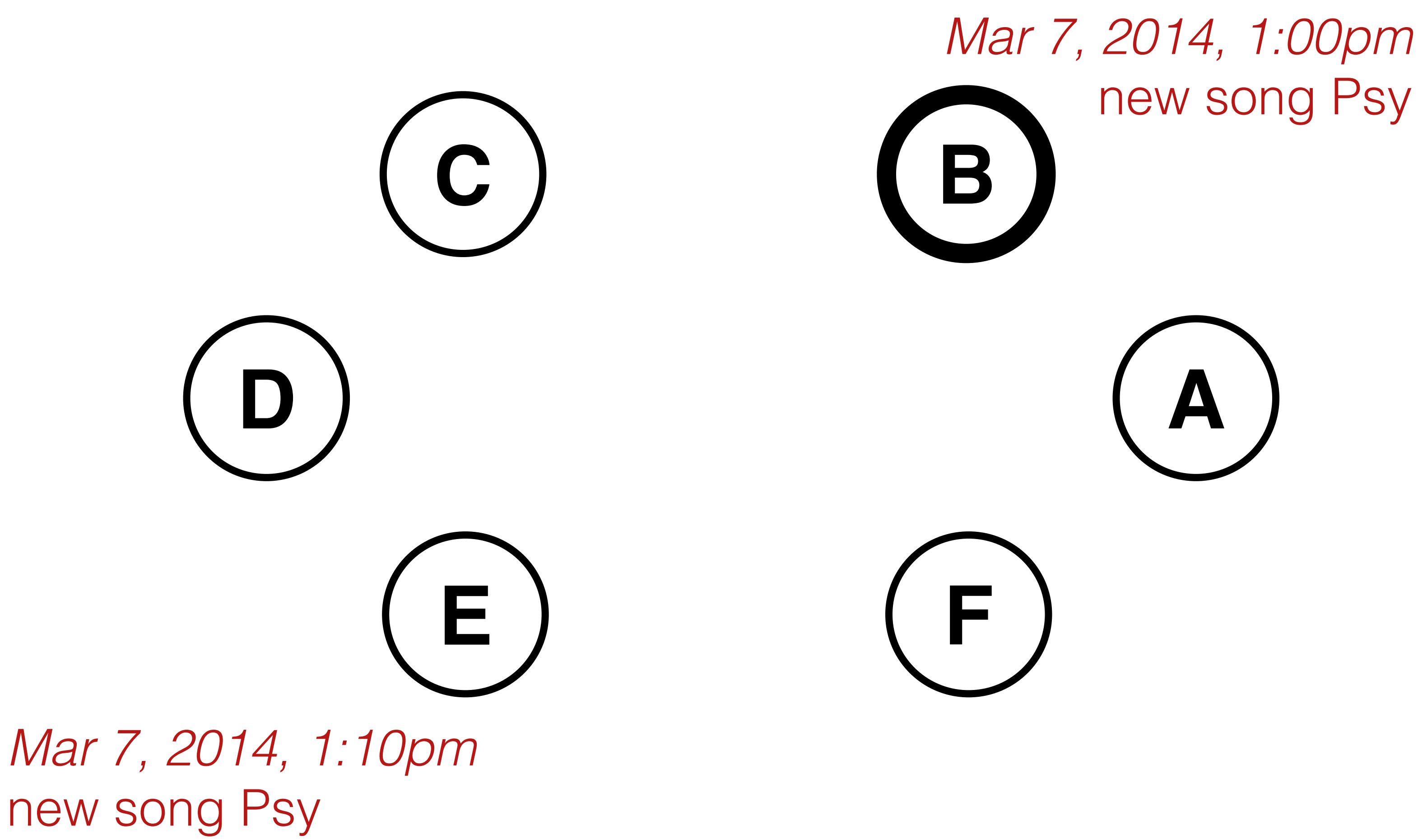


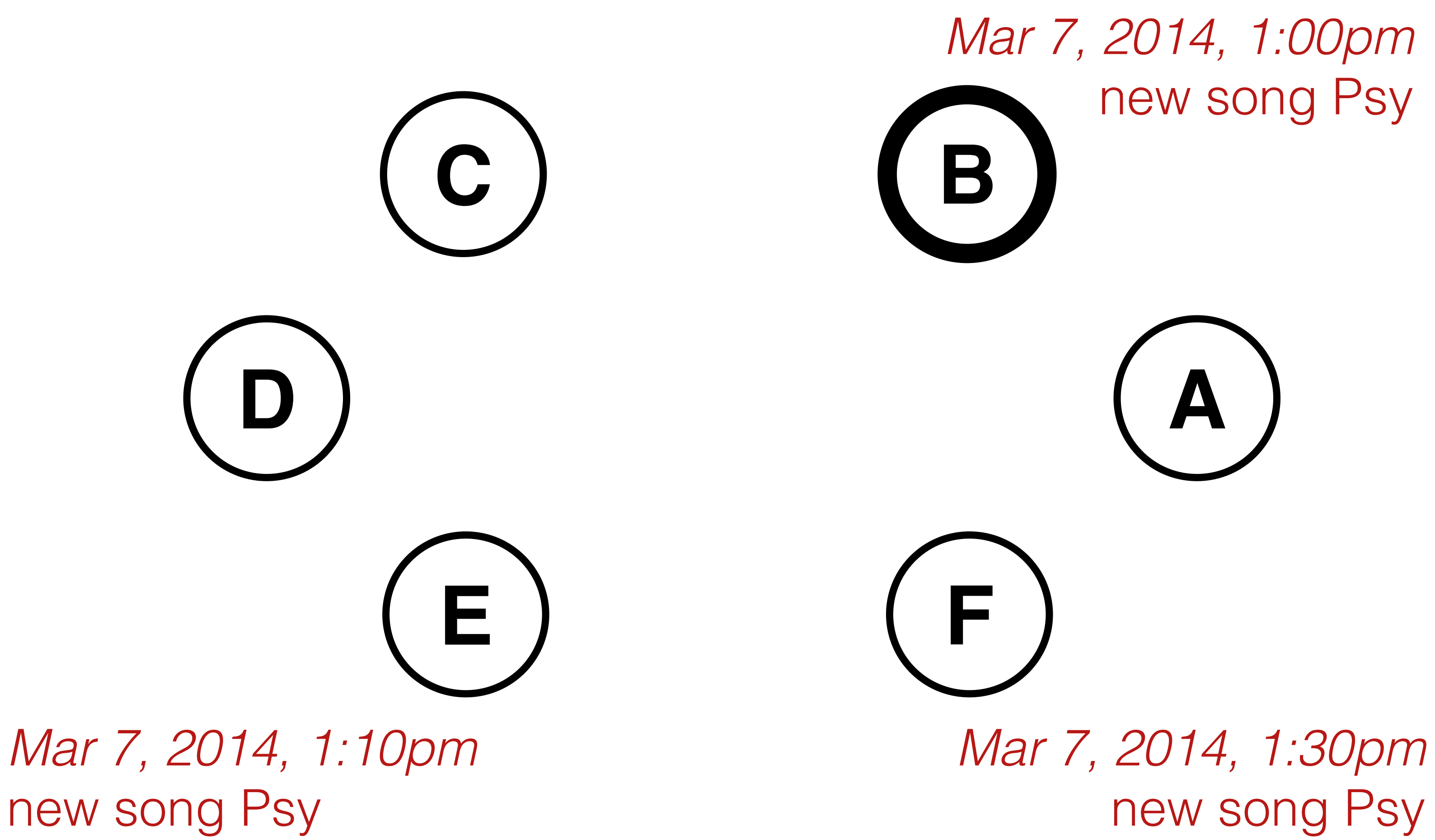
D *35'* A *60'* B *30'* C *30'* F *15'* E
B

Traces



D *35'* A *60'* B *30'* C *30'* F *15'* E
B *10'* E

Traces



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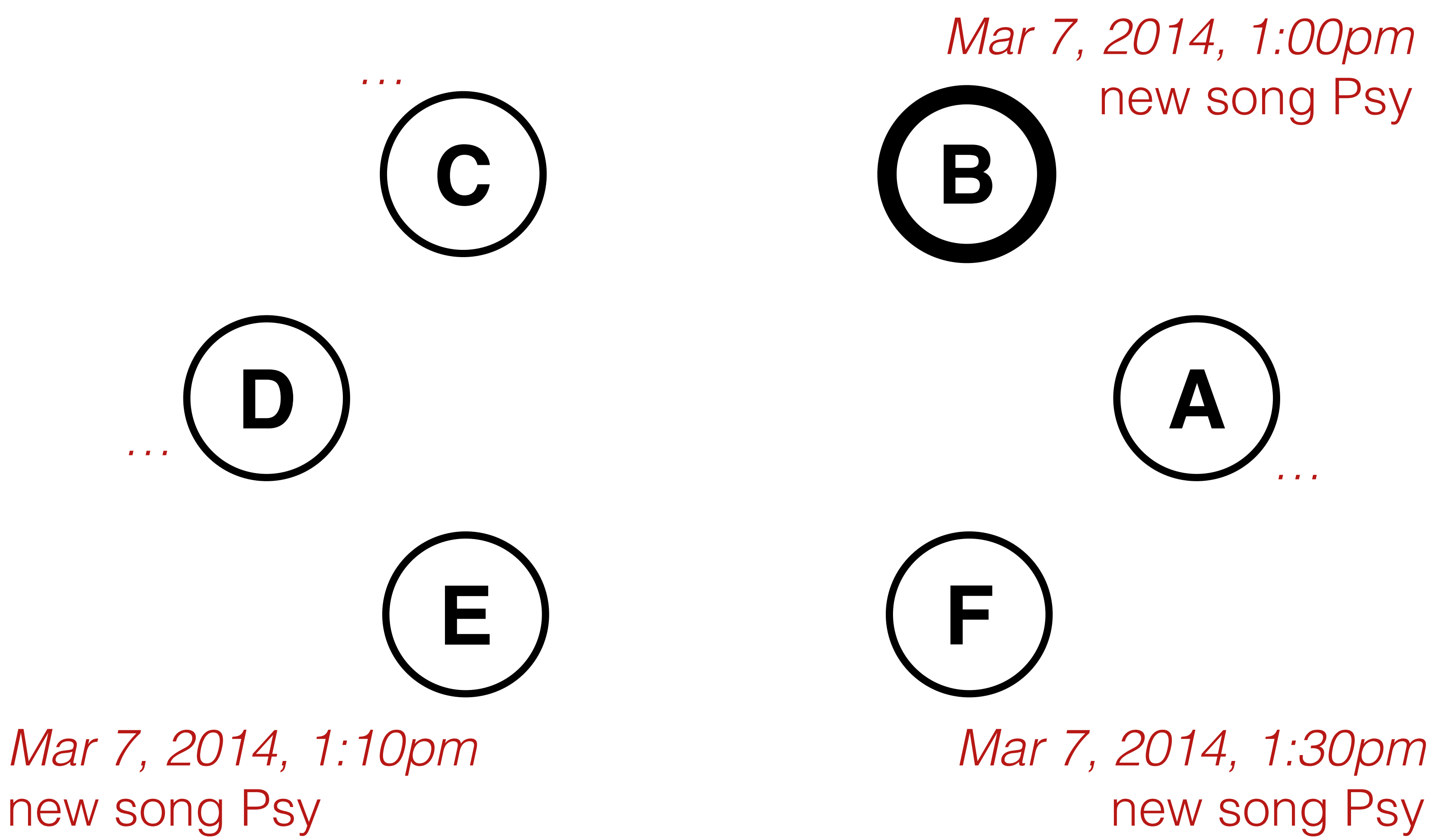
B *10’* E *20’* F

Traces



D *35'* A *60'* B *30'* C *30'* F *15'* E

B *10'* E *20'* F *15'* A *15'* D *30'* C

Inference

Given a set of traces

D	*35'*	A	*60'*	B	*30'*	C	*30'*	F	*15'*	E
B	*10'*	E	*20'*	F	*15'*	A	*15'*	D	*30'*	C
B	*12'*	D	*15'*	A	*10'*	C	*25'*	E	*30'*	F

...

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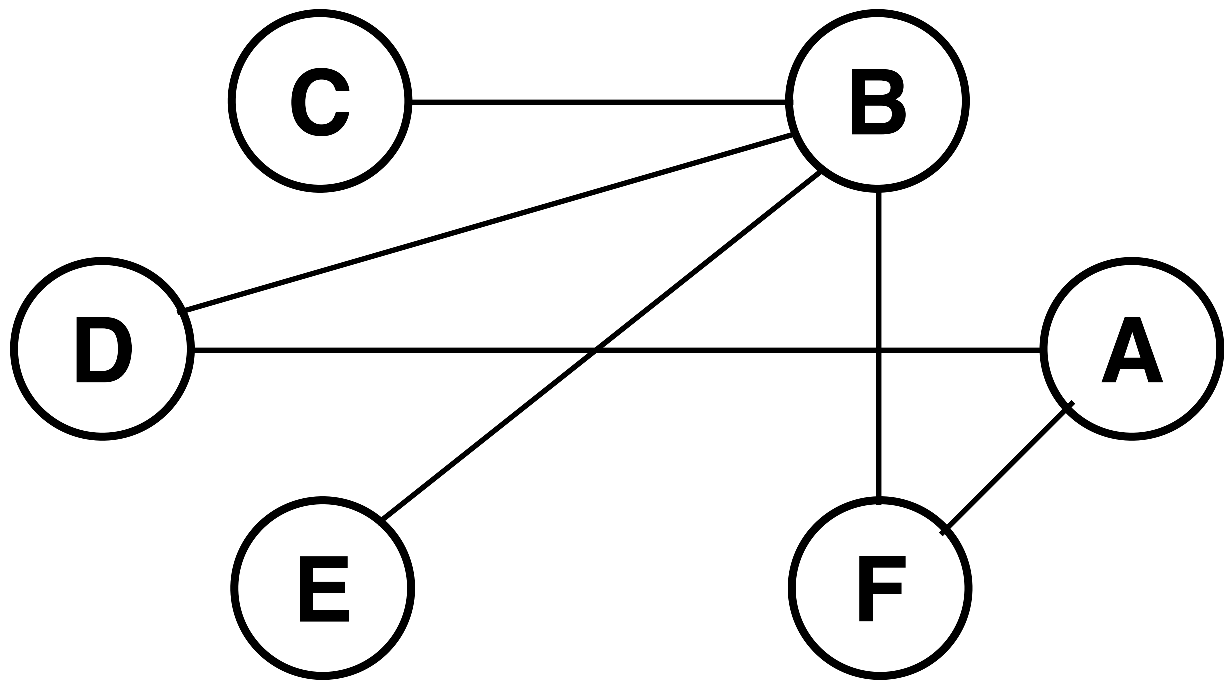
...

we would like to infer the
unknown blogger graph *G*

Trace Spreading Model

Gomez-Rodriguez, Leskovec, Krause *[KDD'10]*



- The unique source is chosen uniformly at random

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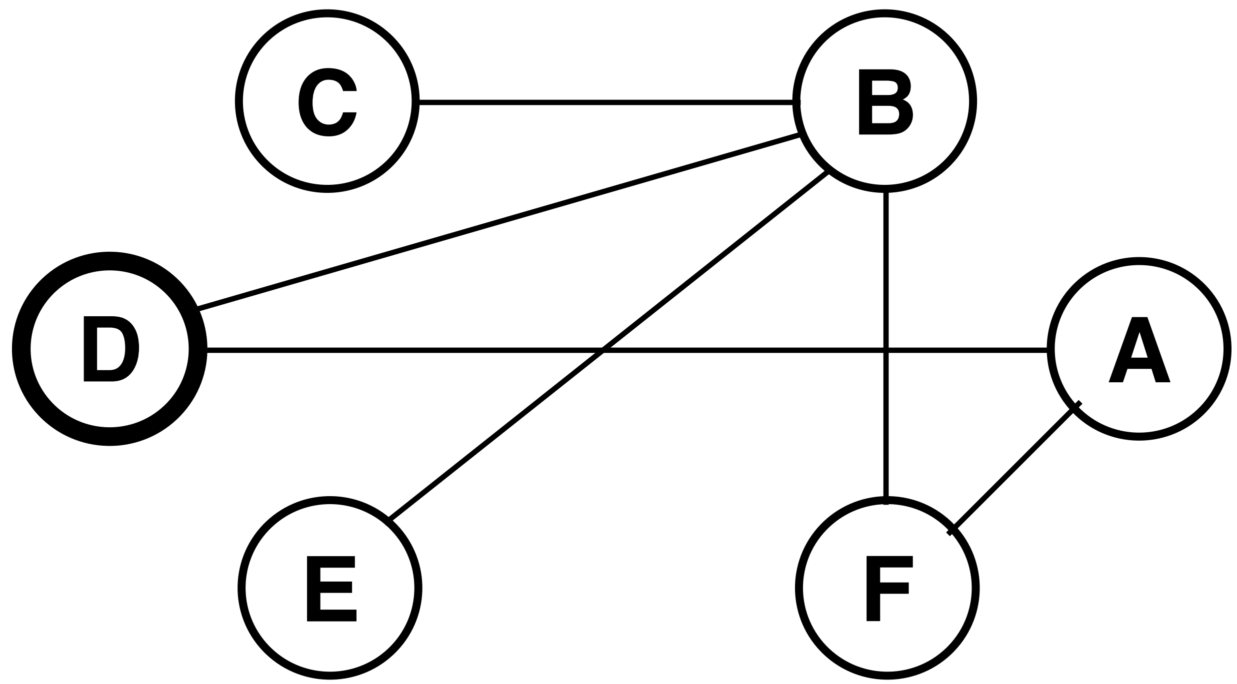

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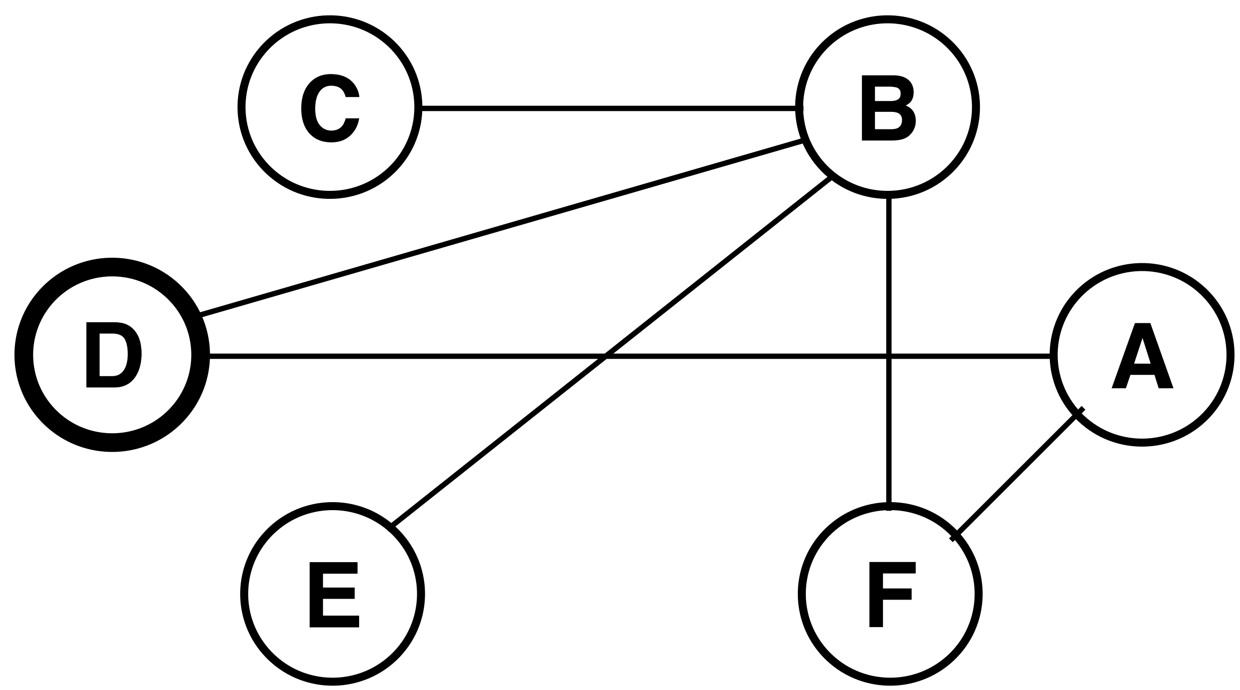

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- The time to traverse an edge is an iid sample of $\mathrm{Exp}(\lambda)$

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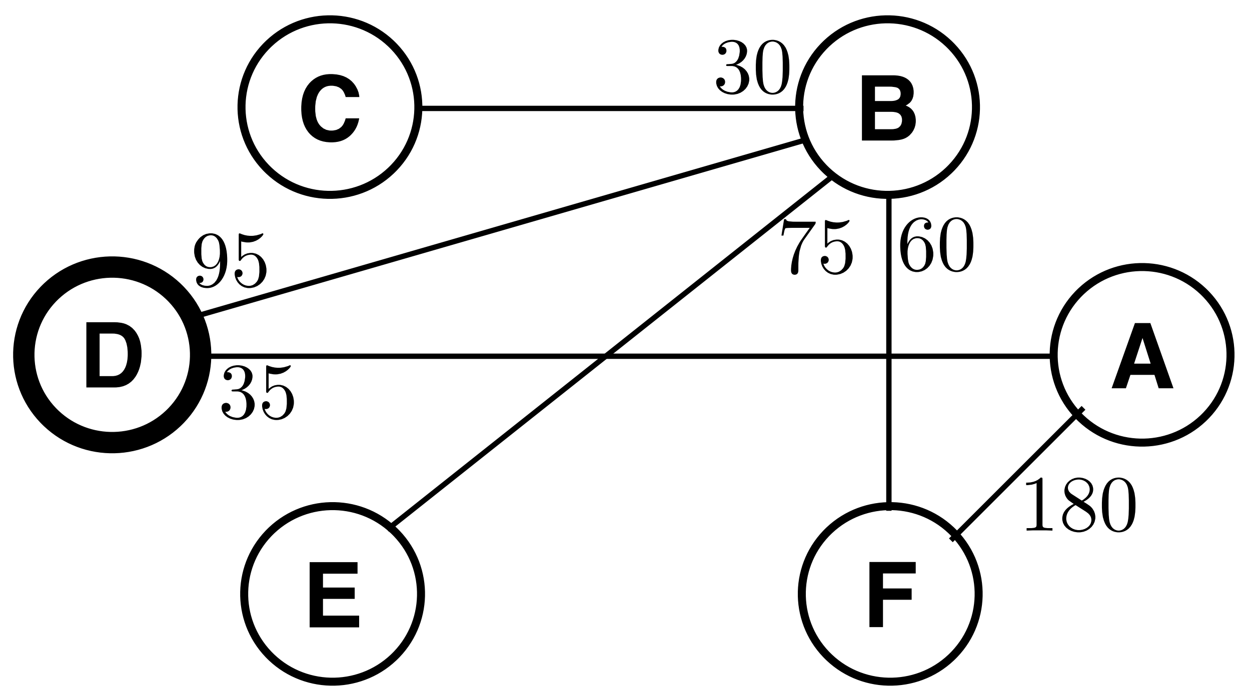

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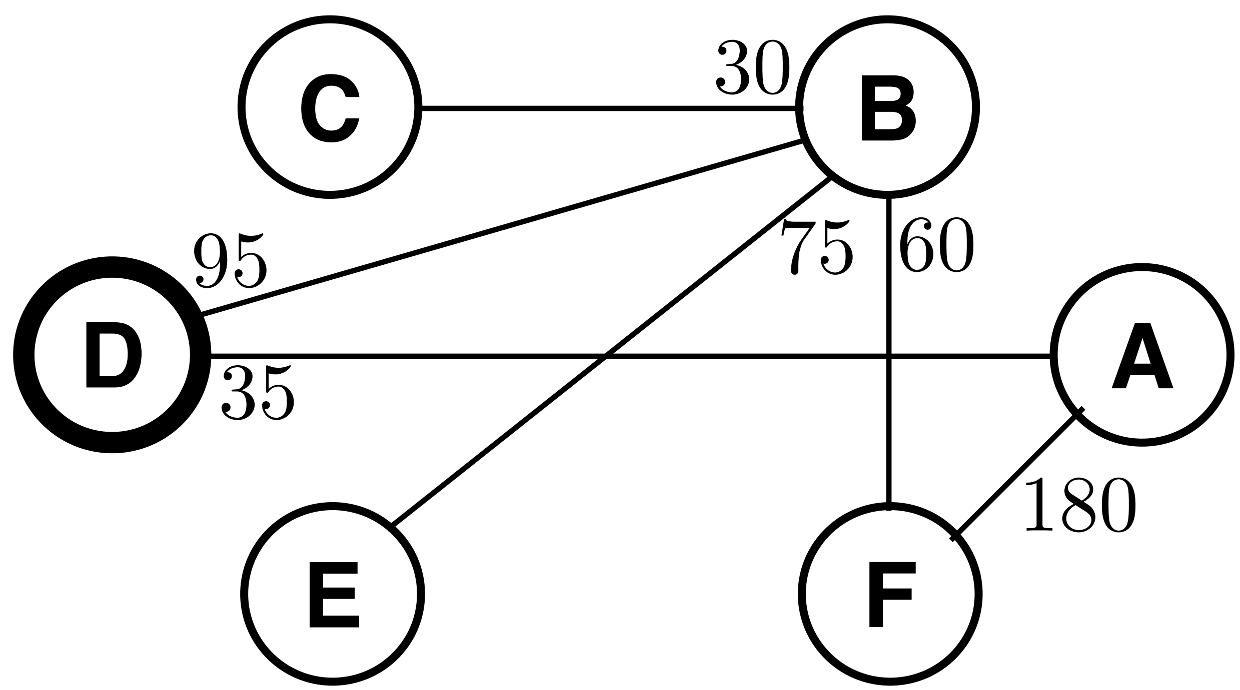

- The unique source is chosen uniformly at random
- The time to traverse an edge is an iid sample of $\mathrm{Exp}(\lambda)$
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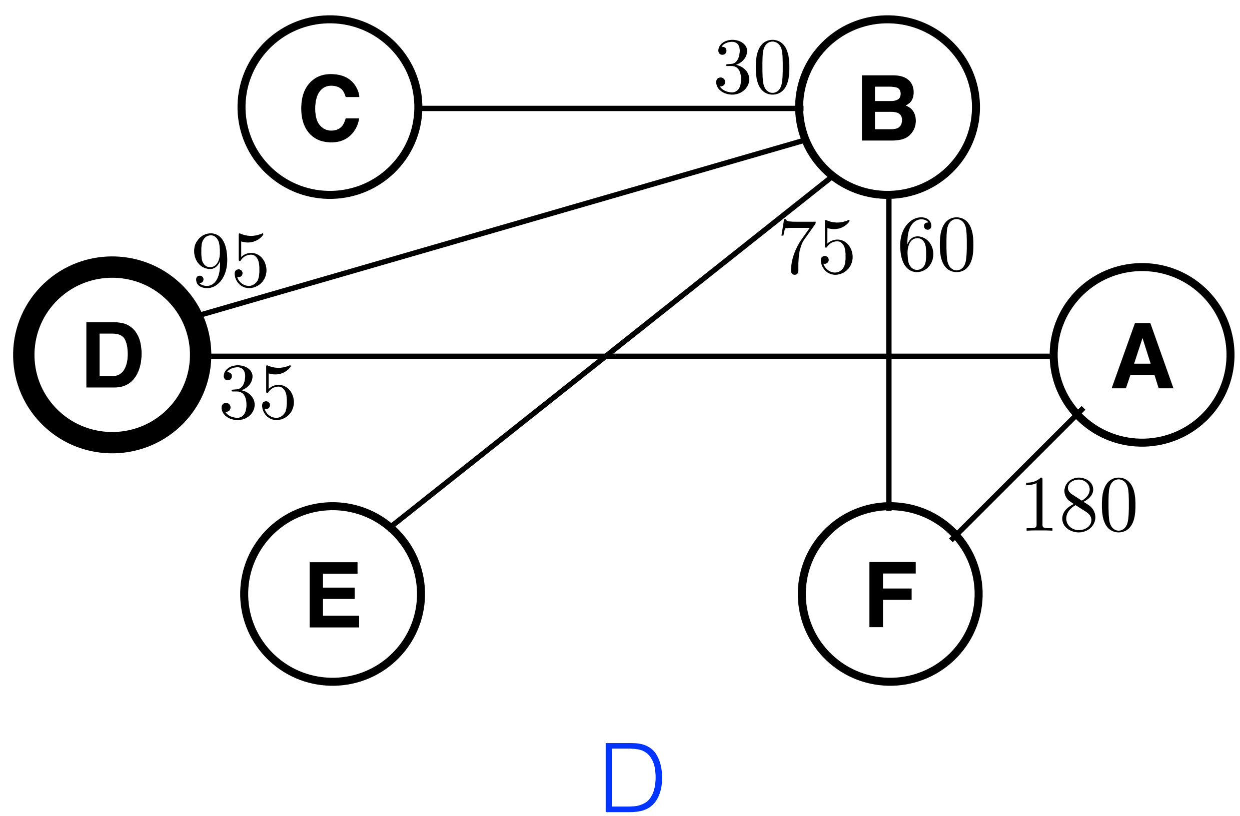

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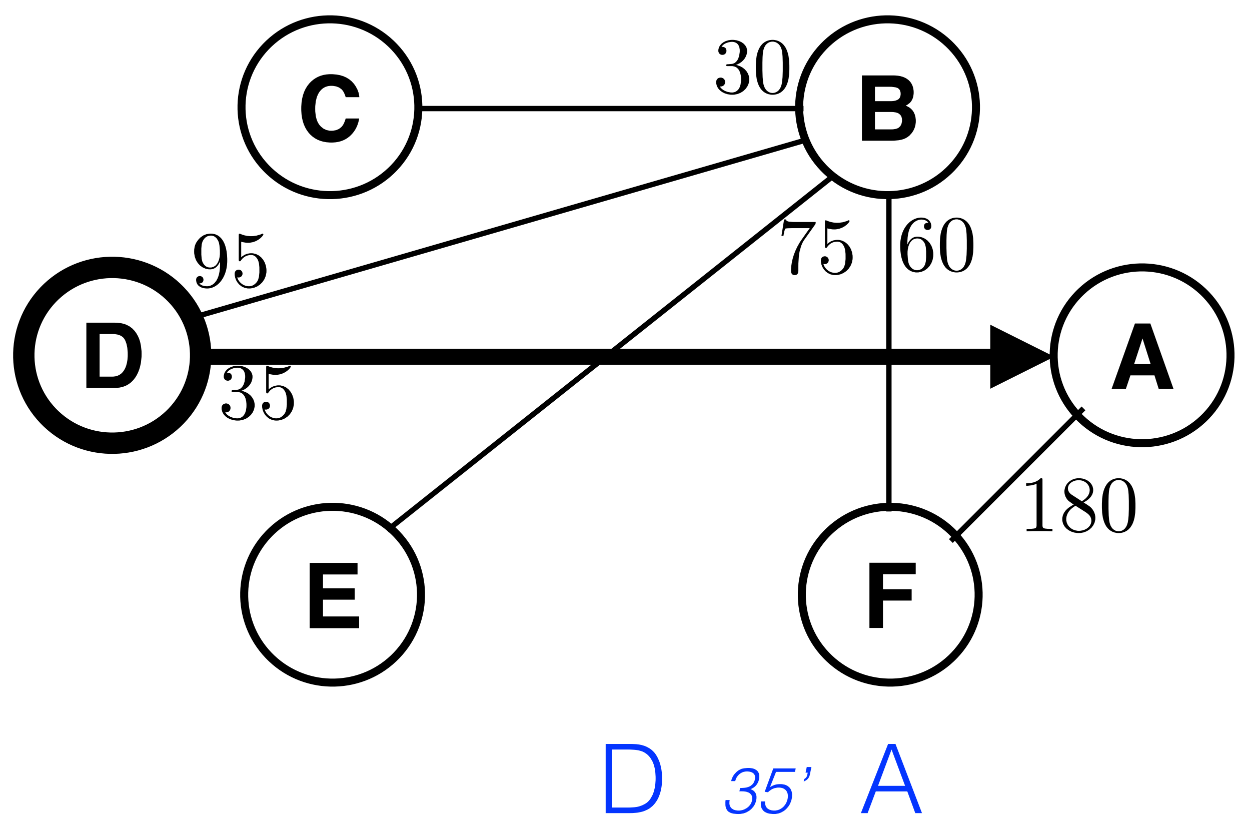

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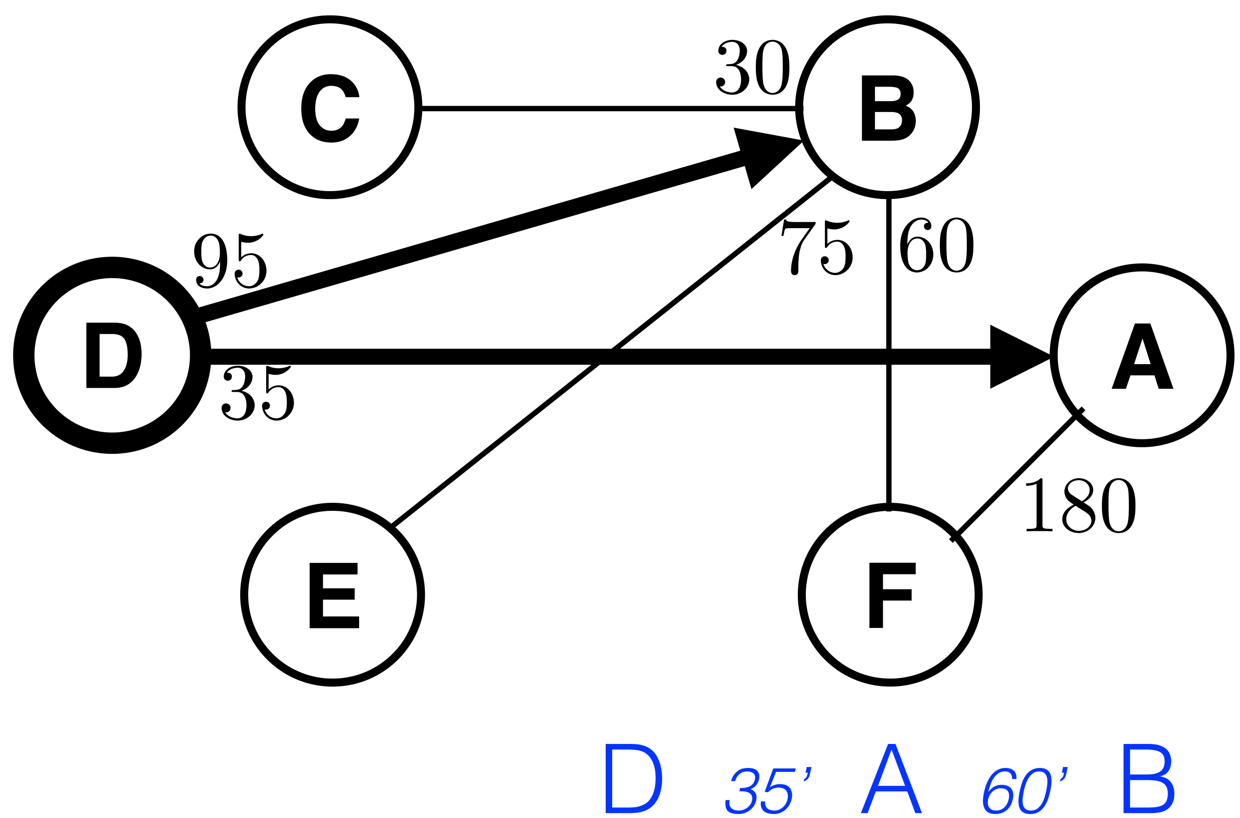

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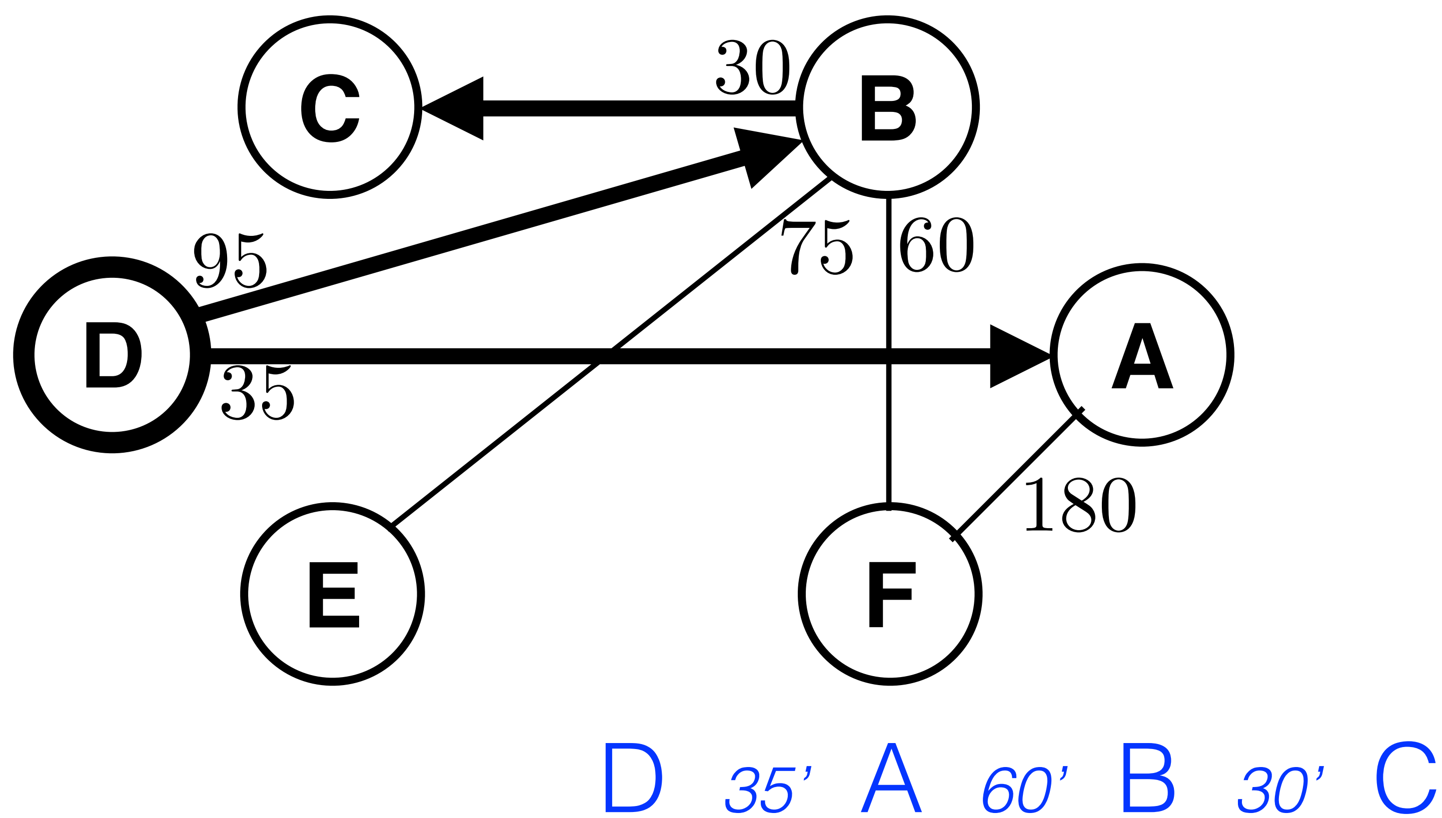

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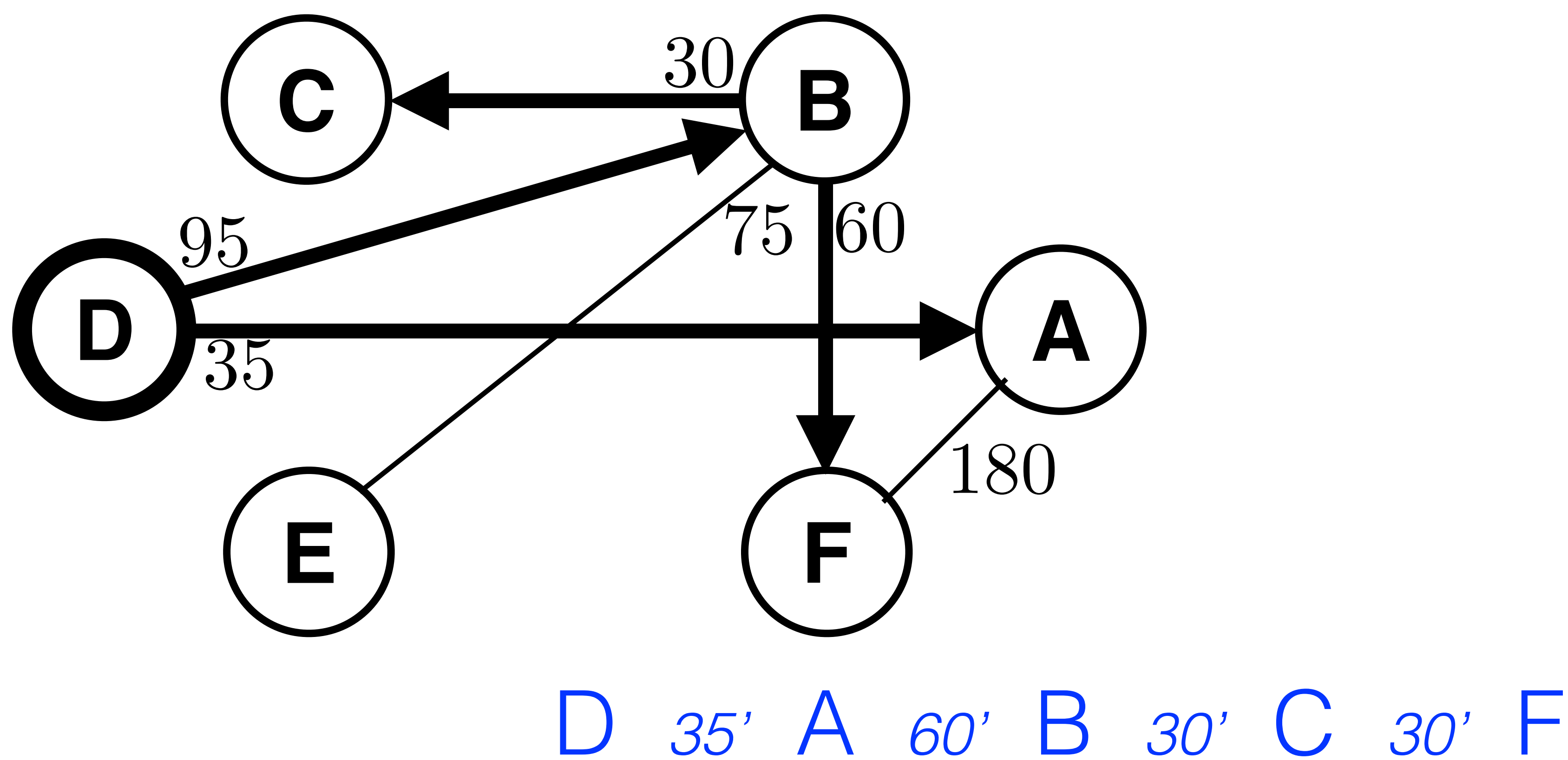

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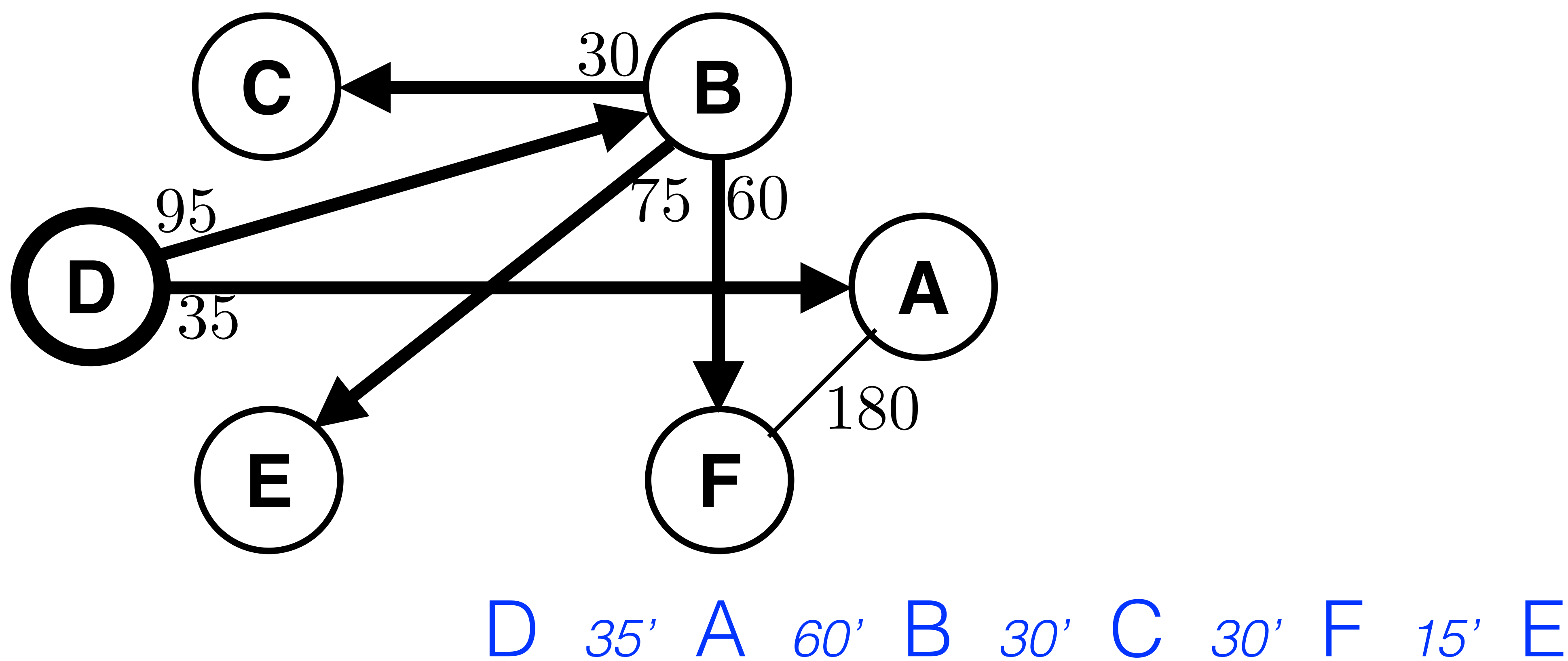

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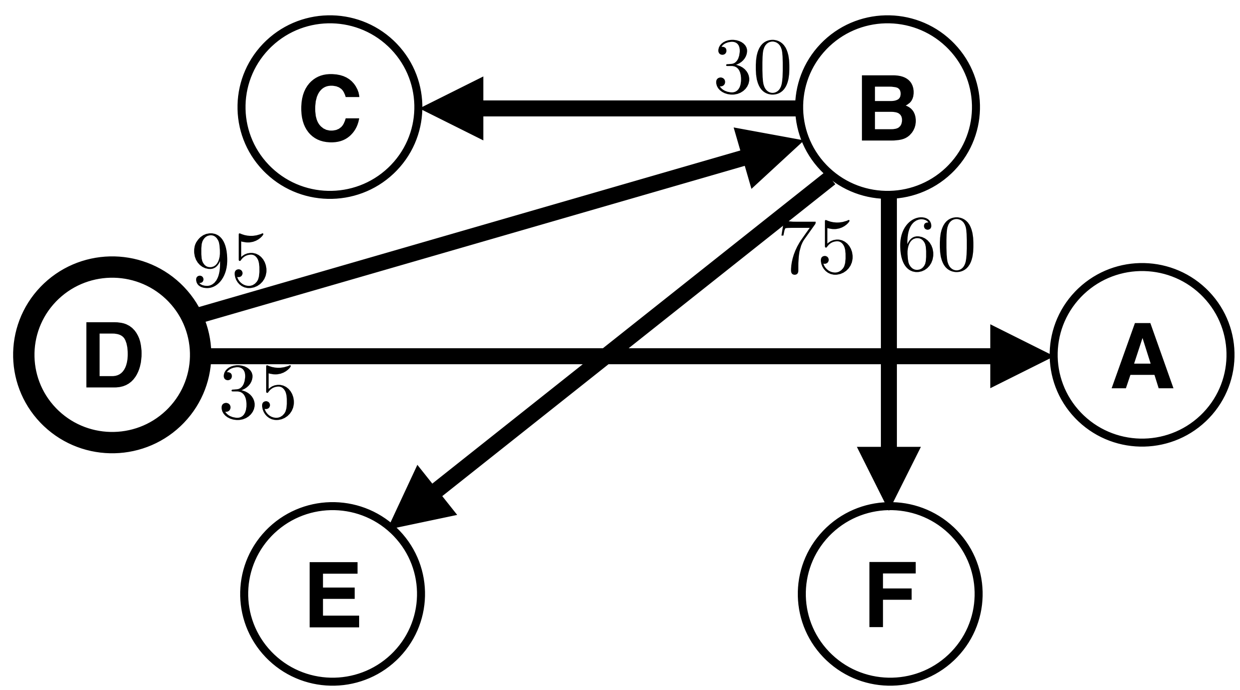

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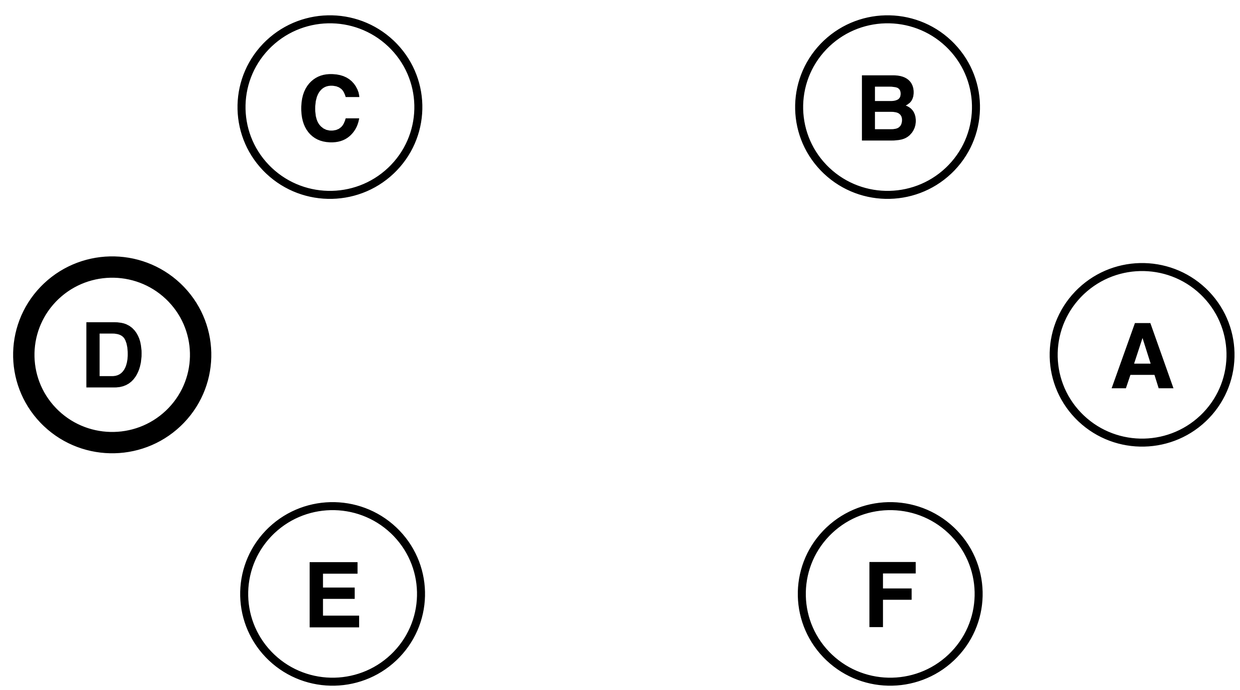

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Can we make a sensible guess of the network?

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How many traces would we need?

Our Contributions

Abrahao, Chierichetti, Kleinberg, Panconesi *[KDD'13]*

- We show that $O\left(n\Delta \log n\right)$ traces are sufficient for reconstruction.
- We show that, in some cases, $\Omega\left(\frac{n\Delta}{\log^2 n}\right)$ traces are necessary.

n is the number of nodes of the unknown graph,
Δ is its maximum degree.

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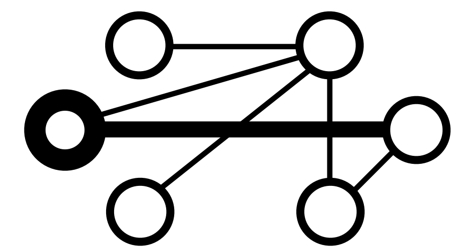
Abrahao, Chierichetti, Kleinberg, Panconesi *[KDD'13]*

- We show that $O\left(n\Delta\log n\right)$ traces are sufficient for reconstruction.
- We show that, in some cases, $\Omega\left(\frac{n\Delta}{\log^2 n}\right)$ traces are necessary.
- We also show that $O\left(\mathrm{poly}(\Delta)\cdot\log n\right)$ traces are sufficient, that $O(\log n)$ traces are sufficient for trees…

A simple algorithm

- **Observation**
  If u and v are the first two nodes of a trace then the edge $\{u,v\}$ is in the unknown graph.

A simple algorithm



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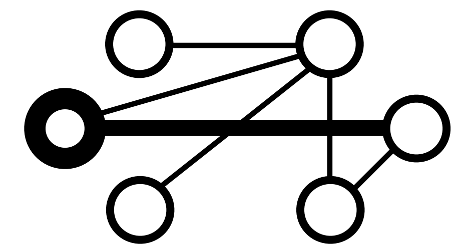
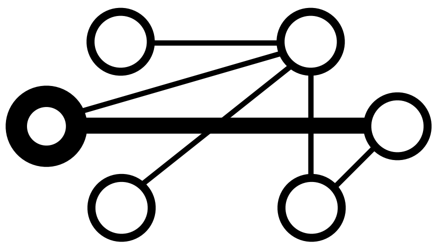


- **Observation**
  If *u* and *v* are the first two nodes of a trace then the edge *{u,v}* is in the unknown graph.

- We can then perfectly reconstruct if, for each unknown edge *{u,v}*, there exists a trace that begins with its two endpoints.

A simple algorithm



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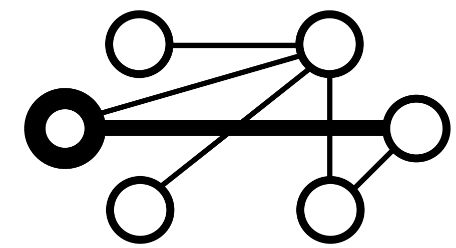
B *12'* D *15'* A *10'* C *25'* E *30'* F

A simple algorithm



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A simple algorithm

- The probability that a specific node is the first one in a trace is ${}^{1}\!/_{n}$.

- The probability that the edge {u,v} is spanned by the first two nodes of the trace is then at least

$$\frac{1}{n \cdot \min(\deg(u), \deg(v))}$$

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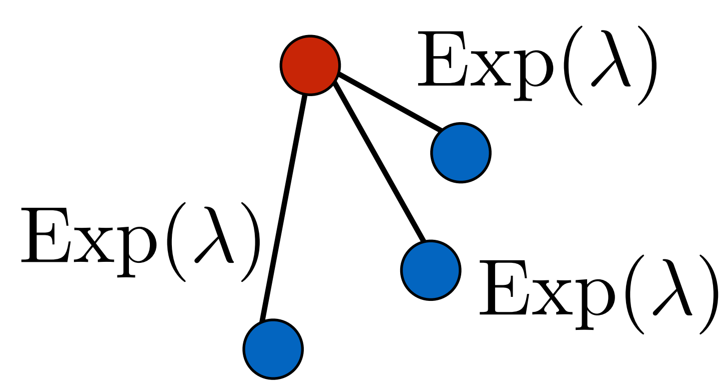


A simple algorithm

- The probability that a specific node is the first one in a trace is $1/n$.

- The probability that the edge {u,v} is spanned by the first two nodes of the trace is then at least
$$\frac{1}{n \cdot \min(\deg(u), \deg(v))}$$

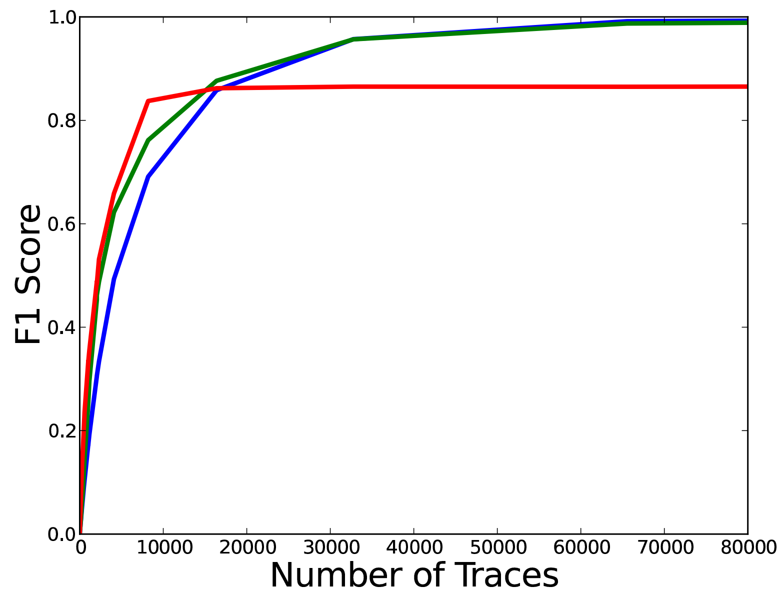
- Any classical tail bound then proves that $O(n\Delta \log n)$ traces are enough to perfectly reconstruct the graph.

The first-edge algorithm

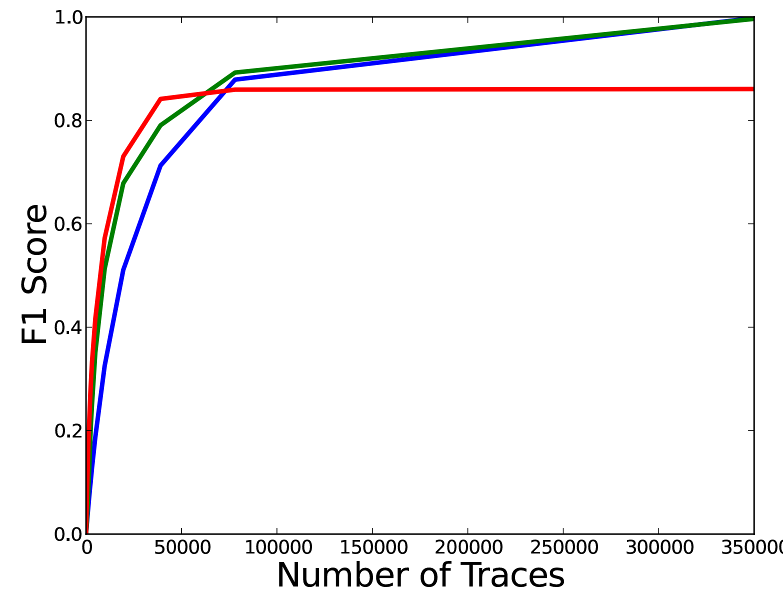
- Works for any (uniform) edge waiting time distribution.

- It is competitive with more complex machine-learning based algorithms (Gomez-Rodriguez, Leskovec, Krause *[KDD'10]*).

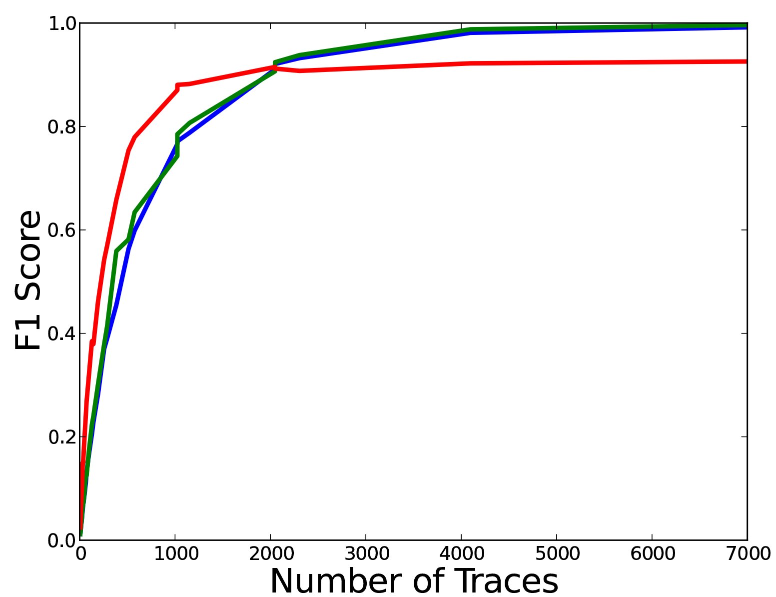
Performances



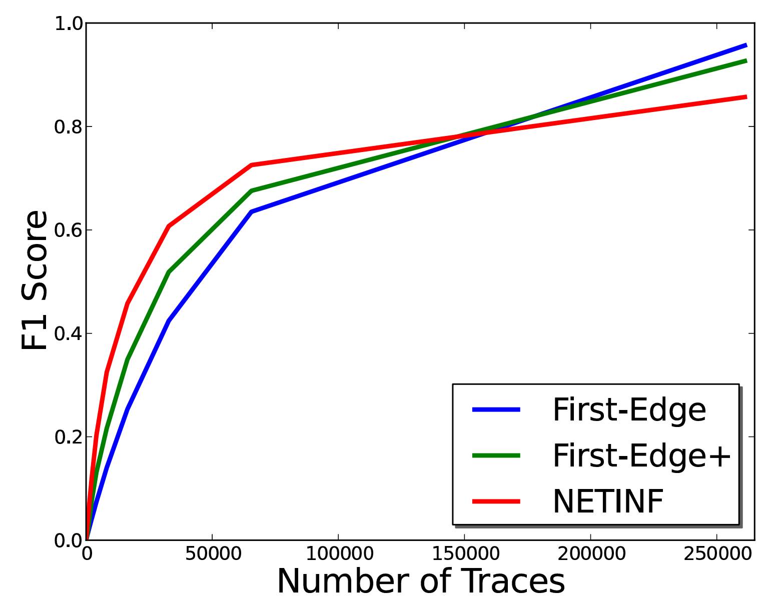
(a) Barabasi-Albert



(b) Facebook-Rice Undergrad



(c) Power-Law Tree



(d) $G_{n,p}$

Our Contributions

Abrahao, Chierichetti, Kleinberg, Panconesi *[KDD'13]*

- We show that $O\left(n\Delta\log n\right)$ traces are sufficient for reconstruction.
- We show that, in some cases, $\Omega\left(\frac{n\Delta}{\log^2 n}\right)$ traces are necessary.

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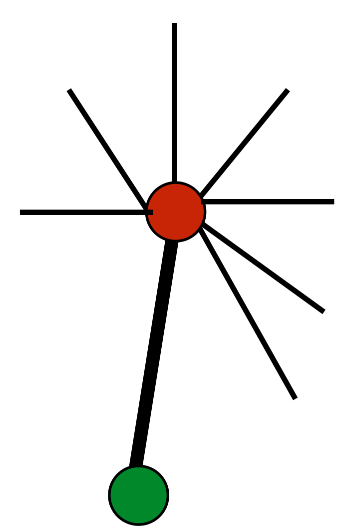

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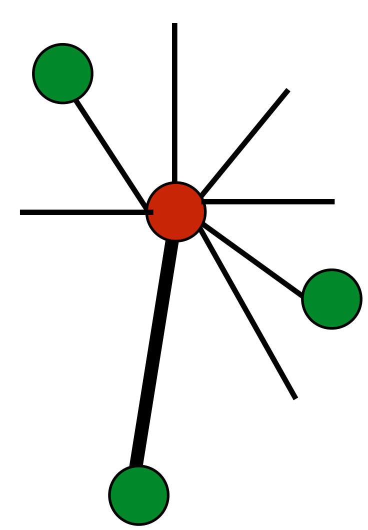
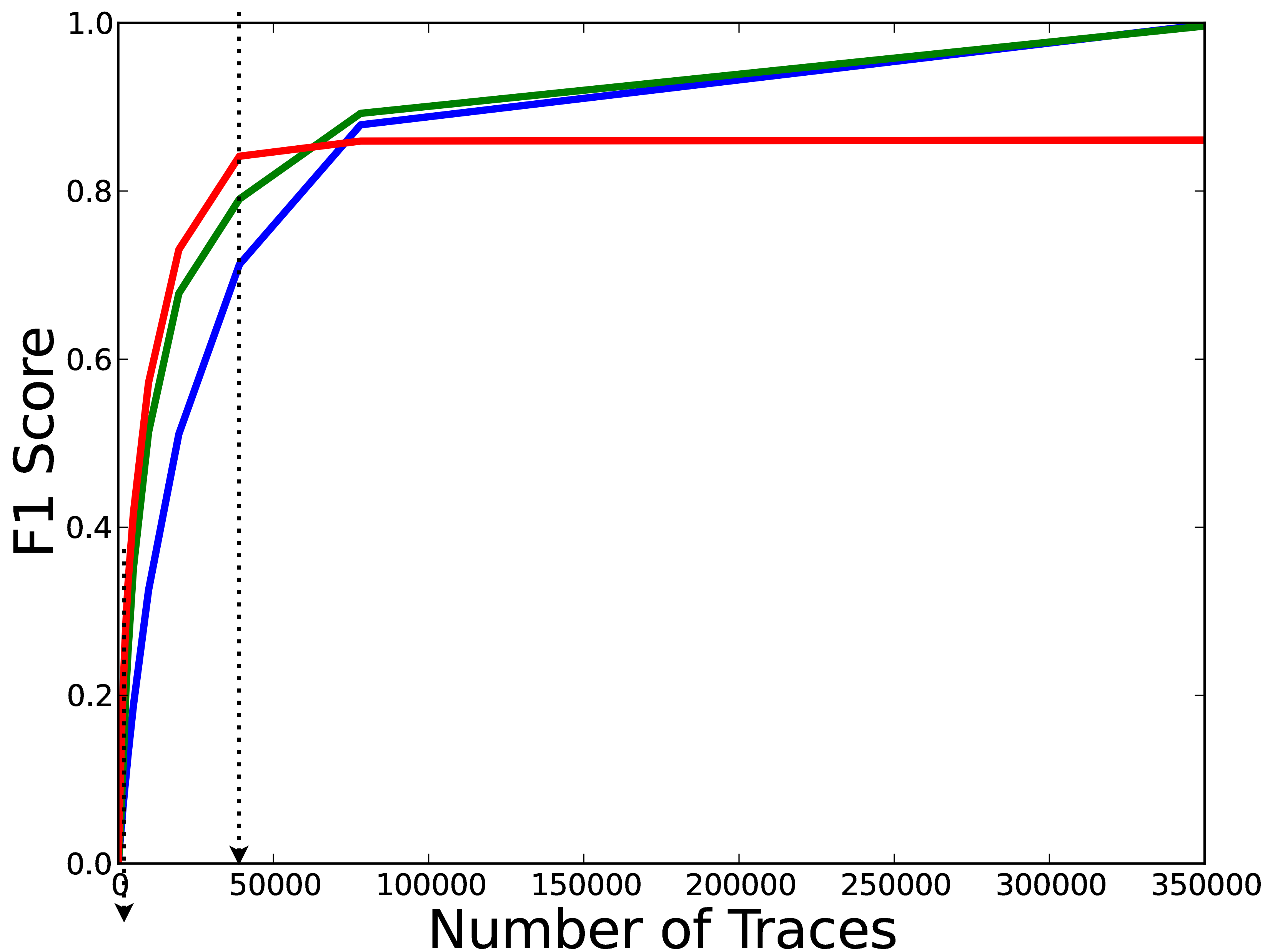


Why ~ 40K traces?

Most edges are incident on nodes having small degree



Facebook-Rice Undergrad

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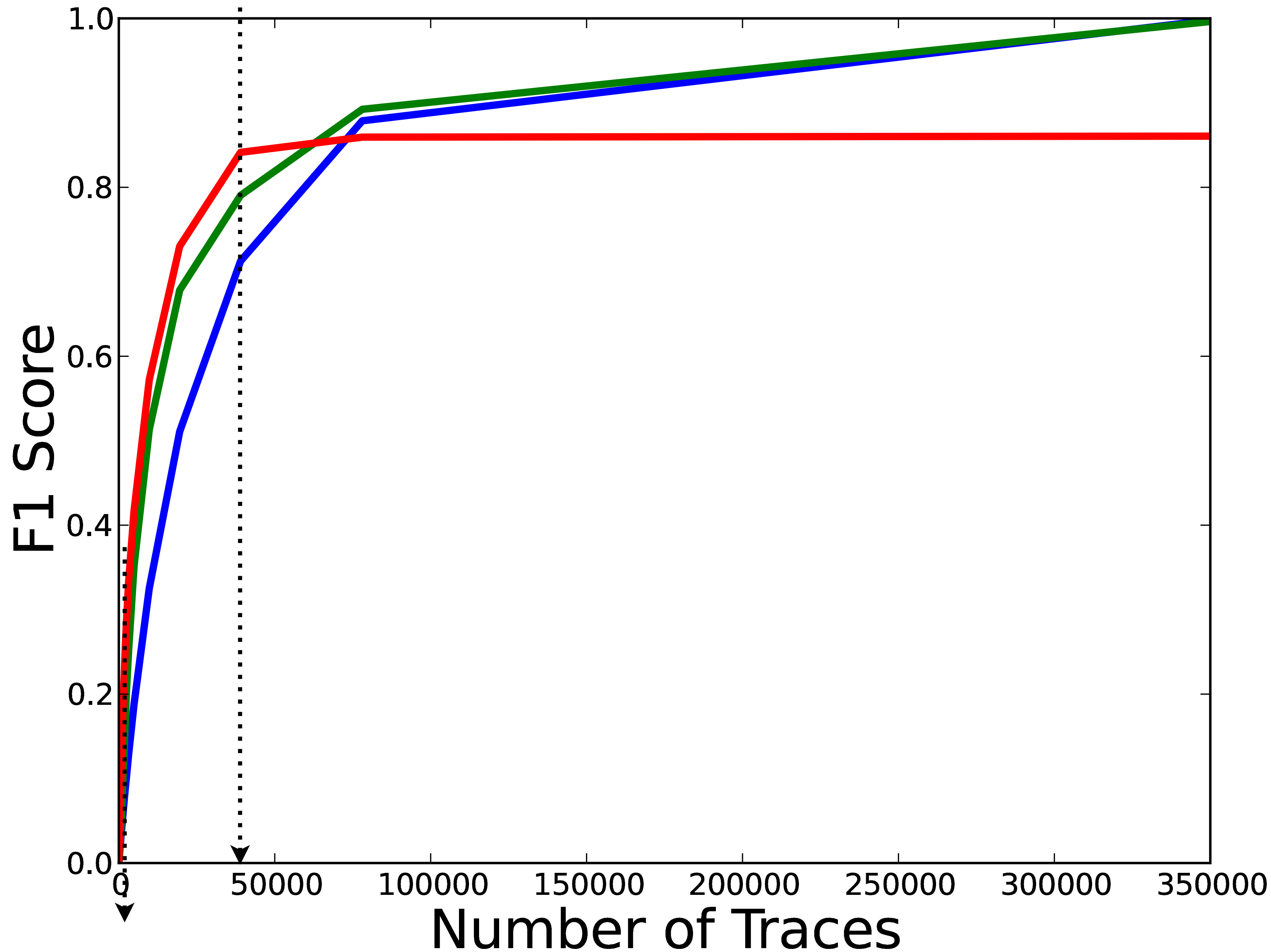

Facebook-Rice Undergrad

Back-of-envelope calculation

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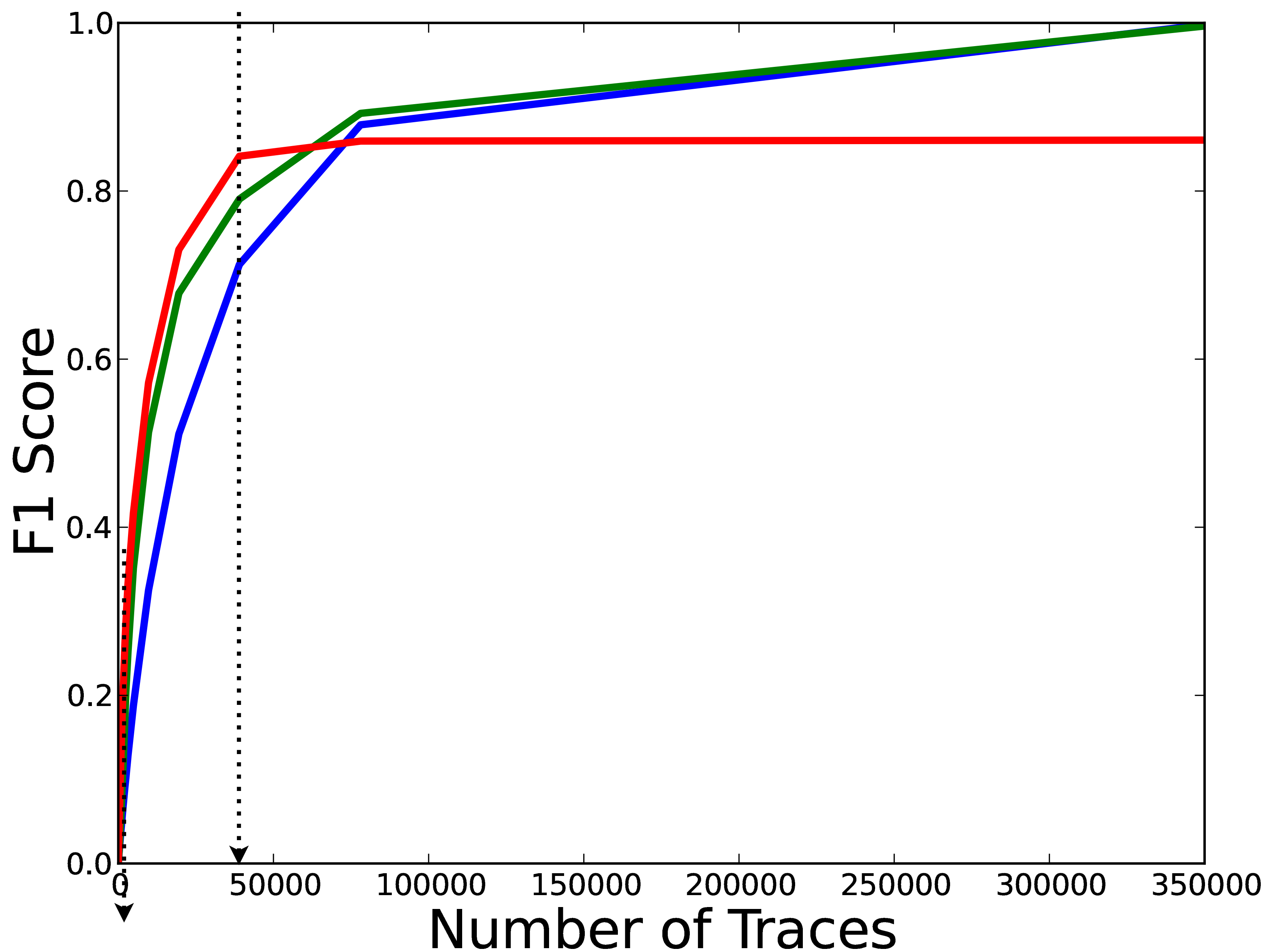

Facebook-Rice Undergrad

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The graph has 1220 nodes and average degree equal to 35.41…

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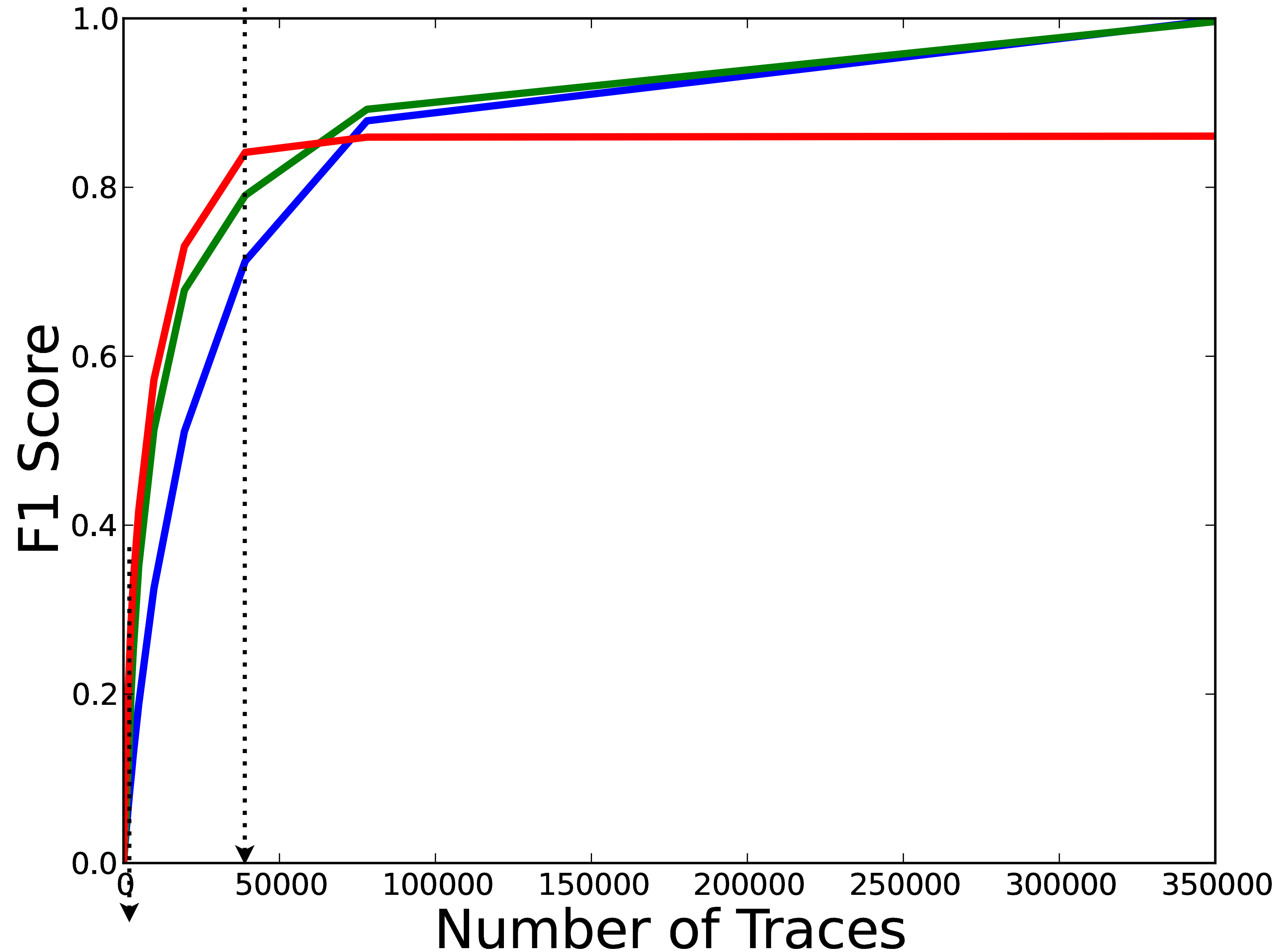

Facebook-Rice Undergrad

Back-of-envelope calculation

The graph has 1220 nodes and average degree equal to 35.41…

1220 x 35.41 = 43200.2

Too many traces?

- In some cases such a large number of traces is not available.

- In those cases, one can still try to reconstruct some (less precise) missing information about the network.

E-mail Activisim

Joint with

Jon Kleinberg, David Liben-Nowell

Internet Activism

- Very important phenomenon
- Incomplete Traces
- Chain Letter Petitions:
  how to estimate the reach?

NPR Chain Letter

PBS, NPR (National Public Radio), and the arts are facing major cutbacks in funding. In spite of the efforts of each station to reduce spending costs and streamline their services, the government officials believe that the funding currently going to these programs is too large a portion of funding for something which is seen as "unworthwhile."

[...]

When this issue comes up in 1996, the funding will be determined for fiscal years 1996-1998.

The only way that our representatives can be aware of the base of support or PBS and funding for these types of programs is by making our voices heard.

Please add your name to this list if you believe in what we stand for. This list will be forwarded to the President of the United States, the Vice President of the United States, the House of Representatives and Congress.

If you happen to be the 50th, 100th, 150th, etc. signer of this petition, please forward to: kubi7975@blue.univnorthco.edu . This way we can keep track of the lists and organize them. Forward this to everyone you know, and help us to keep these programs alive.

Thank you.

1. Elizabeth Weinert, student, University of Northern Colorado, Greeley, Colorado.
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5. Andrew H. Knightly, Los Angeles, CA
6. Aaron C. Yeater, Somerville, MA
7. Tobie M. Cornejo, Washington, DC
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9. Eric W. Fish, Williamstown, MA
10. Courtney E. Estill, Hamilton College, NY
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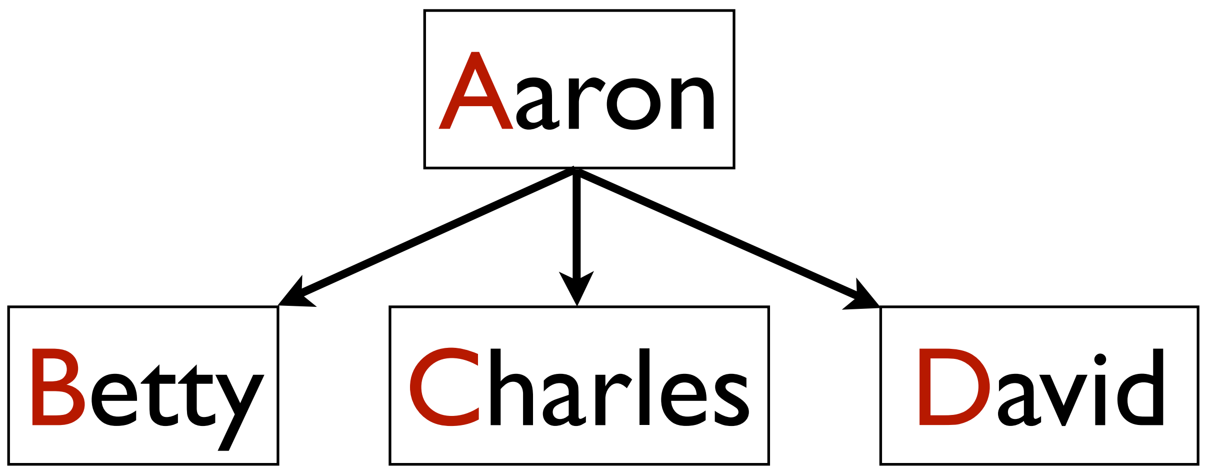
[...]

Chain Letters

Aaron

Chain Letters



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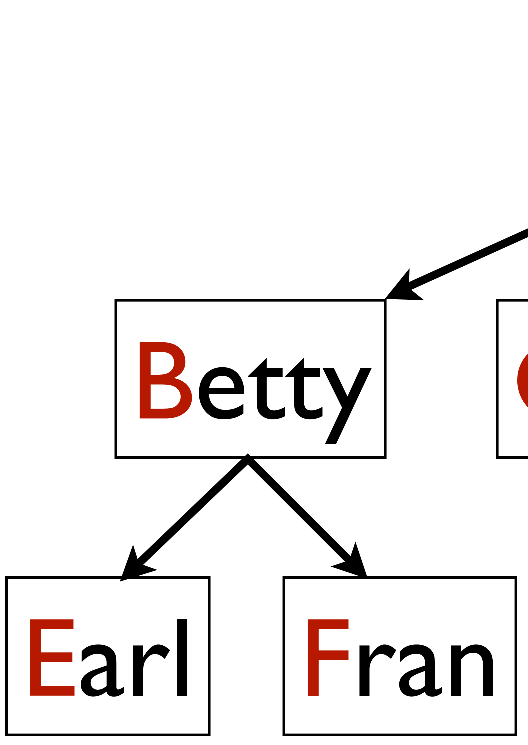

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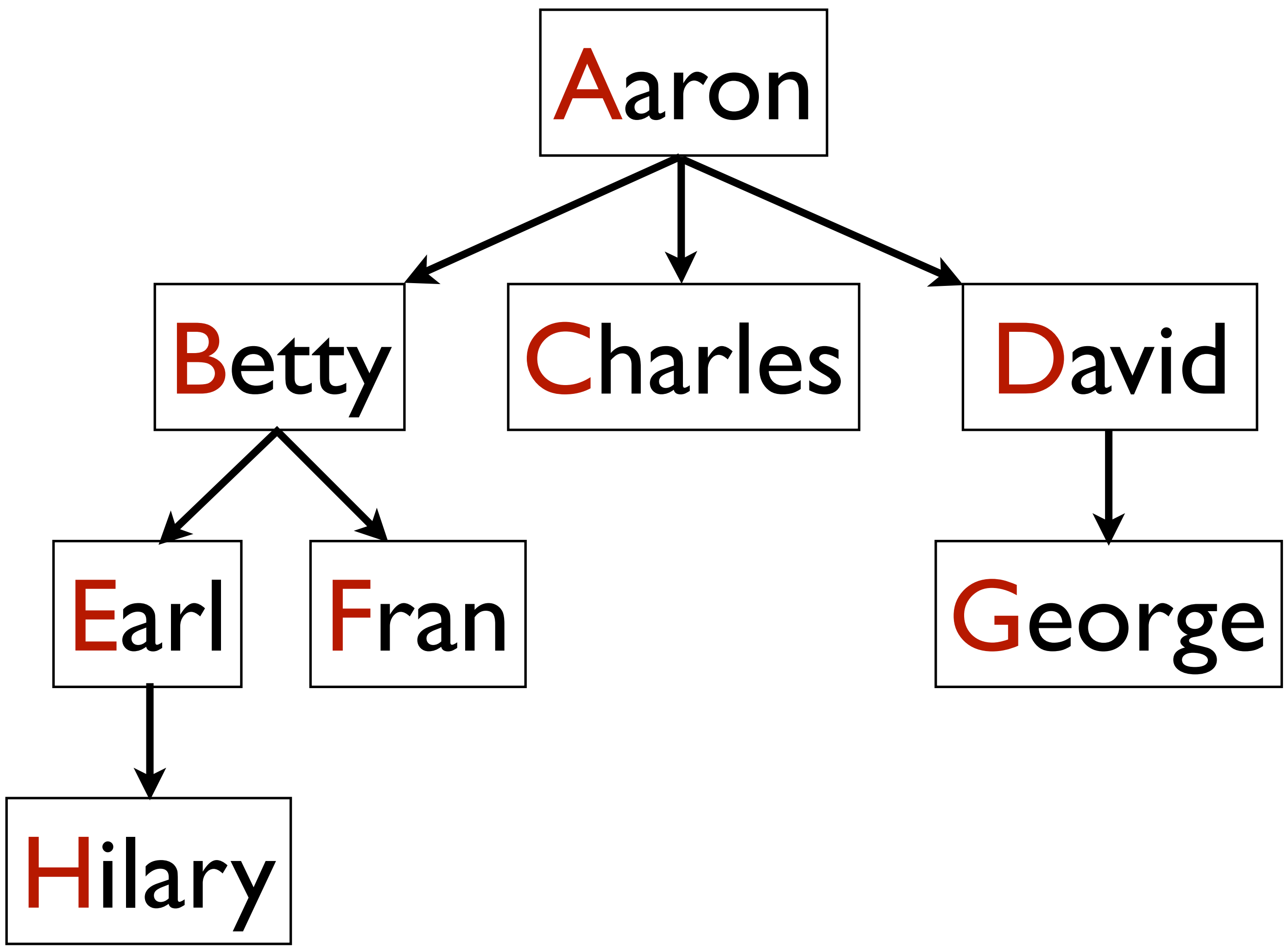
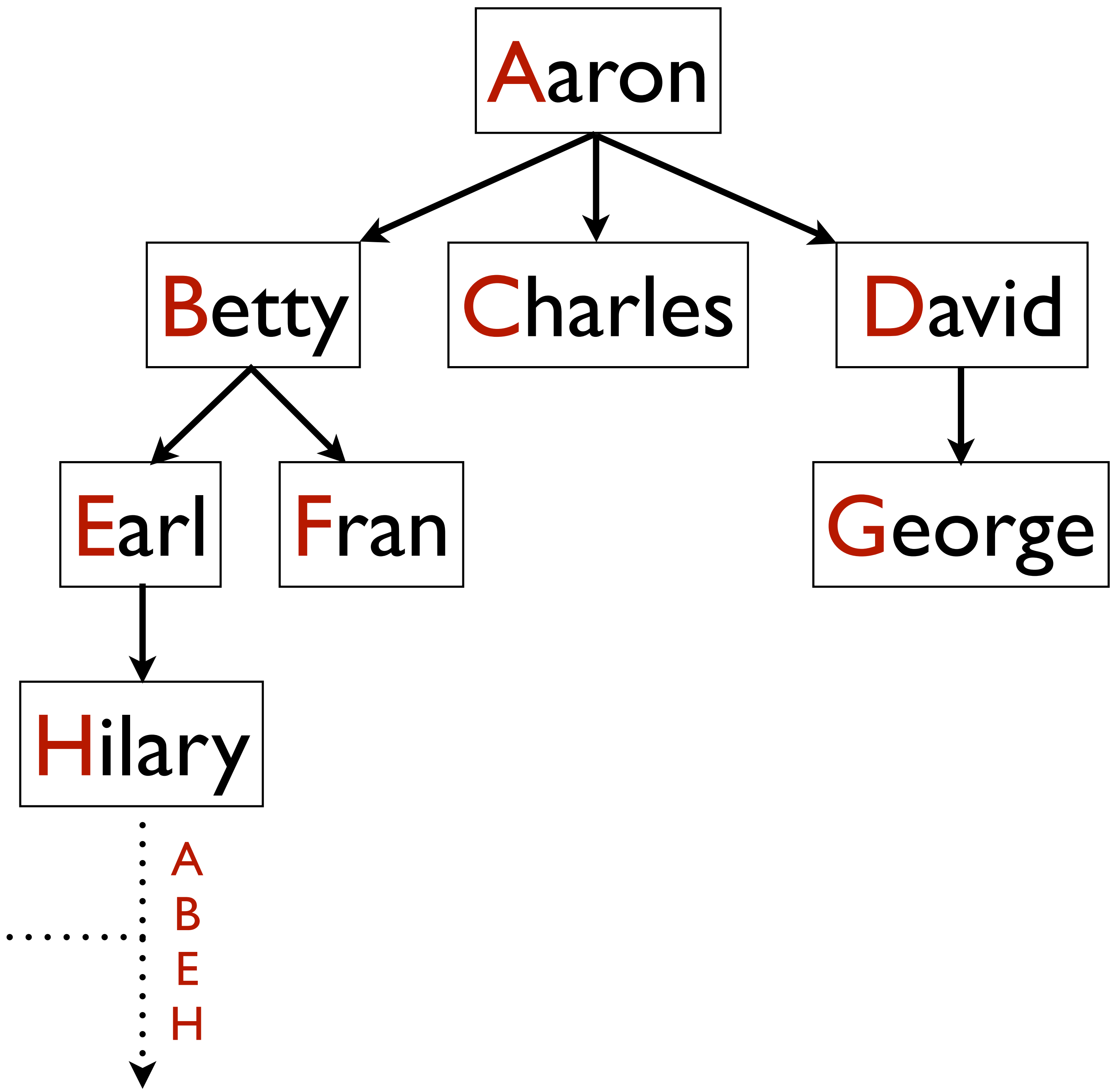


Chain Letters

Dear all,
an important
cause demands
your attention.
[…]
If you care
about this, add
your name and
forward this
letter.
[…]
The signers,
Aaron
Betty
Earl
Hilary



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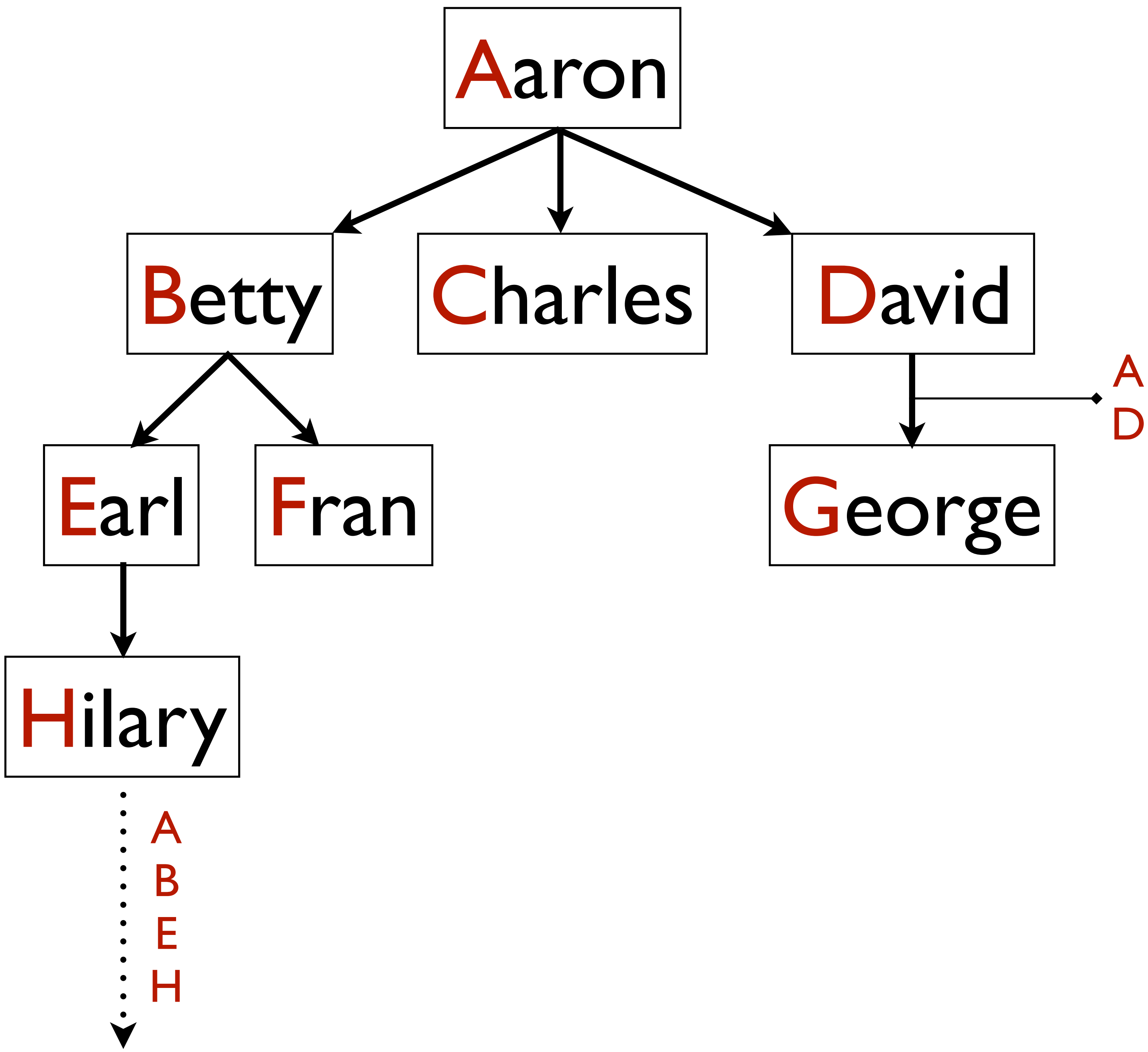

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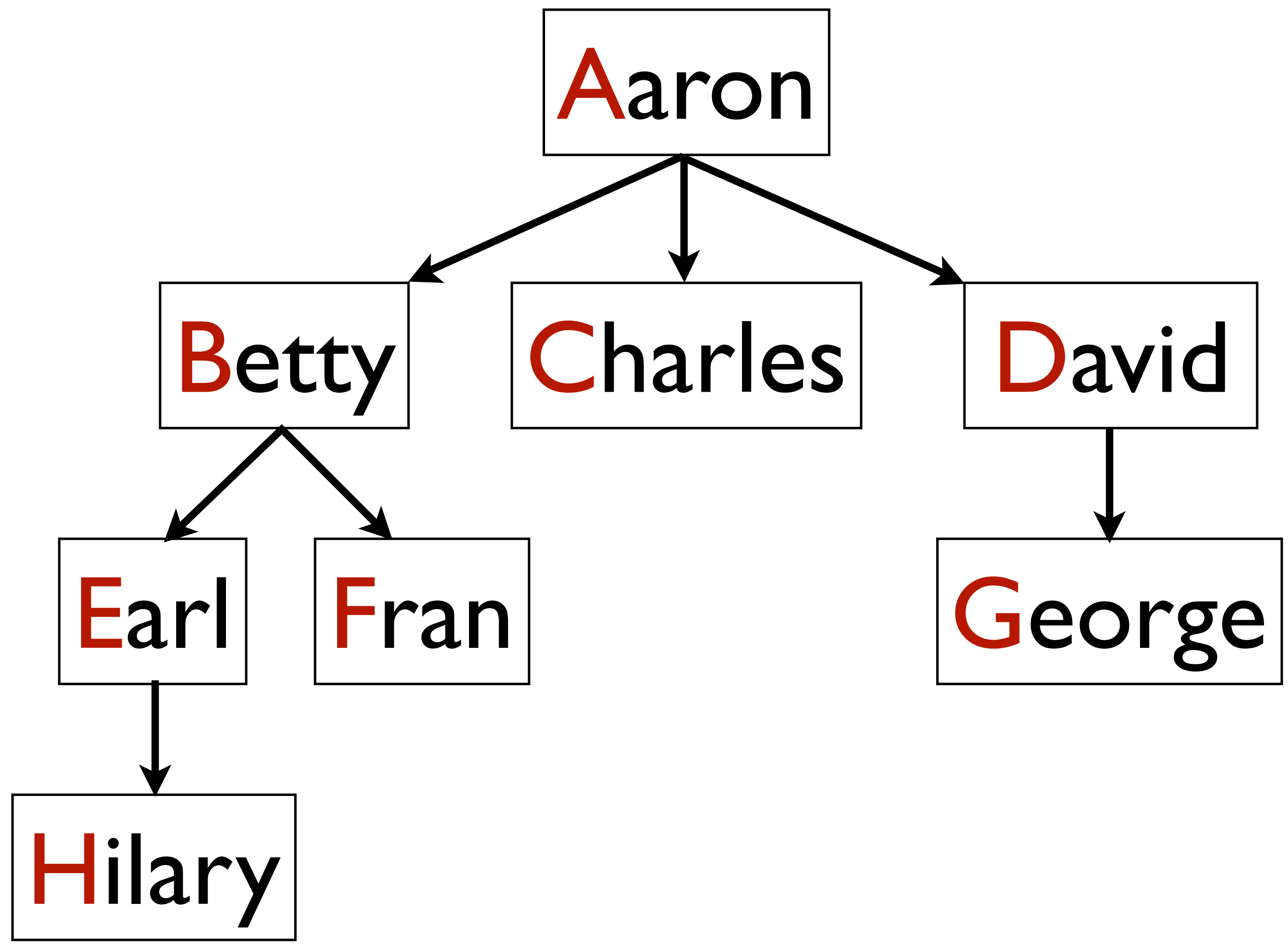

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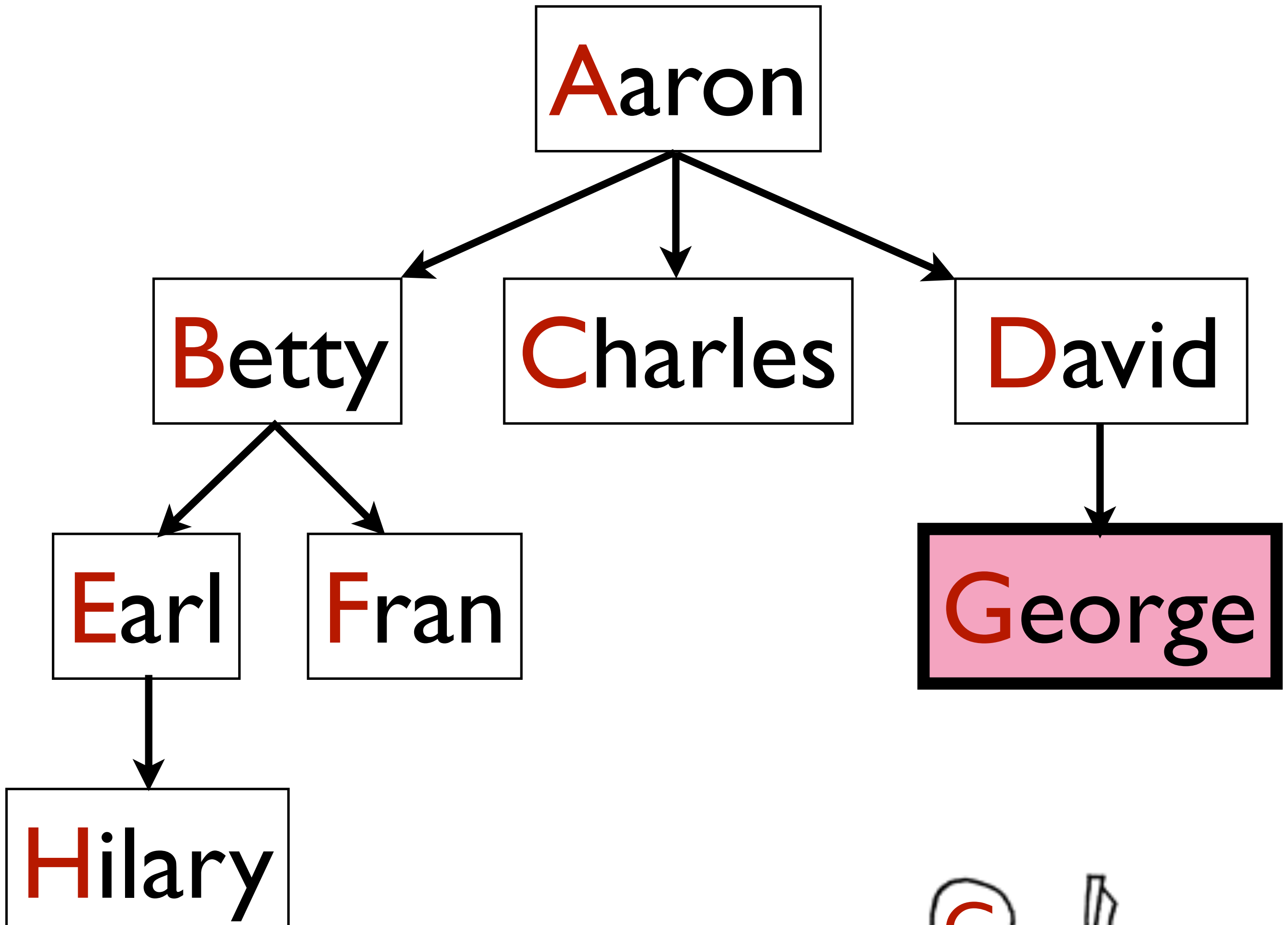

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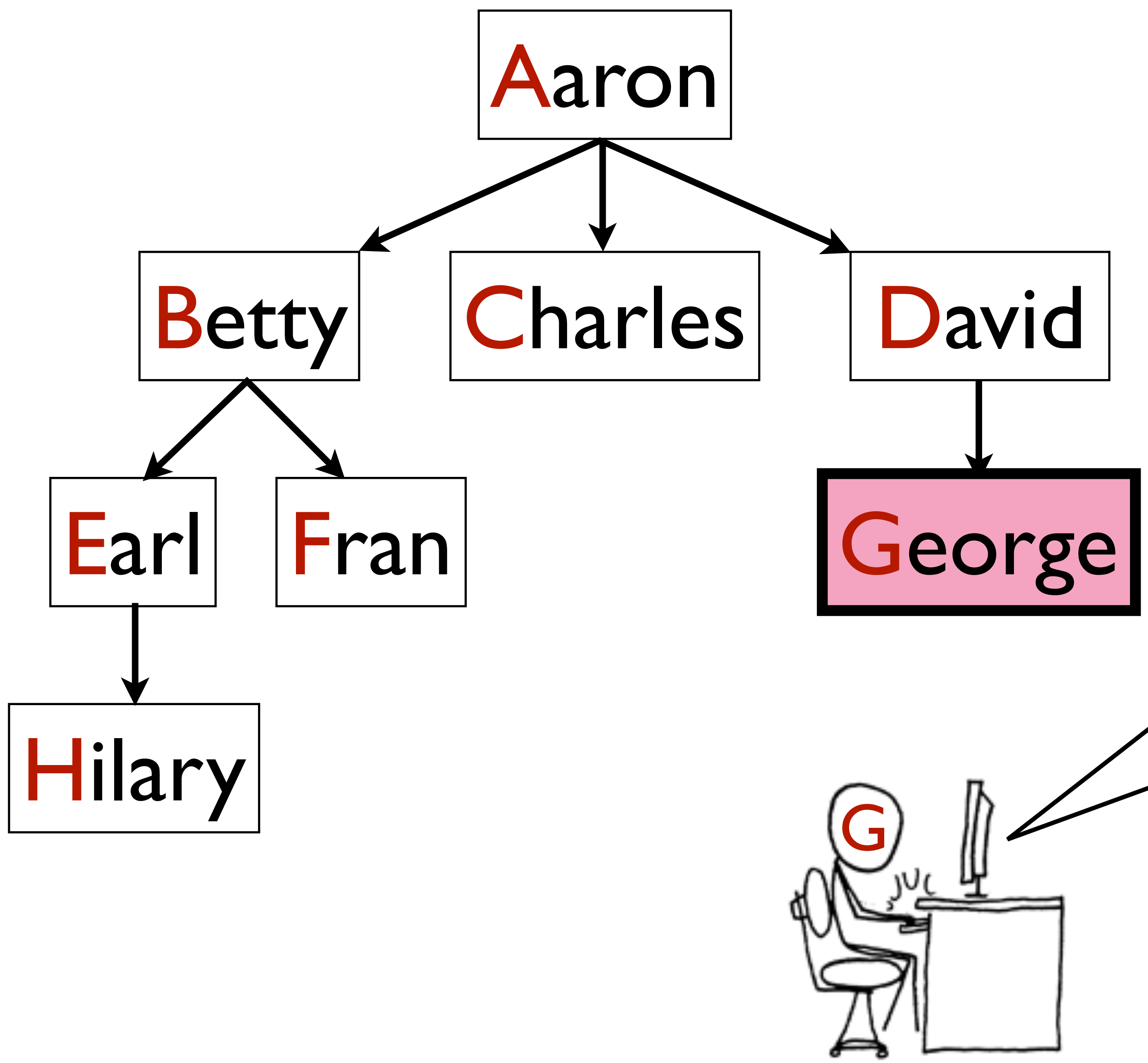
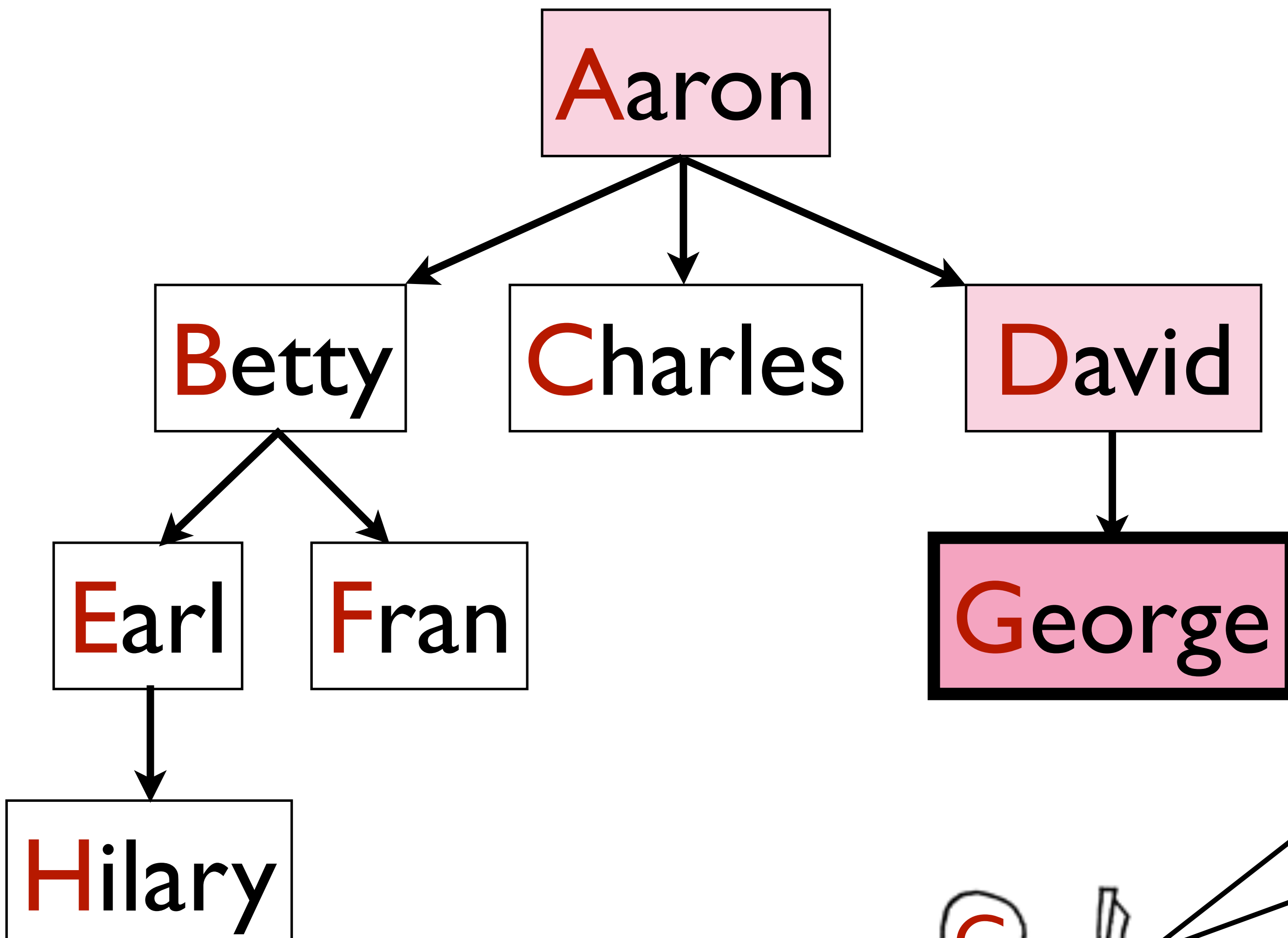


George's Blog

Here is something that I sent to my friends today:

```
Dear all,
an important
cause demands
your attention.
[…]
If you care about
this, add your
name and forward
this letter.
[…]
The signers,
Aaron
David
George
```

Chain Letters



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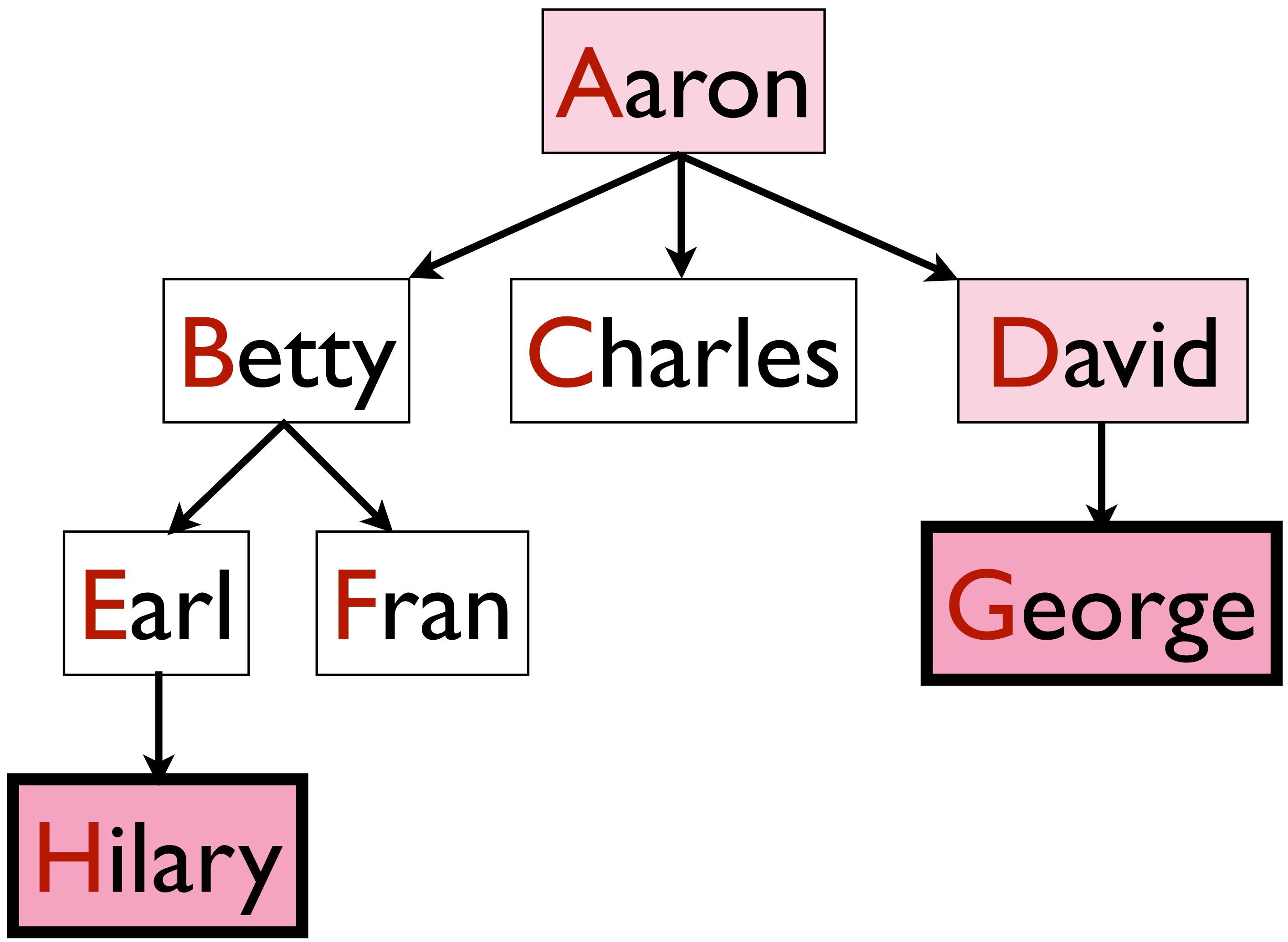

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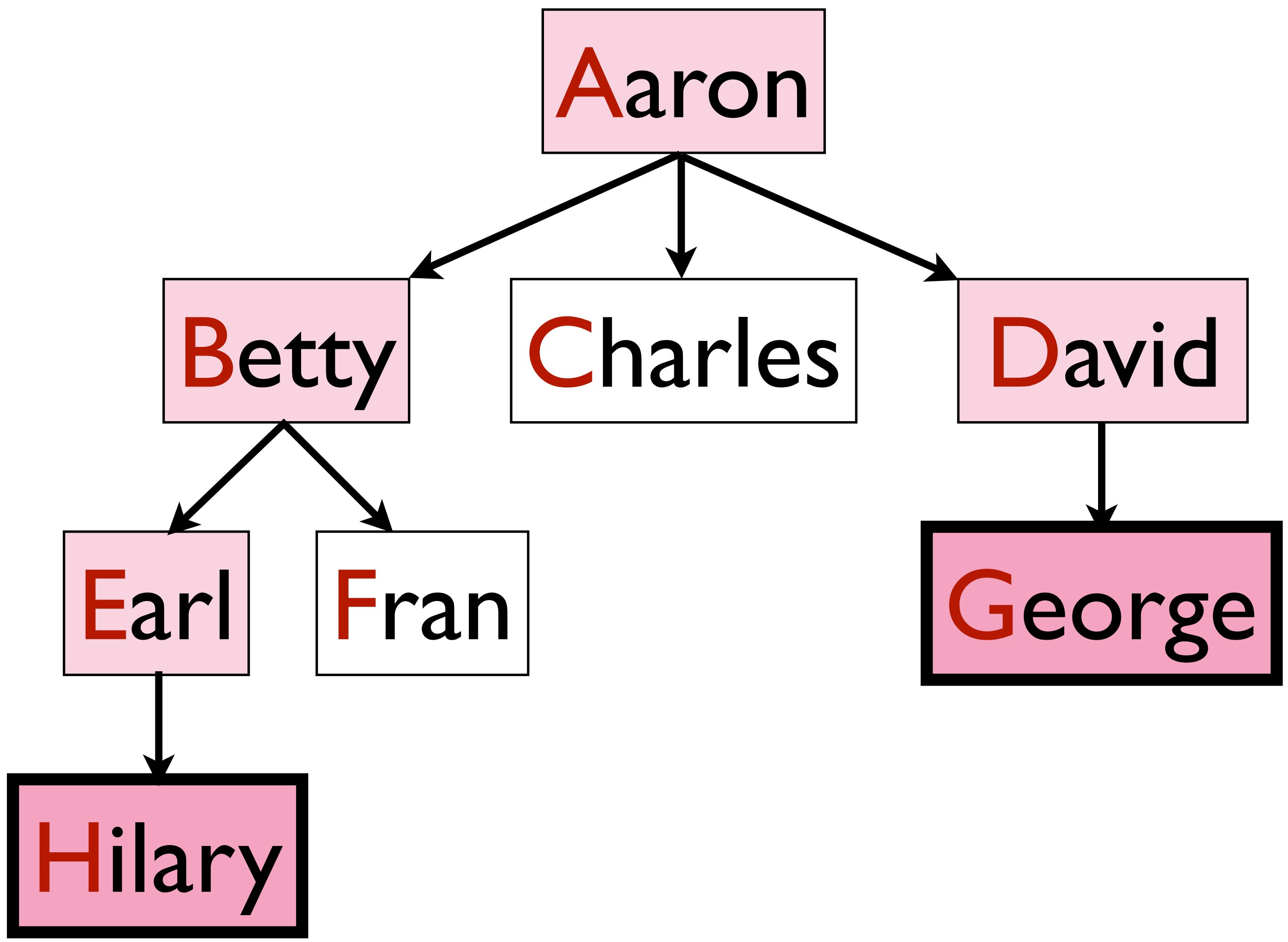

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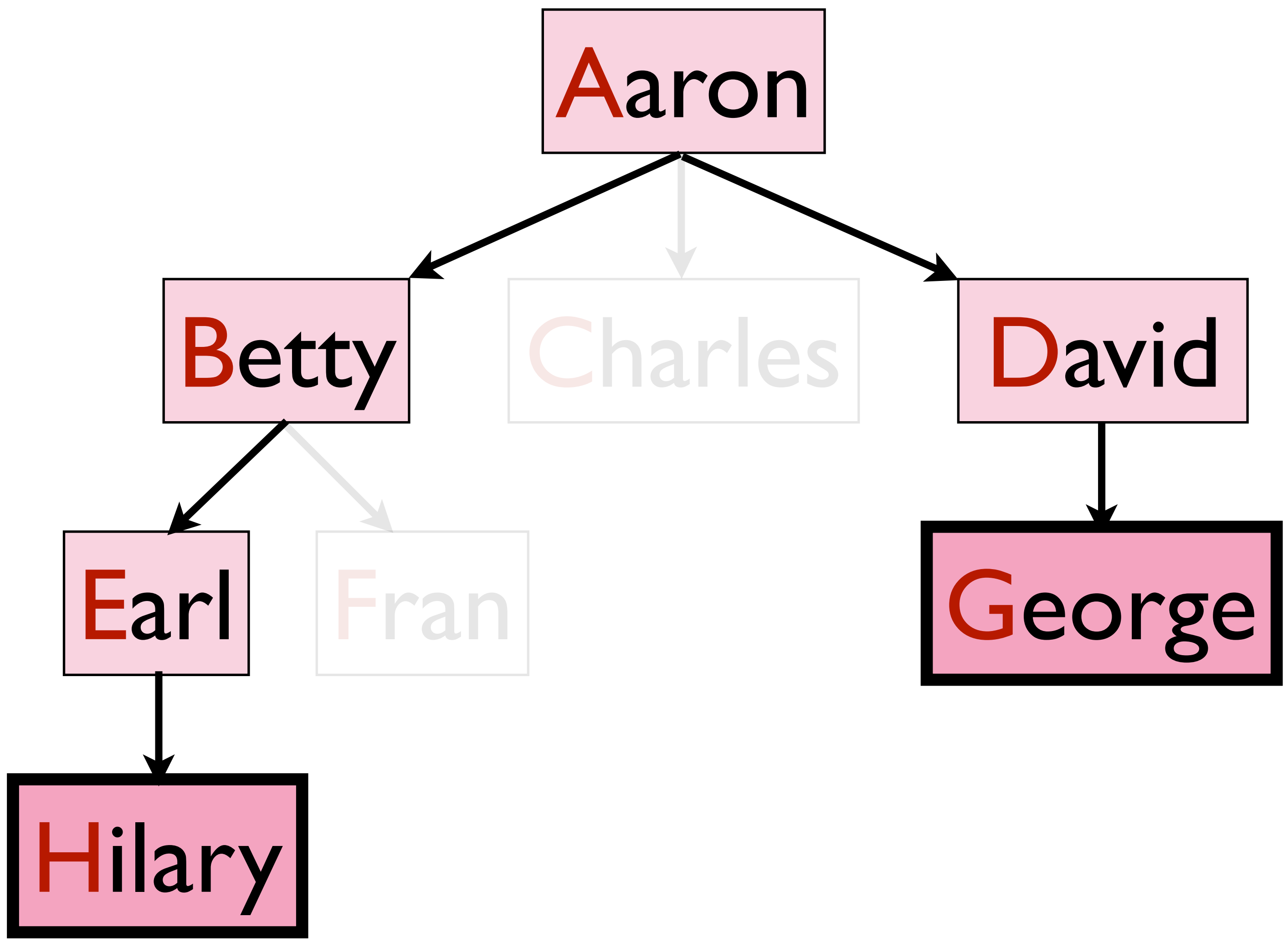


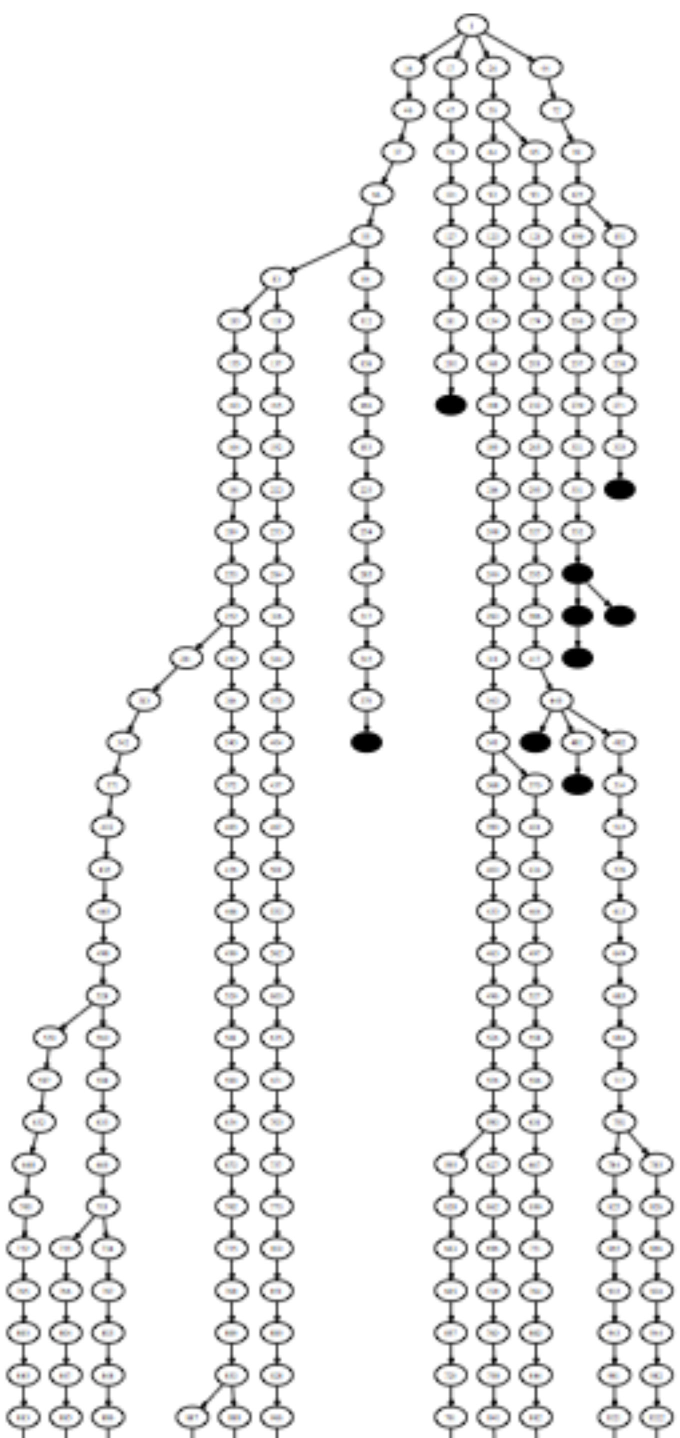
George and Hilary, by *exposing* their emails, *revealed* a subtree of the Chain Letter tree.

Real-World Chain Letters' Tree

- *[Liben-Nowell, Kleinberg, PNAS'08]*, mined
  - web-accessible mailing-lists,
  - blog posts.
- They obtained some *"exposed"* nodes of two Chain Letters' trees, and
- they produced two *"revealed"* trees.

NPR revealed tree

Liben-Nowell, Kleinberg, PNAS'08



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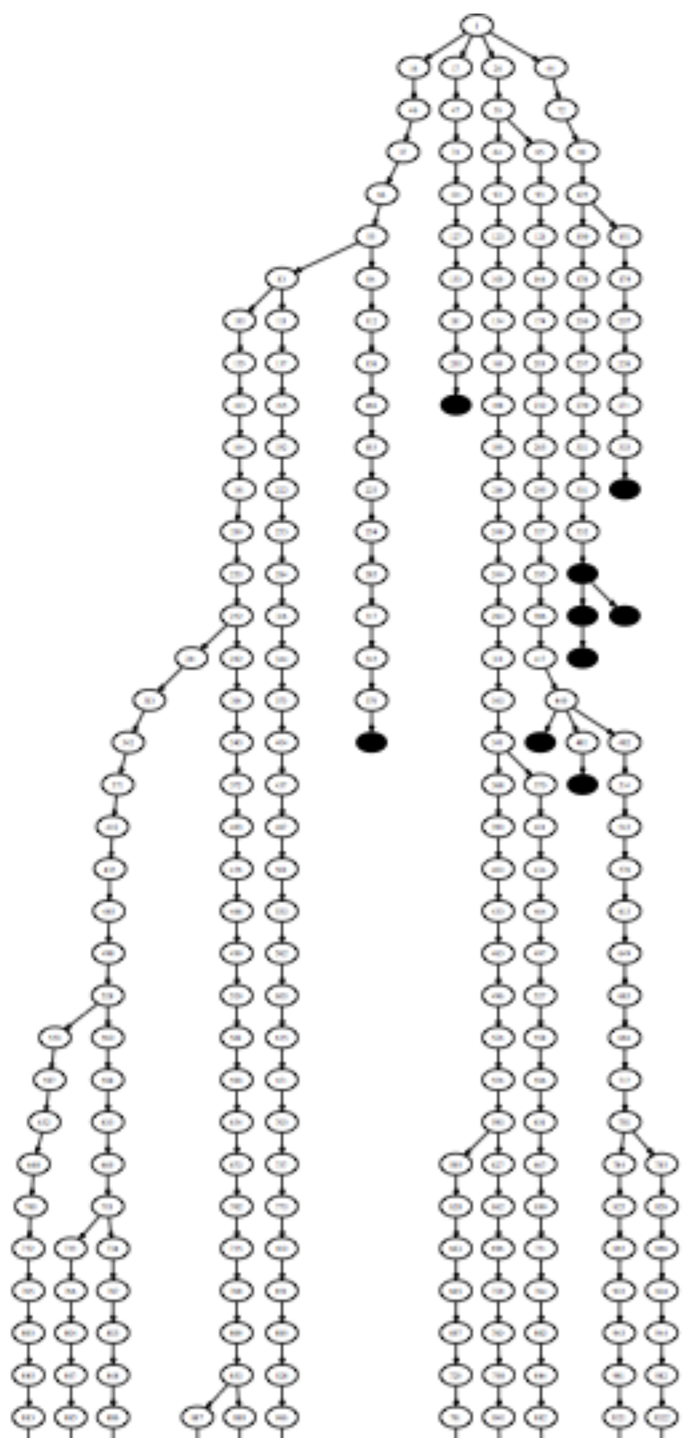

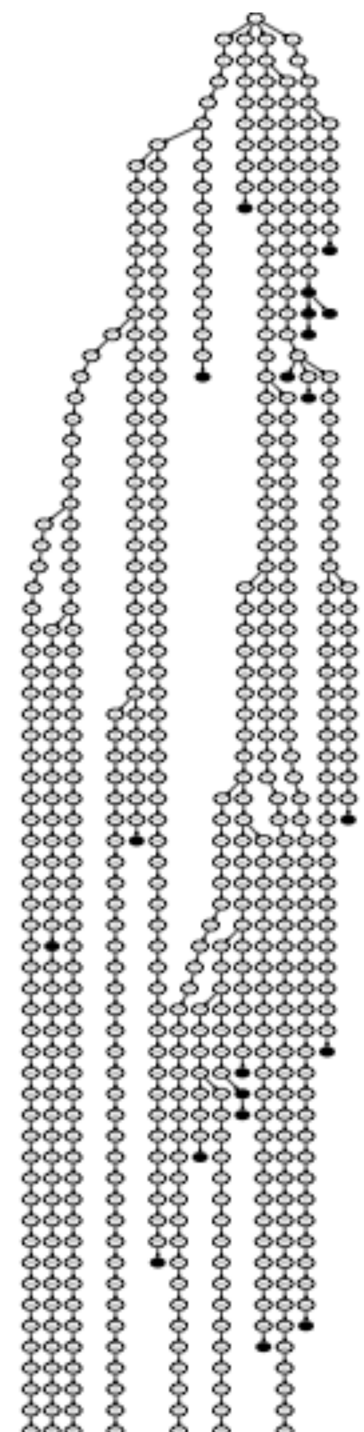

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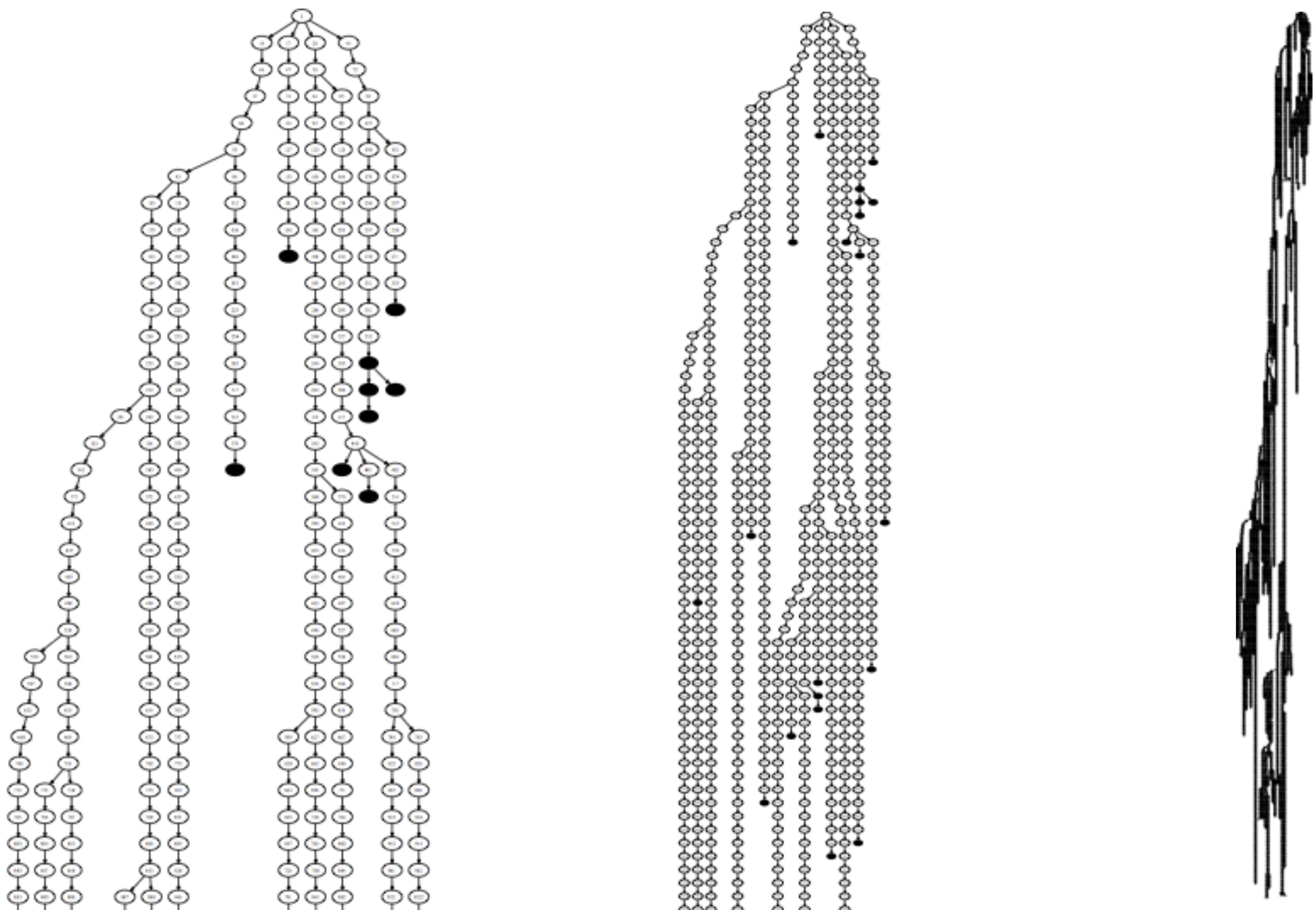

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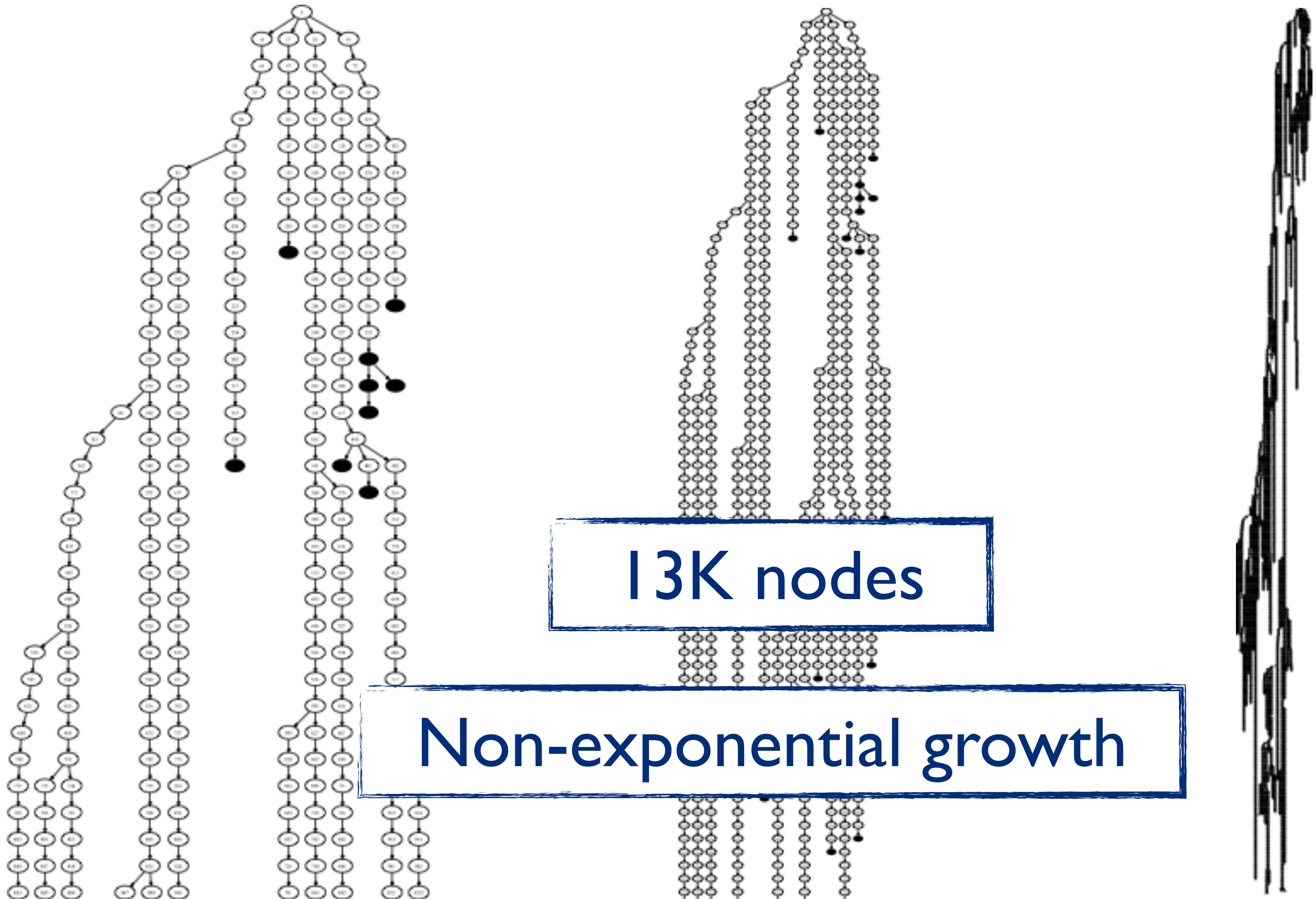

13K nodes

Non-exponential growth

Iraq Chain Letter

Dear all:

The US Congress has just authorized the President of the US to go to war against Iraq. The UN is gathering signatures in an effort to avoid this tragic world event.

Please consider this an urgent request: UN Petition for Peace - Stand for Peace. Islam is not the Enemy.
War is NOT the Answer.

Today we are at a point of imbalance in the world and are moving toward what may be the beginning of a THIRD WORLD WAR.

Please COPY (rather than Forward) this e-mail in a new message, sign at the end of the list, and send it to all the people whom you know.

If you receive this list with more than 500 names signed, please send a copy of the message to:

usa@un.int
president@whitehouse.gov

Even if you decide not to sign, please consider forwarding the petition
on instead of
deleting it.

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10) Tassadite FAVRIE, Grenoble, FRANCE
11) Loic GODARD, St Ismier, FRANCE
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22) Jean IMBER! T, Marseille, FRANCE
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24) Anna BASSOLS, Barcelona, Catalonia
25) Mireia DUNACH, Barcelona, Catalonia
26) Michel VILLAZ, Grenoble, France
27) Pages Frederique, Dijon, France
28) Rodolphe FISCHMEISTER,Chatenay-Malabry, France
29) Francois BOUTEAU, Paris, France
30) Patrick PETER, Paris, France
31) Lorenza RADICI, Paris, France
32) Monika Siegenthaler, Bern, Switzerland
33) Mark Philp,Glasgow,Scotland
34) Tomas Andersson, Stockholm, Sweden
35) Jonas Eriksson, Stockholm, Sweden
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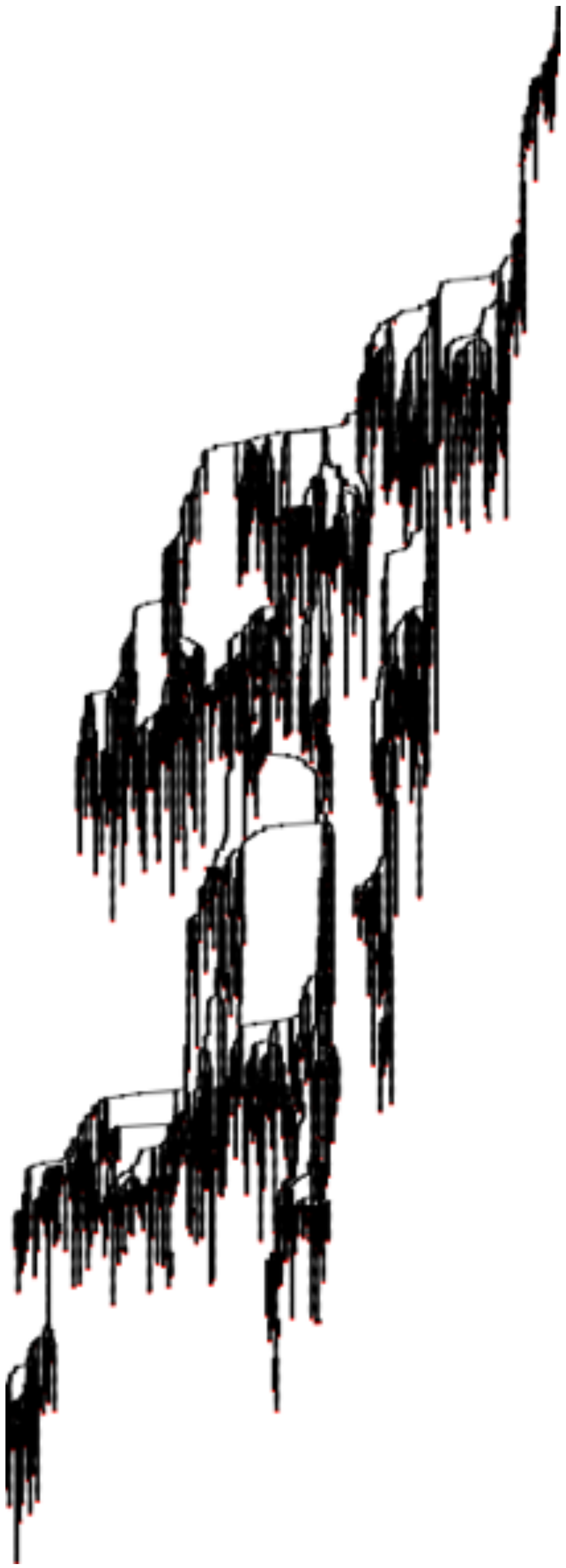
IRAQ revealed tree

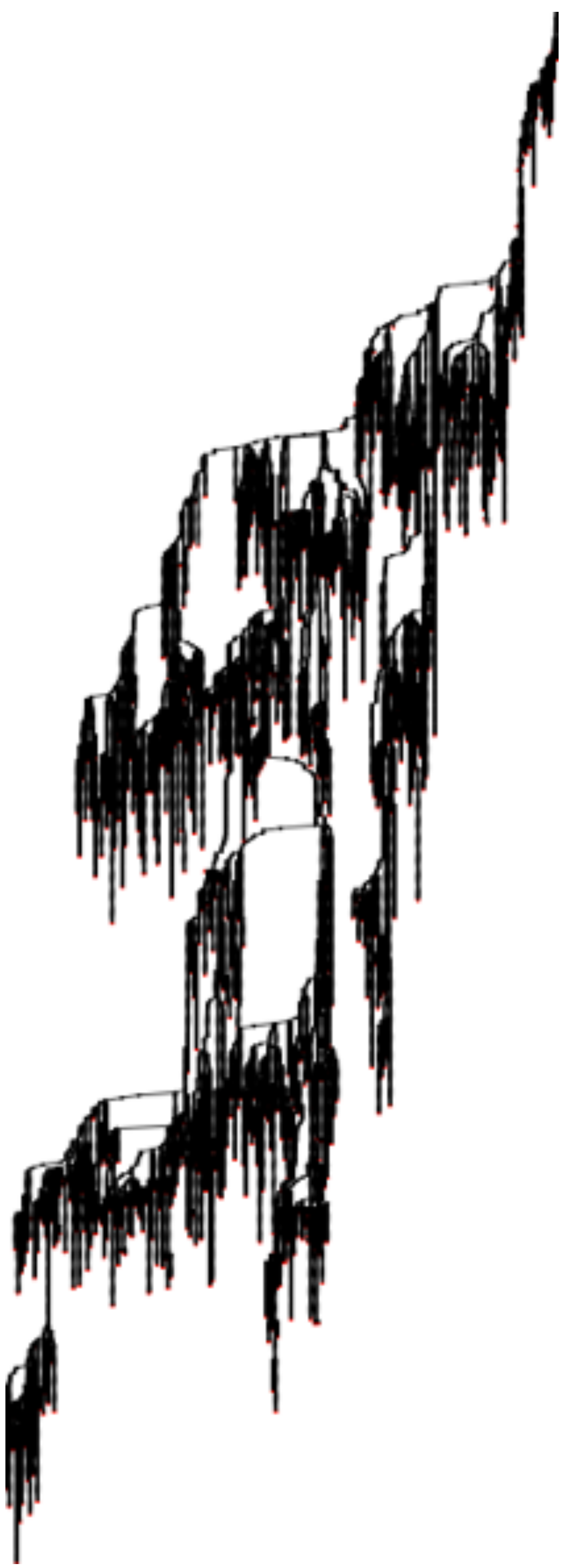
Liben-Nowell, Kleinberg, PNAS'08



18,119 nodes

IRAQ revealed tree

Liben-Nowell, Kleinberg, PNAS'08



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17,079 nodes with one child *(94%)*

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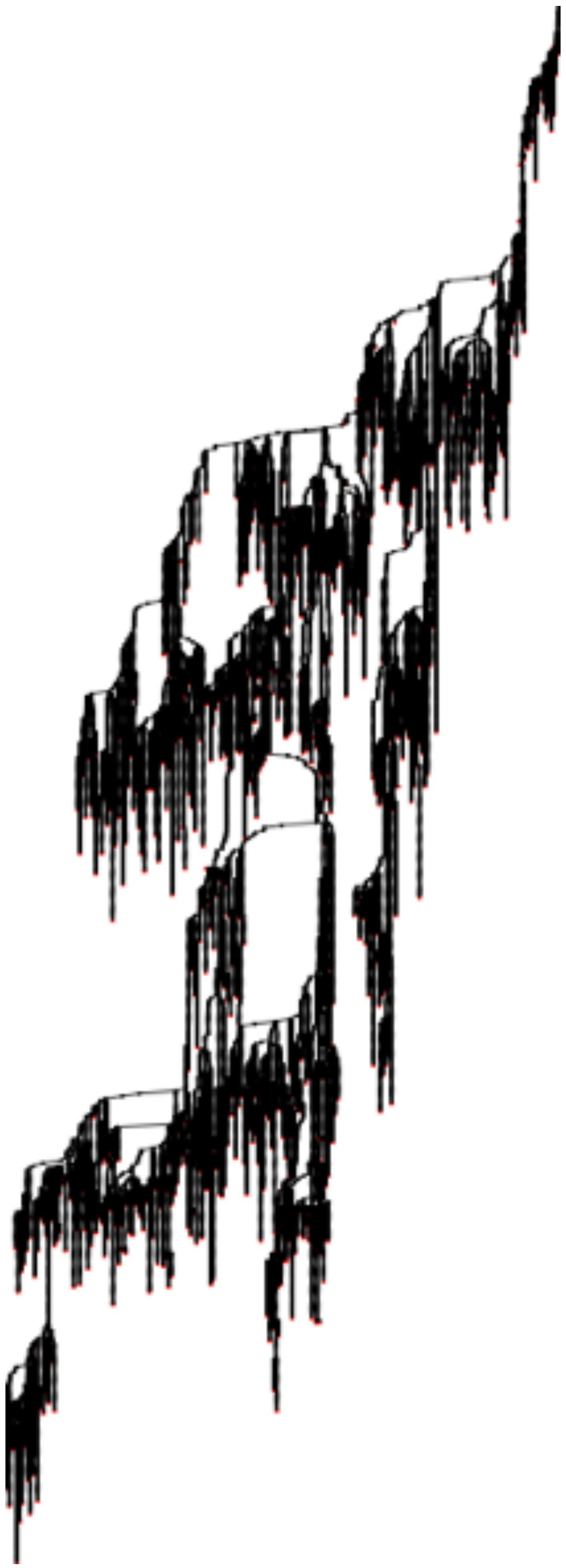

18,119 nodes
17,079 nodes with one child *(94%)*
620 exposed nodes
557 (exposed) leaves

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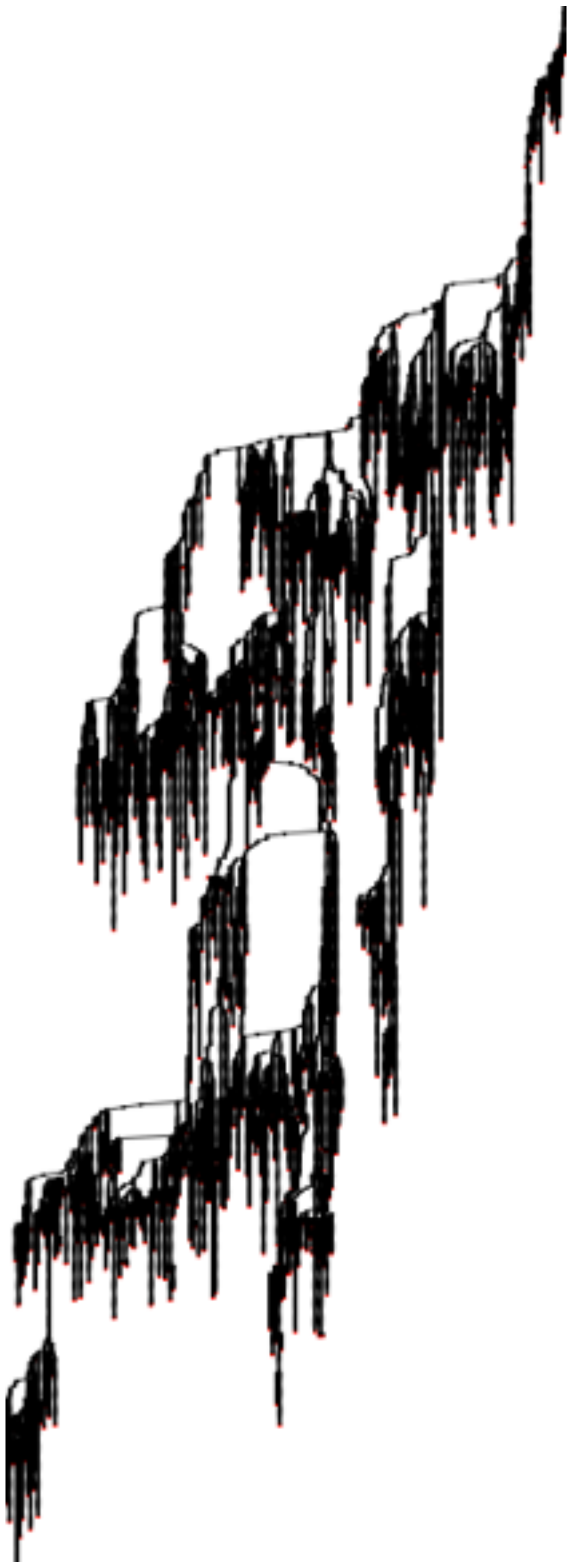

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Why is this fraction so high?

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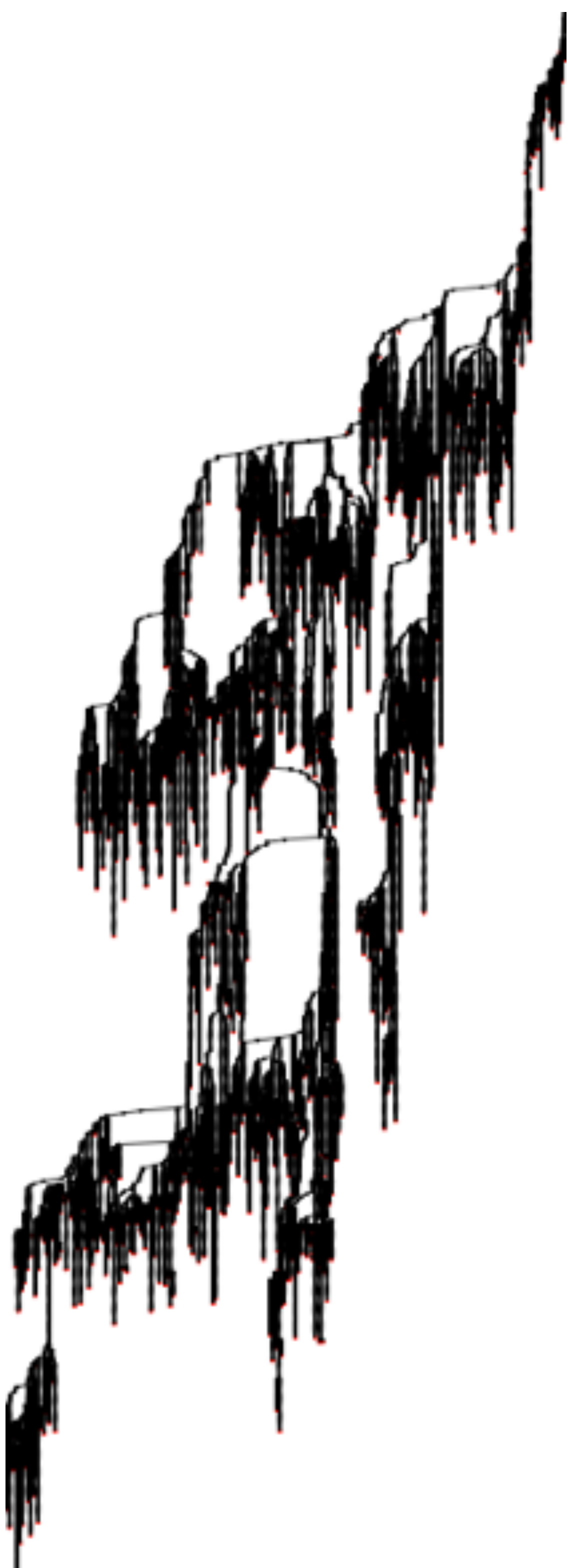

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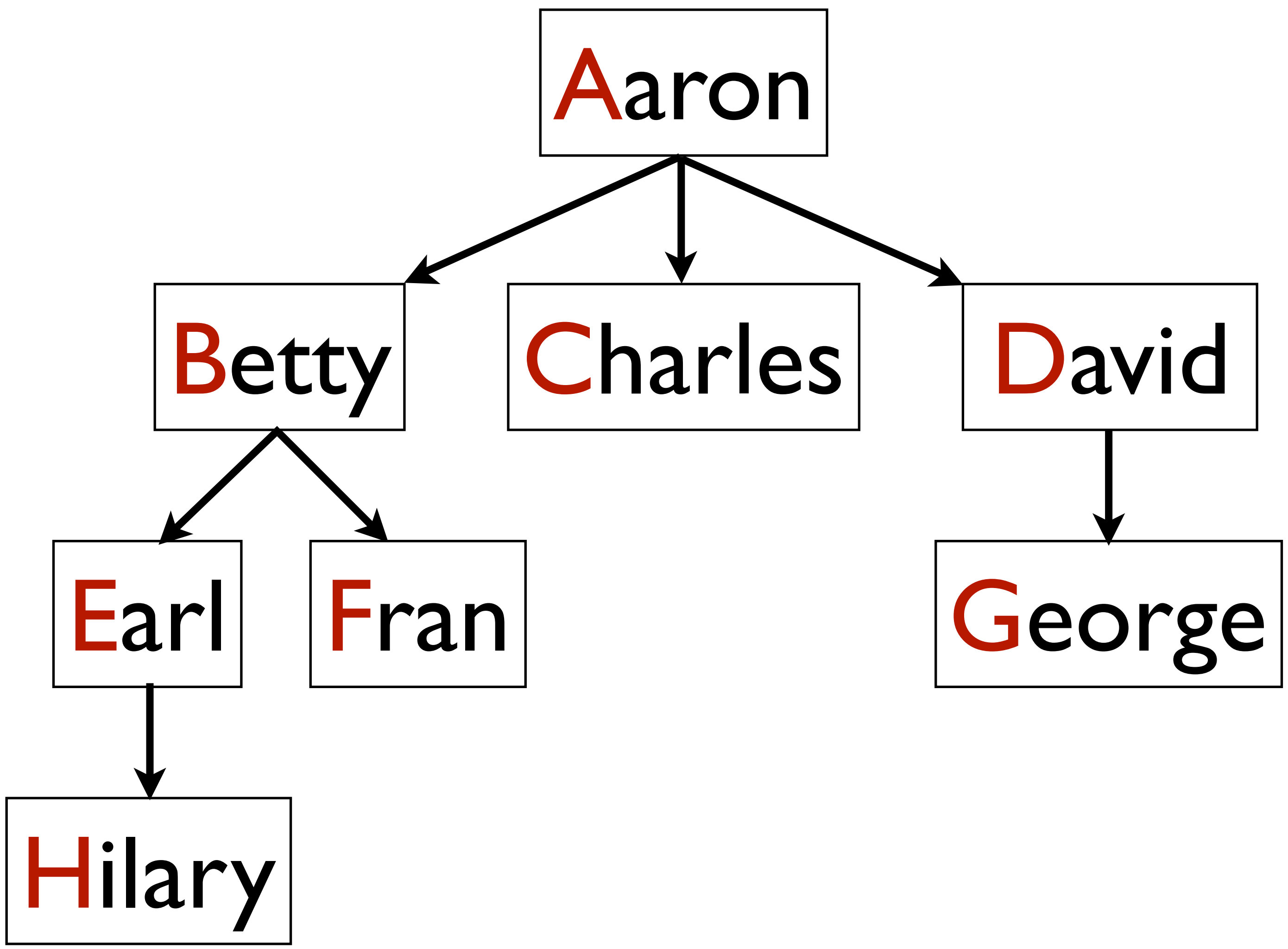
What can we infer about the original, *unknown*, Chain Letter Tree?

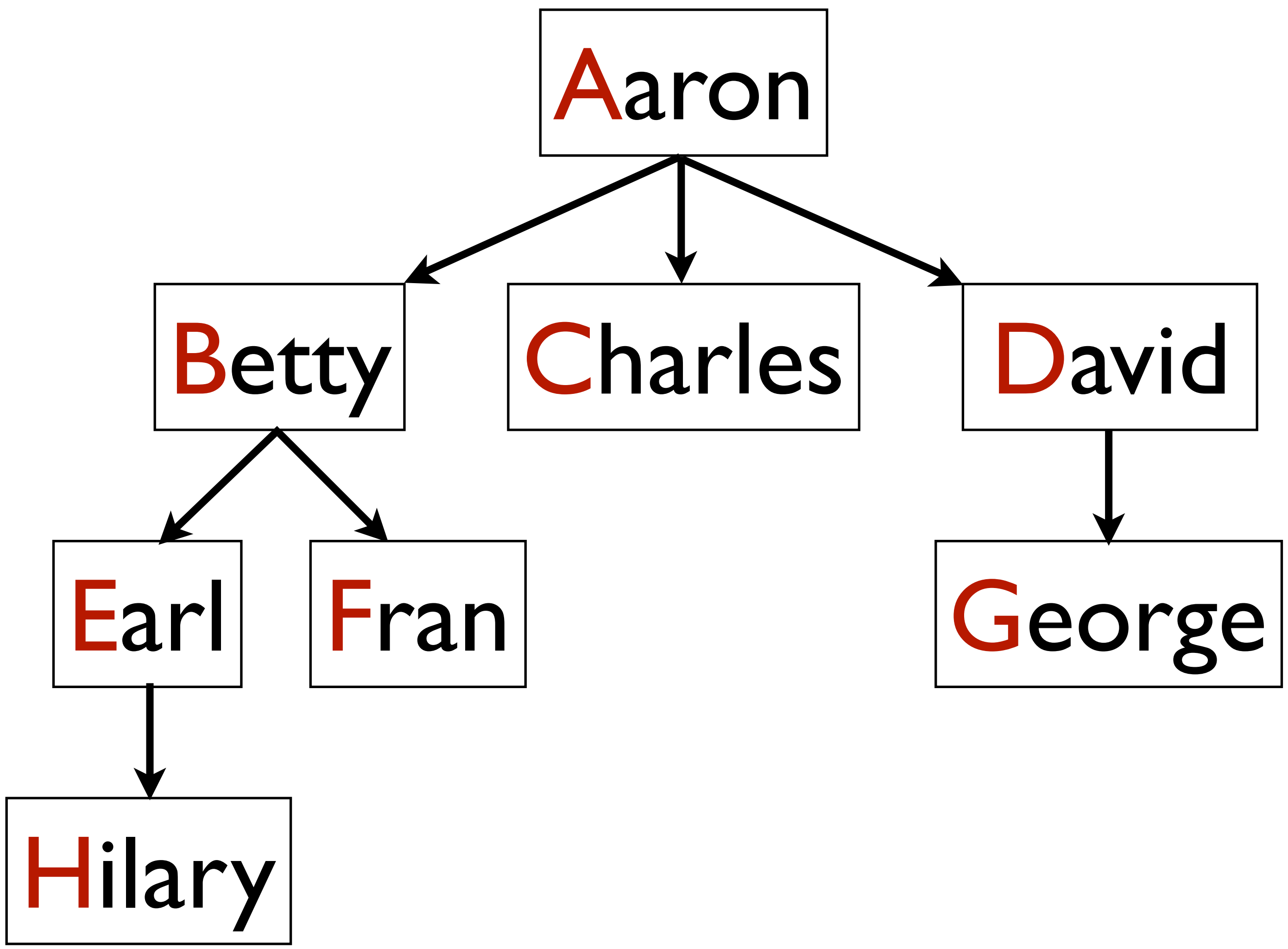
Tree-Revealing Process

Liben-Nowell, Kleinberg, PNAS'08



Tree-Revealing Process

Liben-Nowell, Kleinberg, PNAS'08



Each node is exposed independently with prob. $\delta > 0$

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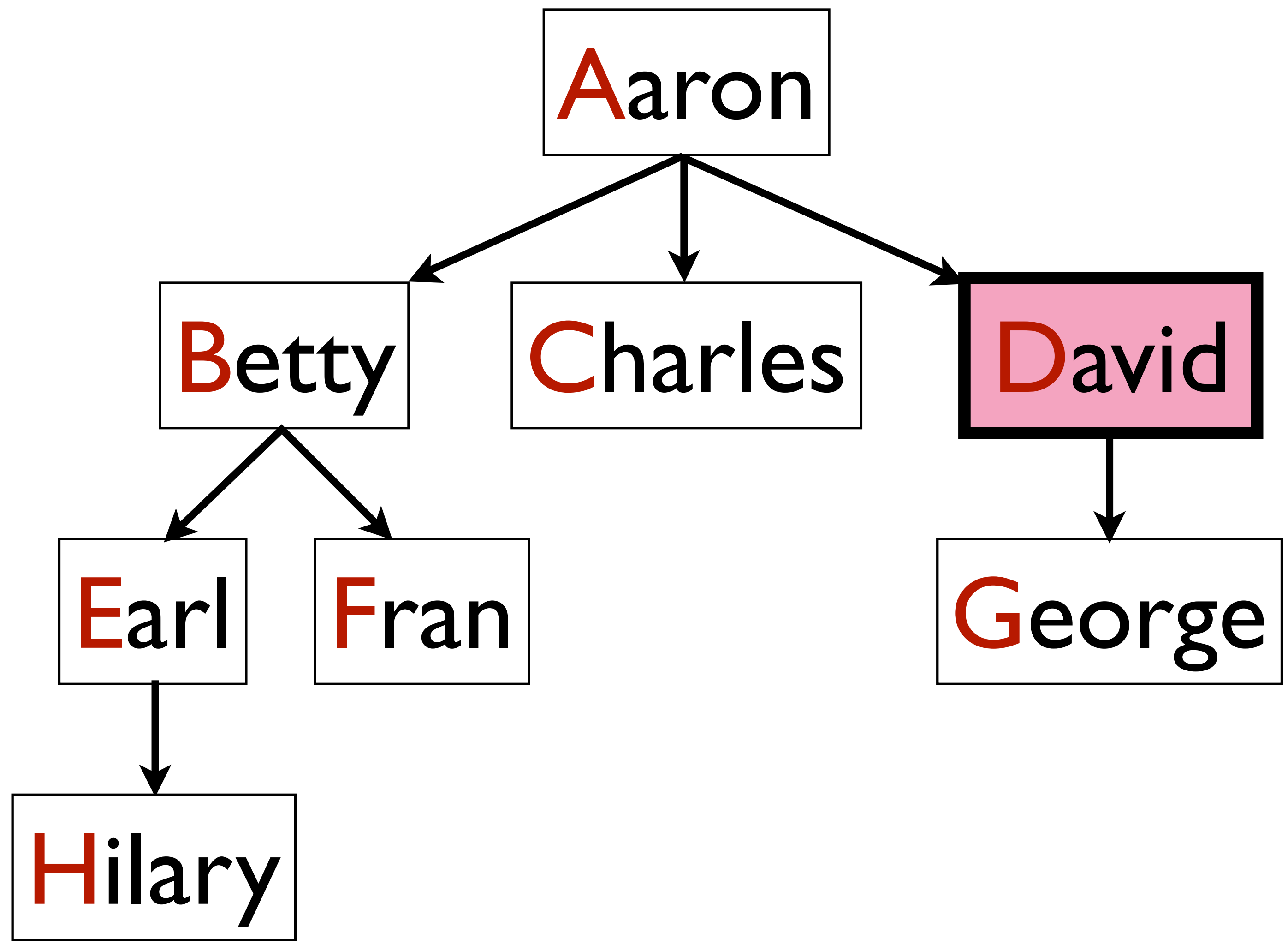

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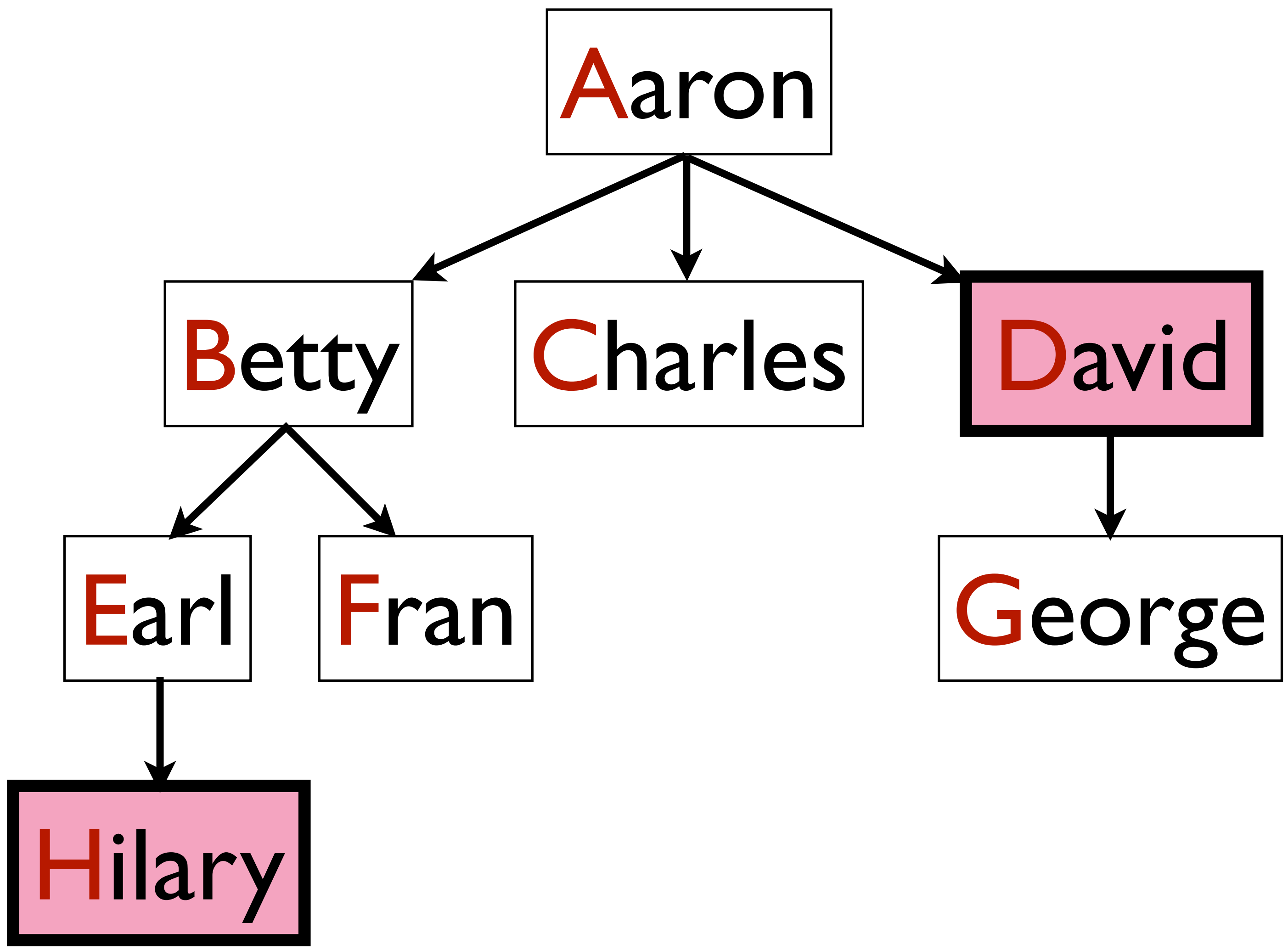

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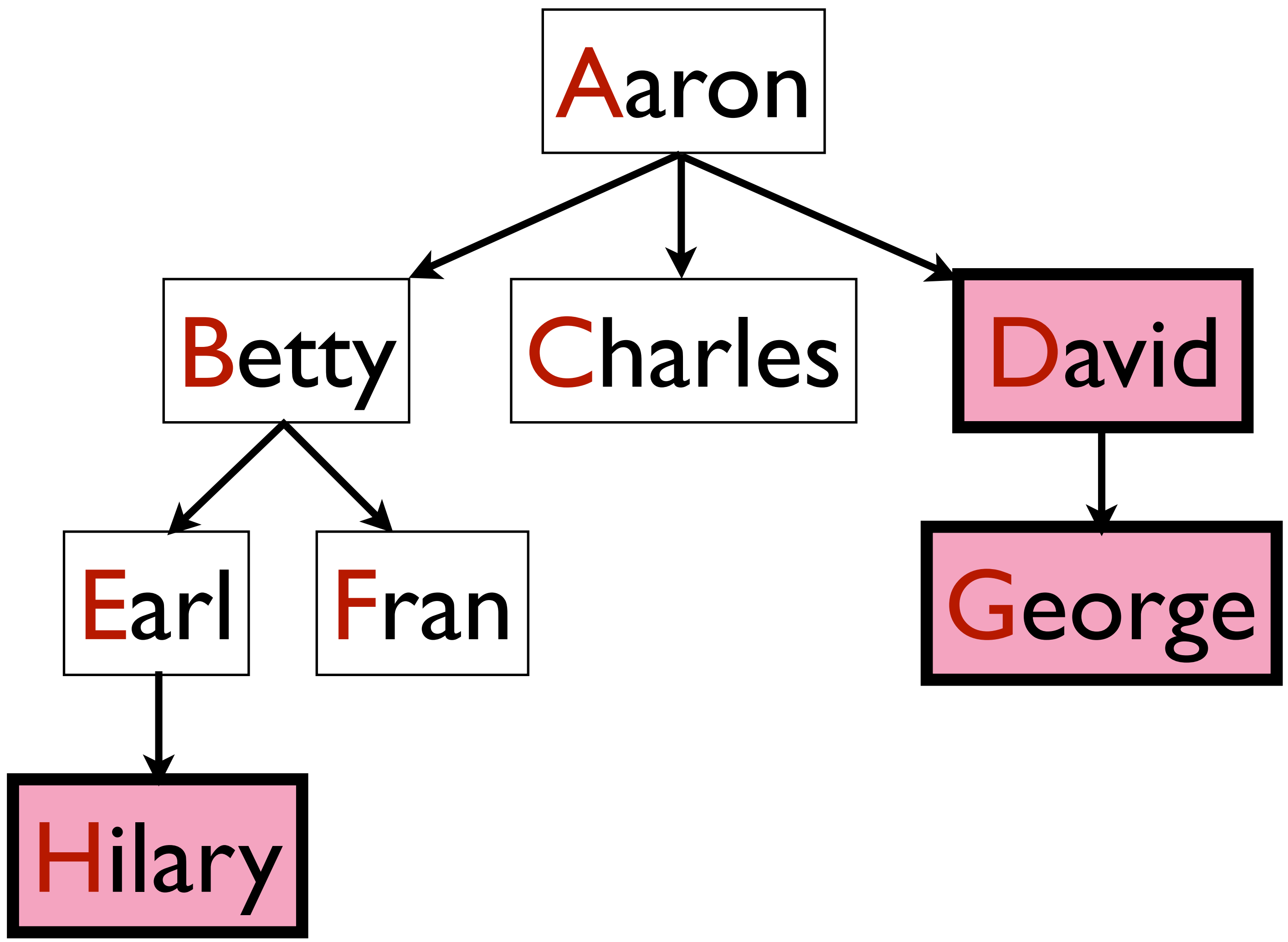

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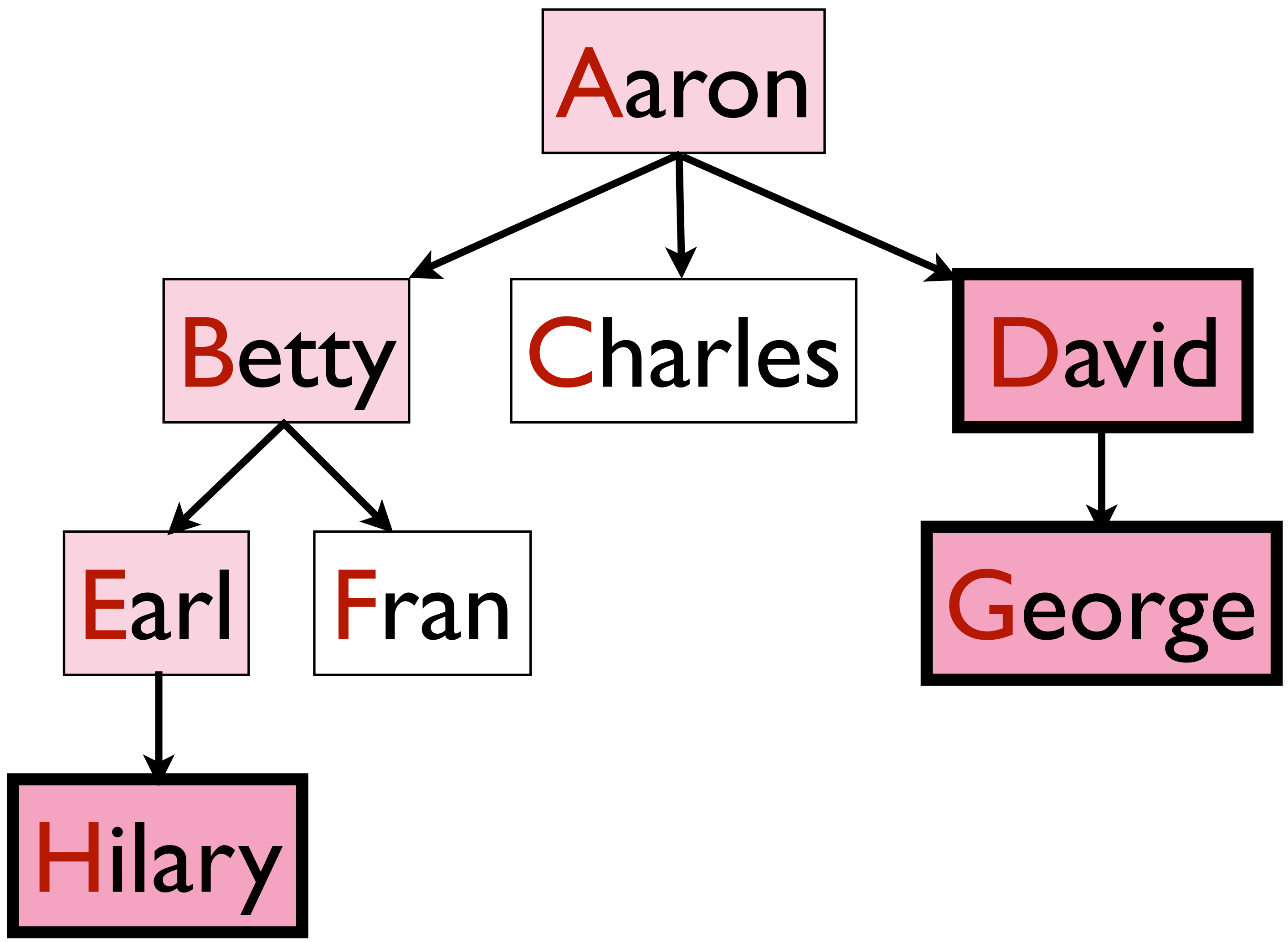

Ancestors of exposed nodes are revealed

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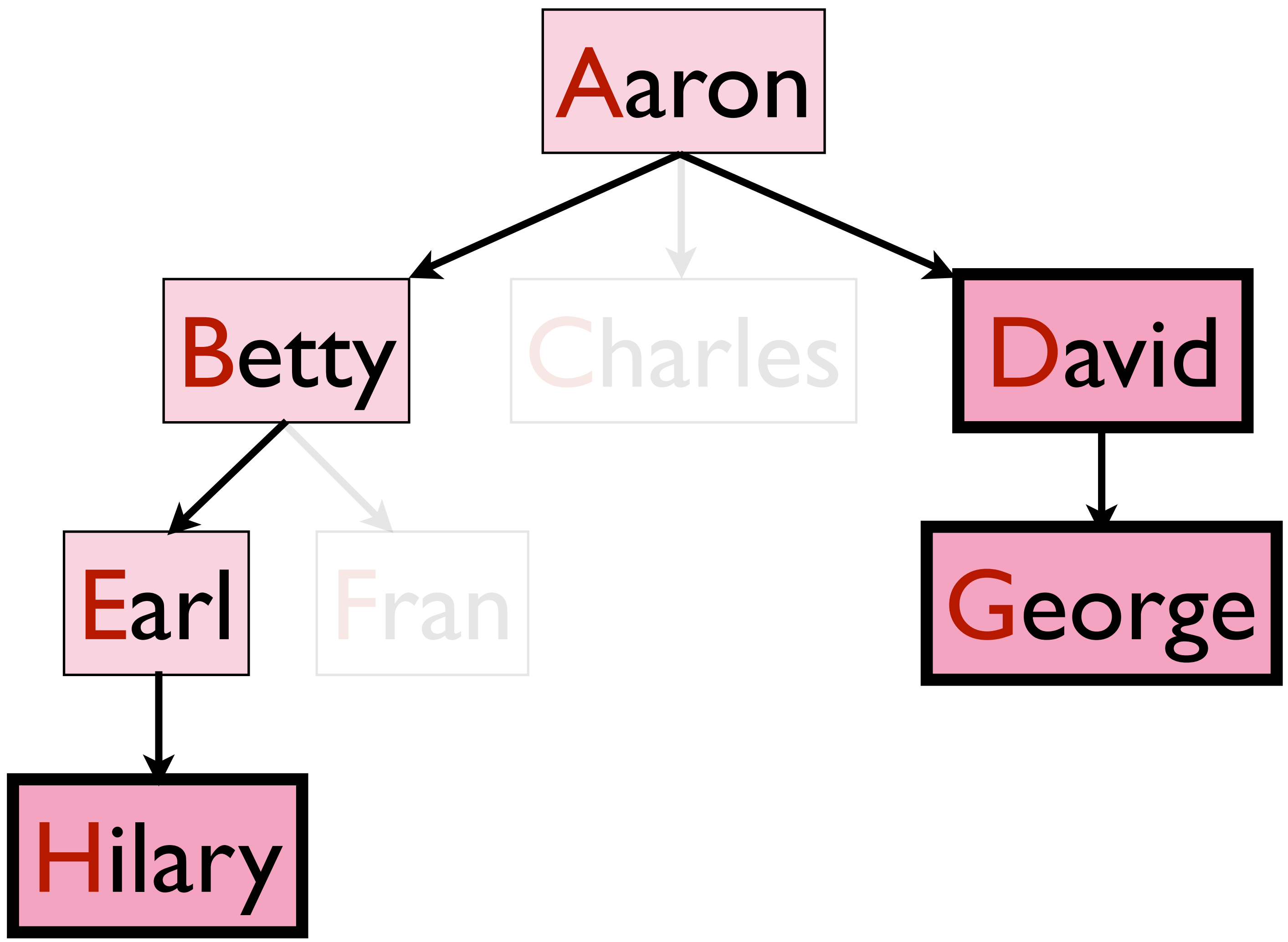

Ancestors of exposed nodes are revealed

Previous Work

- *Golub, Jackson, PNAS'10* perform simulations,
  - using *branching process trees* near the *critical threshold* as the Chain Letter Trees,
  - and *exposing* nodes as in *Kleinberg, Liben-Nowell, PNAS'08*.
- They observe that the *revealed* tree has a high fraction of nodes with only one child (and some other properties).

Our Contribution

- Our 1st result, *informally*, states that the tree-revealing process, is enough to explain the high fraction of single-child nodes

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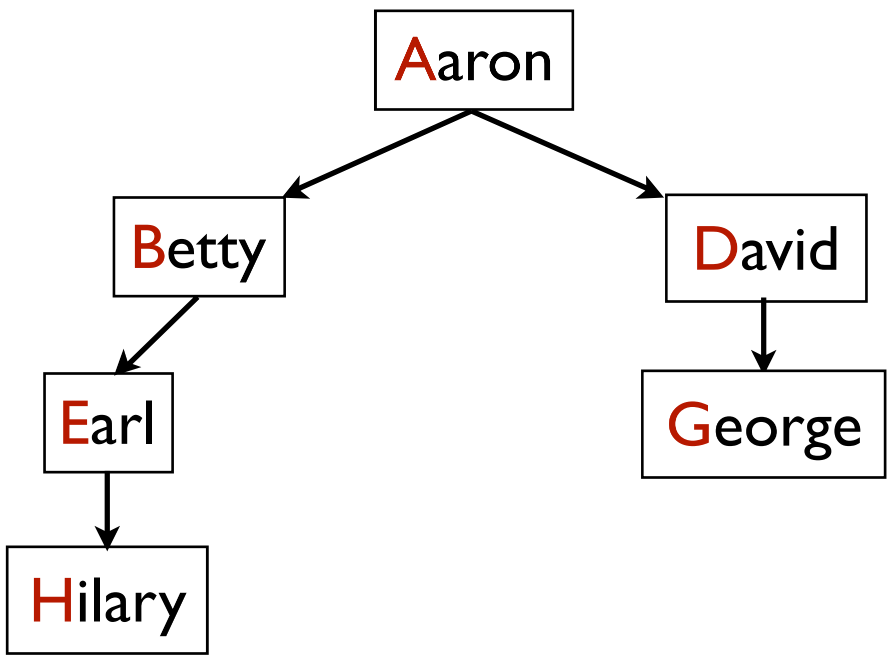
- Our 1st result, *informally*, states that the tree-revealing process, is enough to explain the high fraction of single-child nodes assuming only a degree bound on the unknown chain letter tree.

Revealed vs. Unknown

We see a
“revealed” tree...



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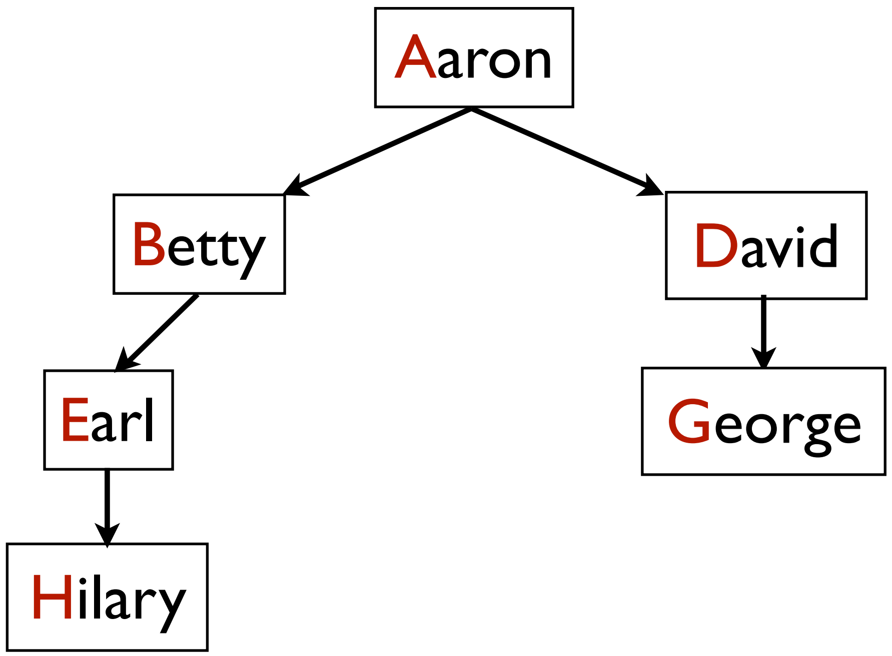
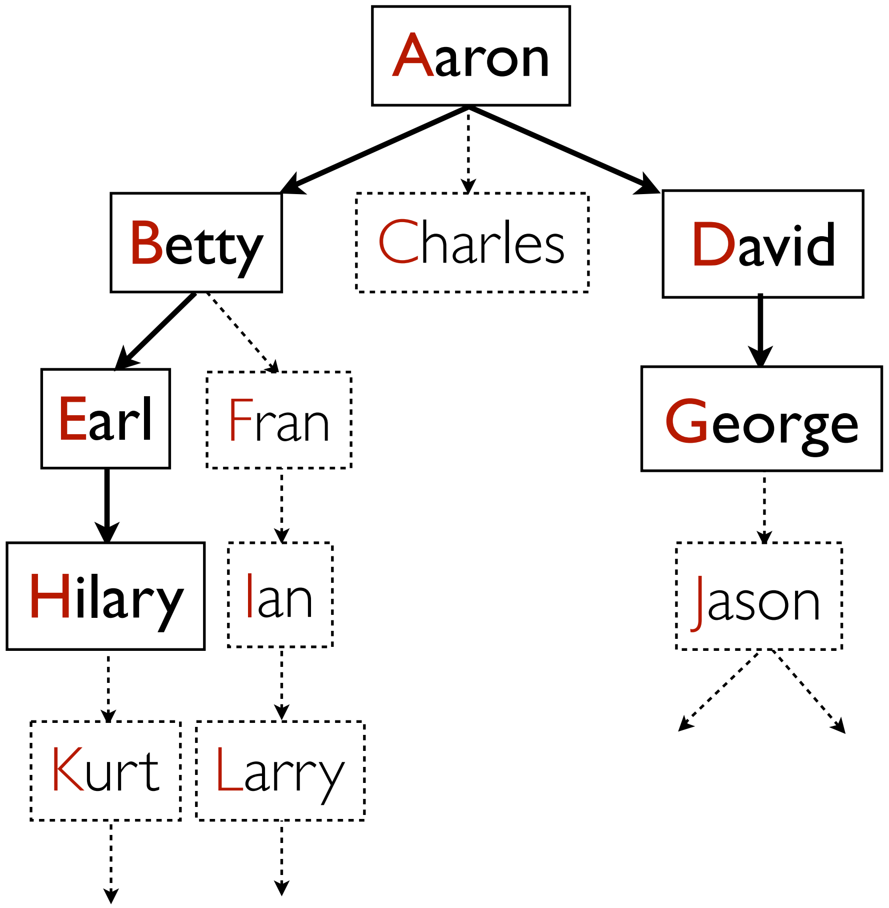


...we would like to study the "unknown" tree!



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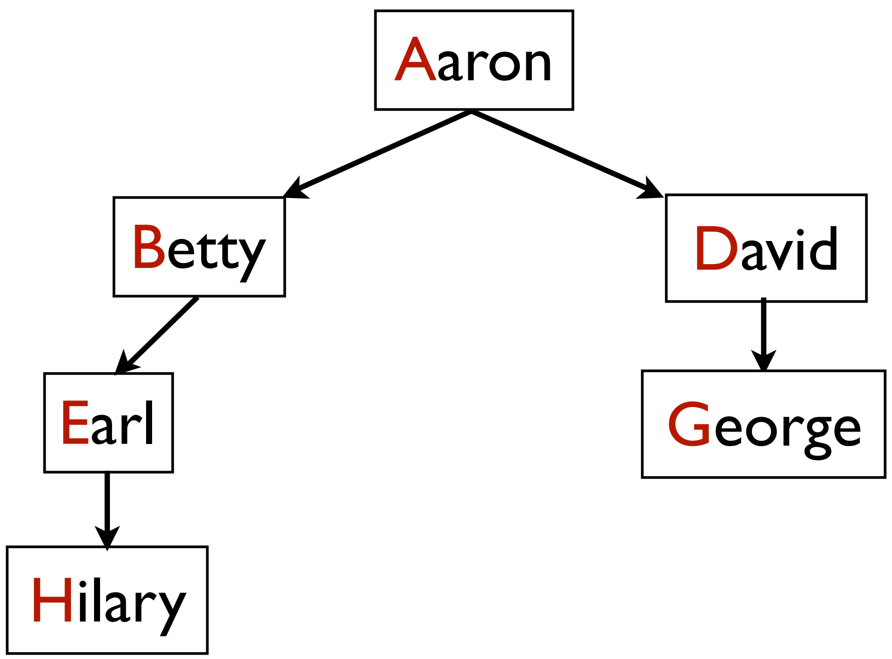

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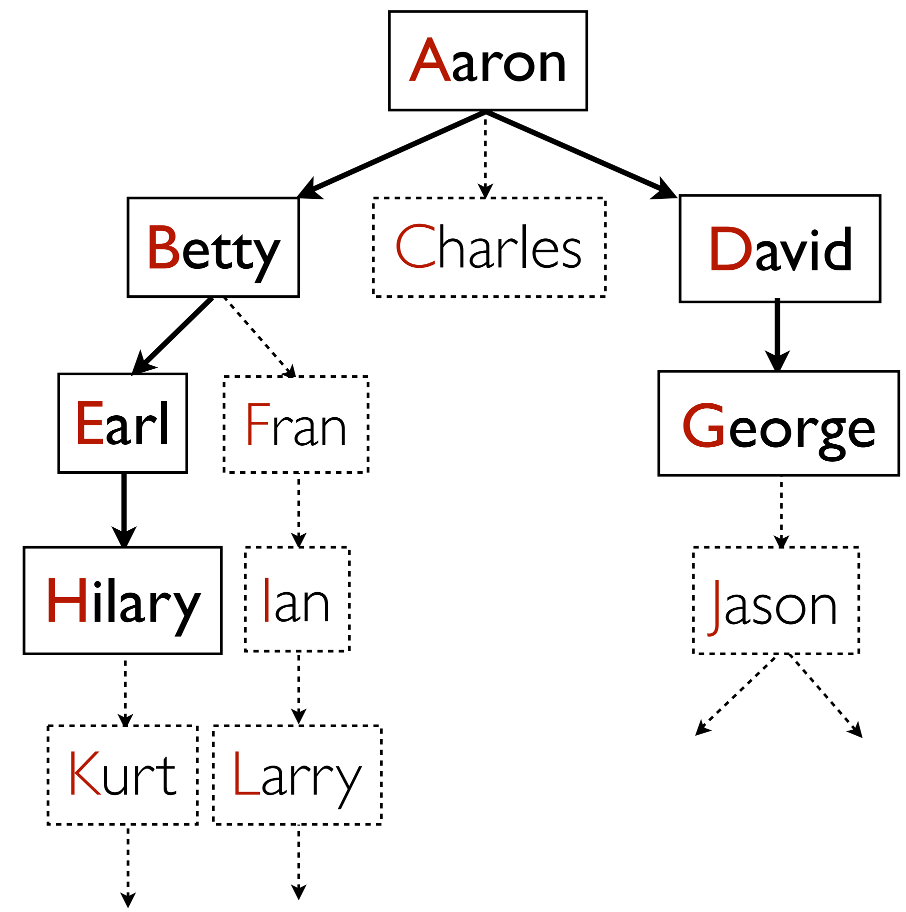

Size? Width? Height? Degree Distribution? ...

Revealed vs. Unknown

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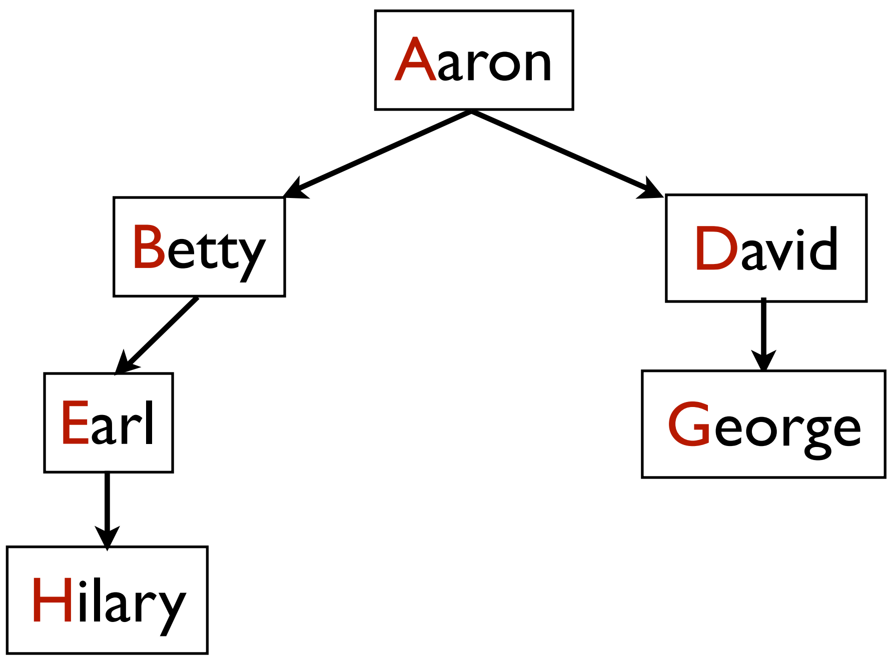

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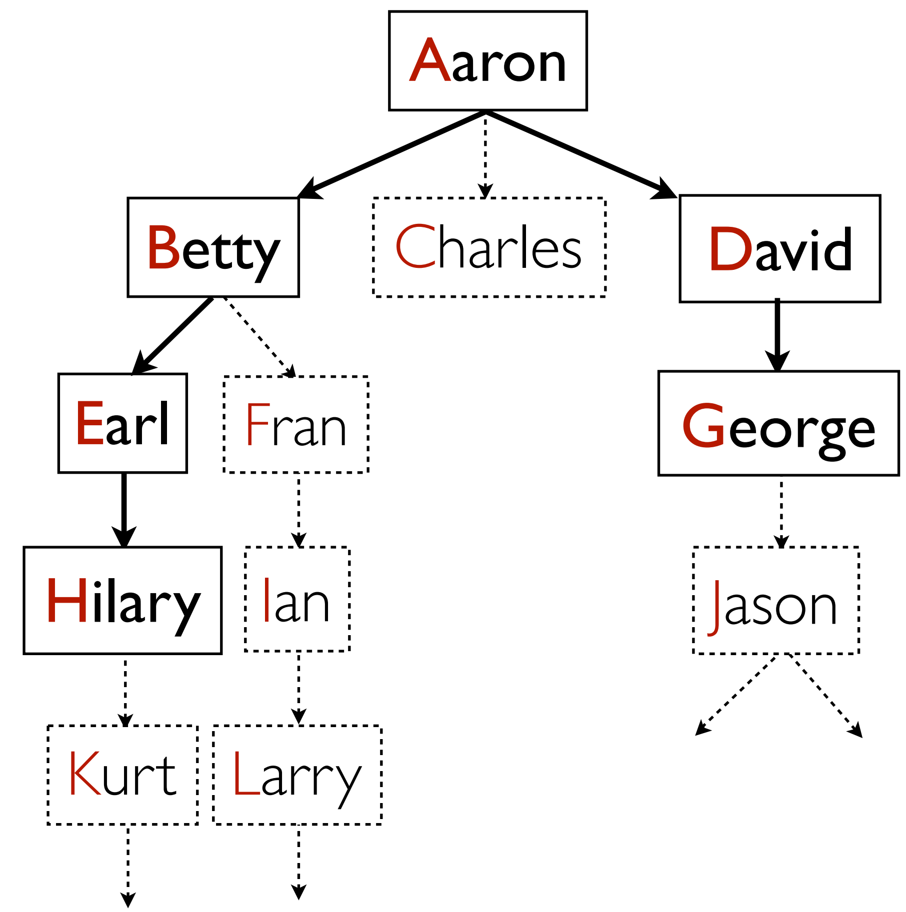

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Observe that we do not know the exposing probability δ

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We use this theorem to estimate that ~ 173k people that signed the IRAQ chain letter

This estimate is backed by a probability bound (on the probability space induced by the revealing process)

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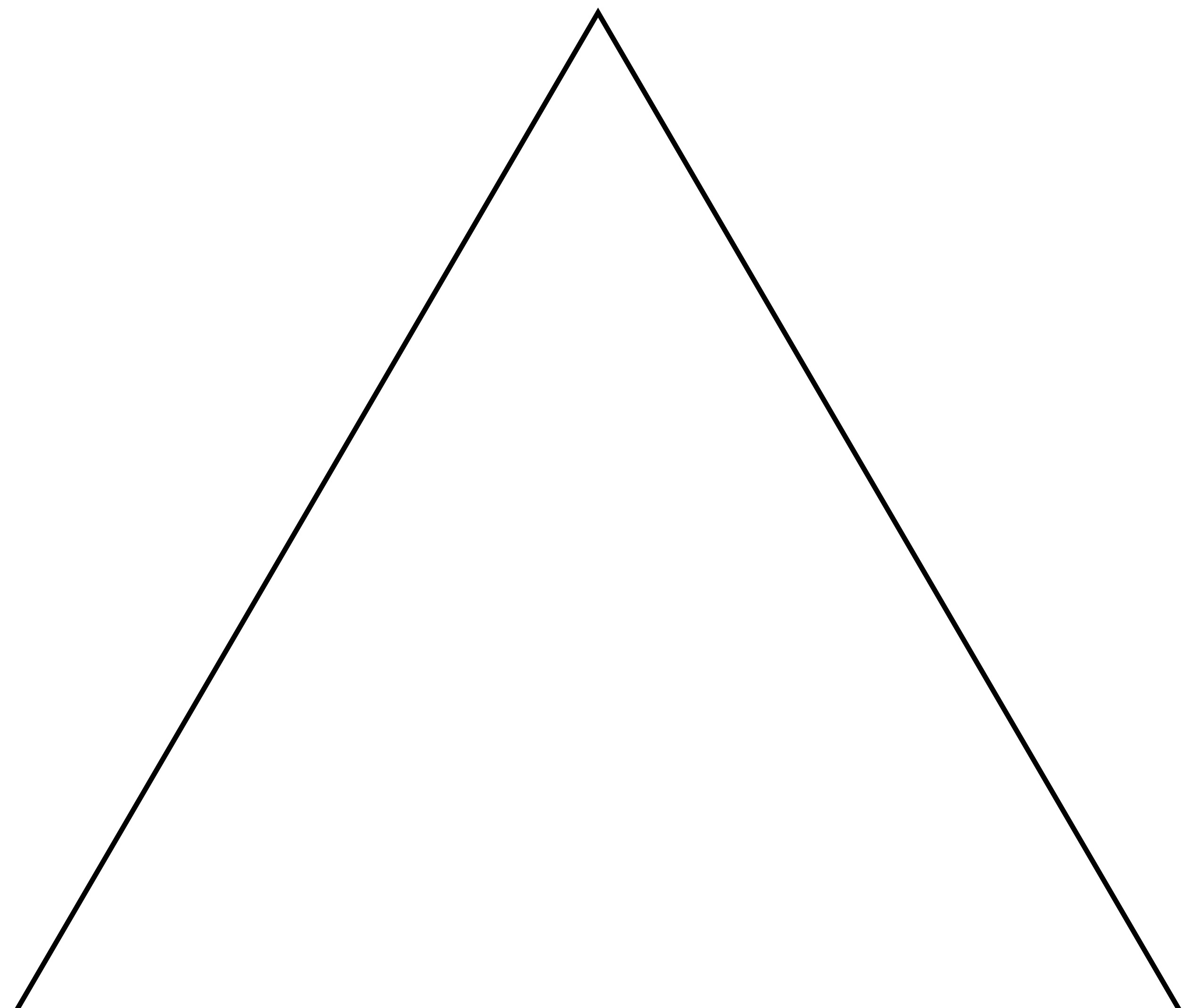
The chain letter generated ~ 3.5M emails

Single-Child Fraction

- Nodes are exposed with probability $\delta > 0$
- We assume that the unknown tree's maximum degree is at most k

Single-Child Fraction



We partition the tree into subforests

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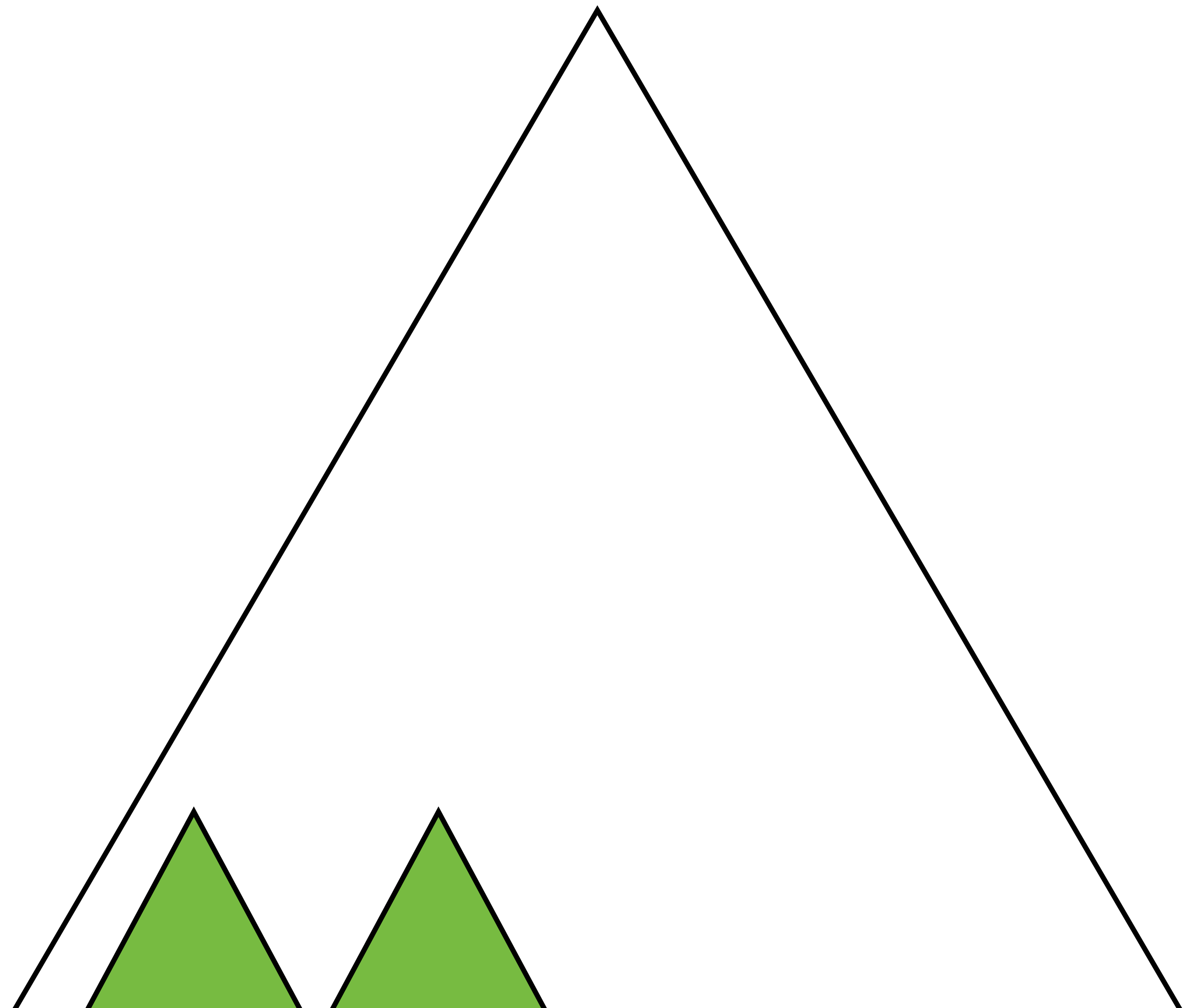

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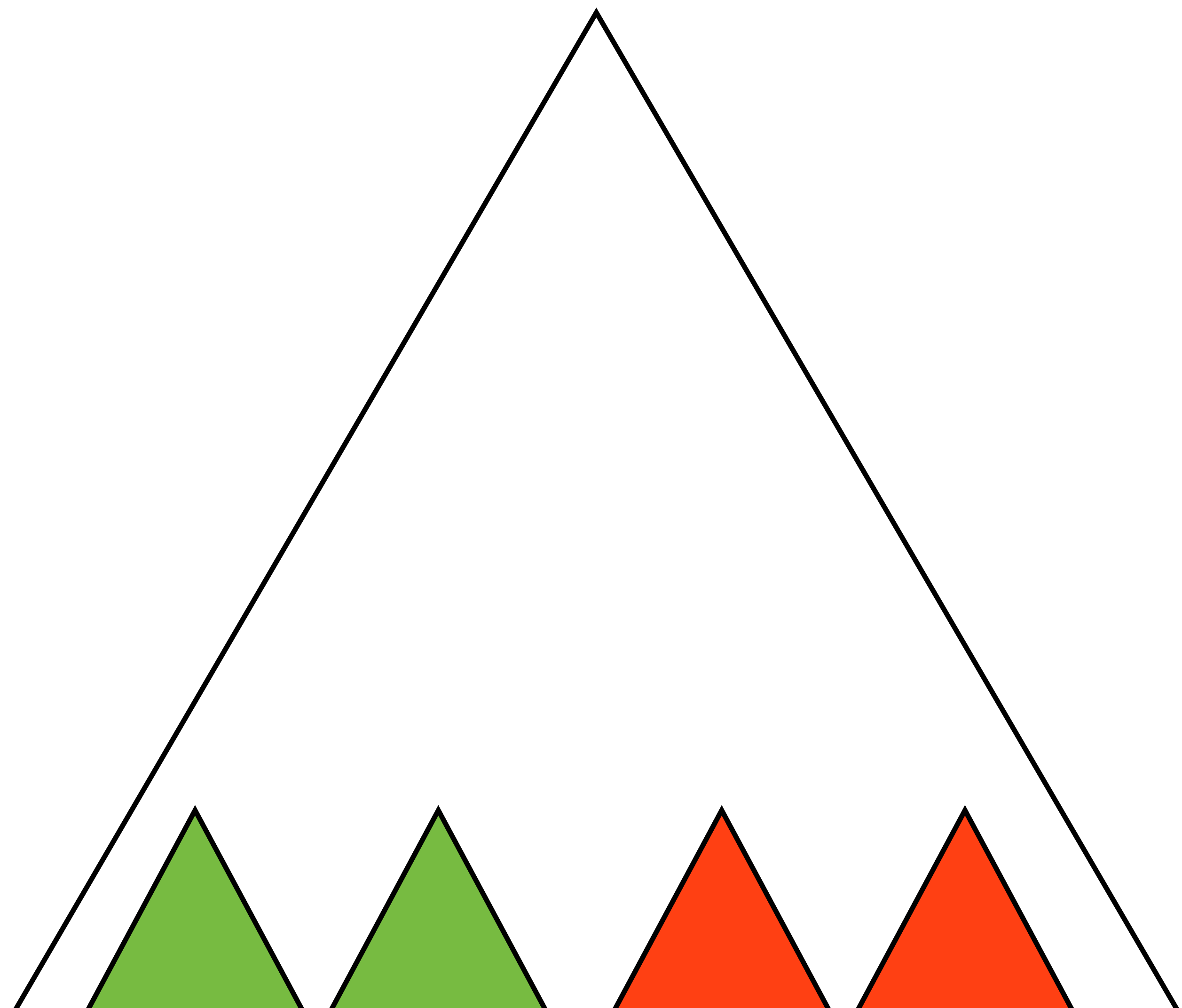

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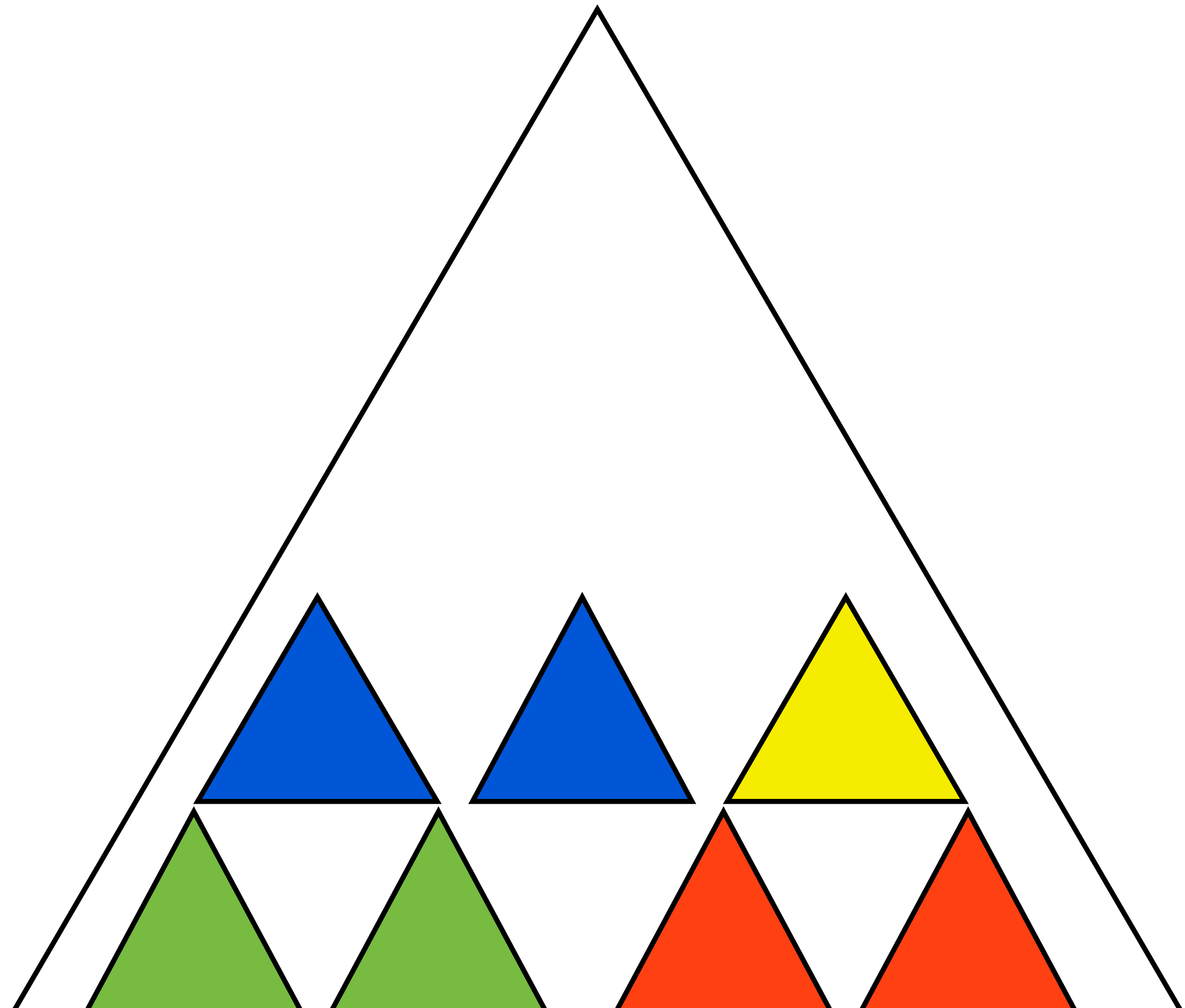

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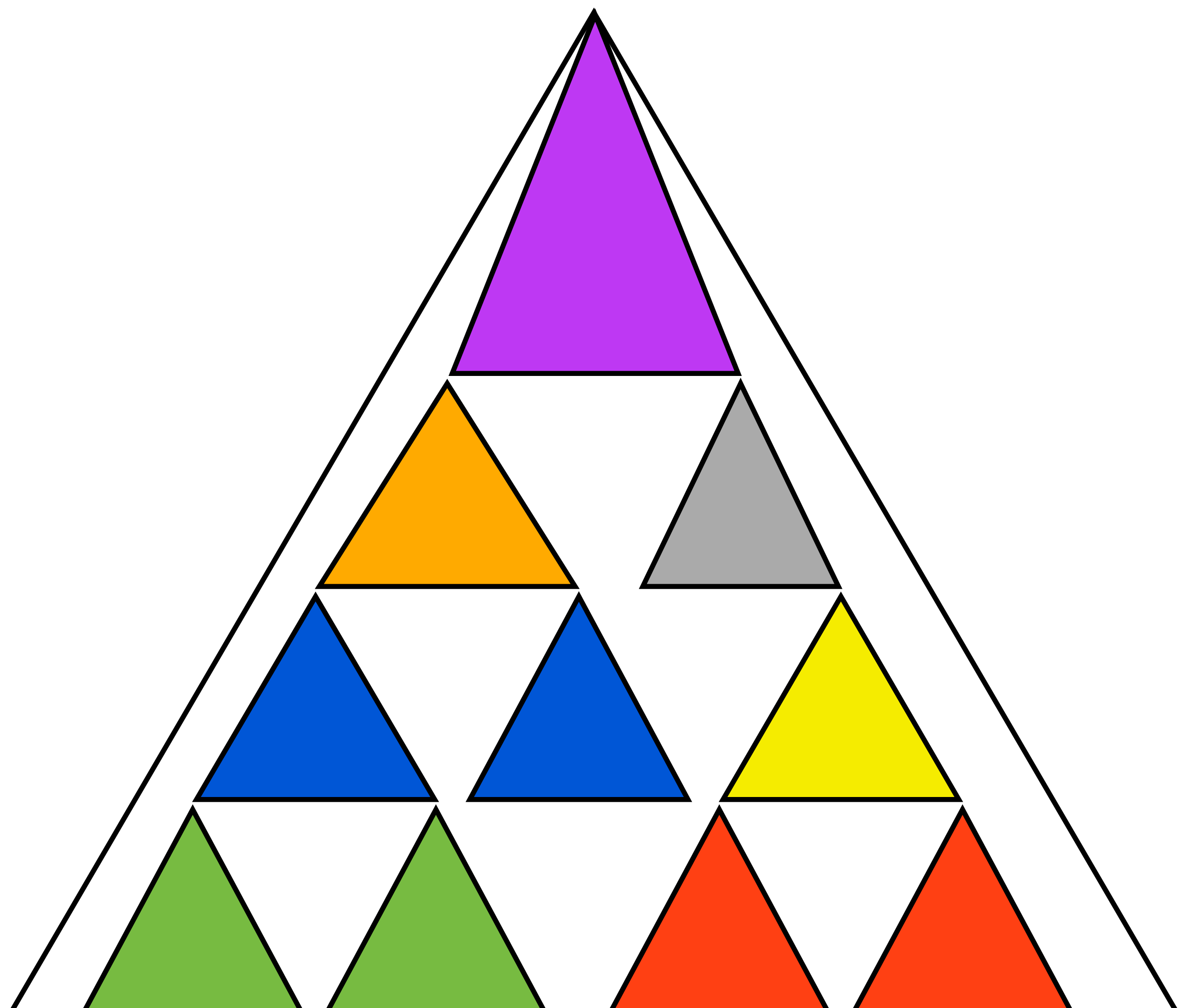

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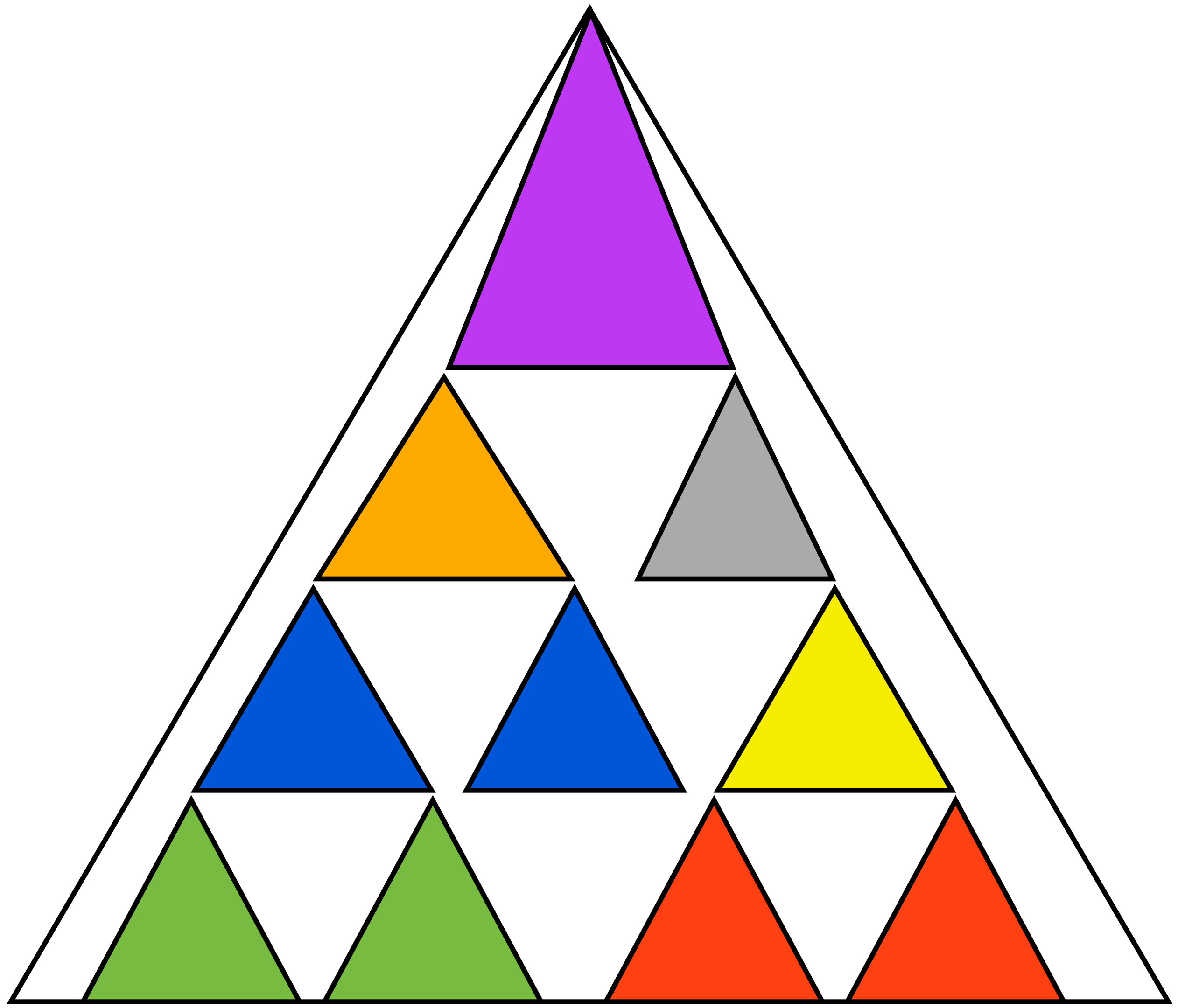

We partition the tree into subforests,
in such a way that each subforest has $\simeq \delta^{-1}$ nodes
and the median height in the subforest is $\Omega\left(\log_{k-1} \delta^{-1}\right)$.

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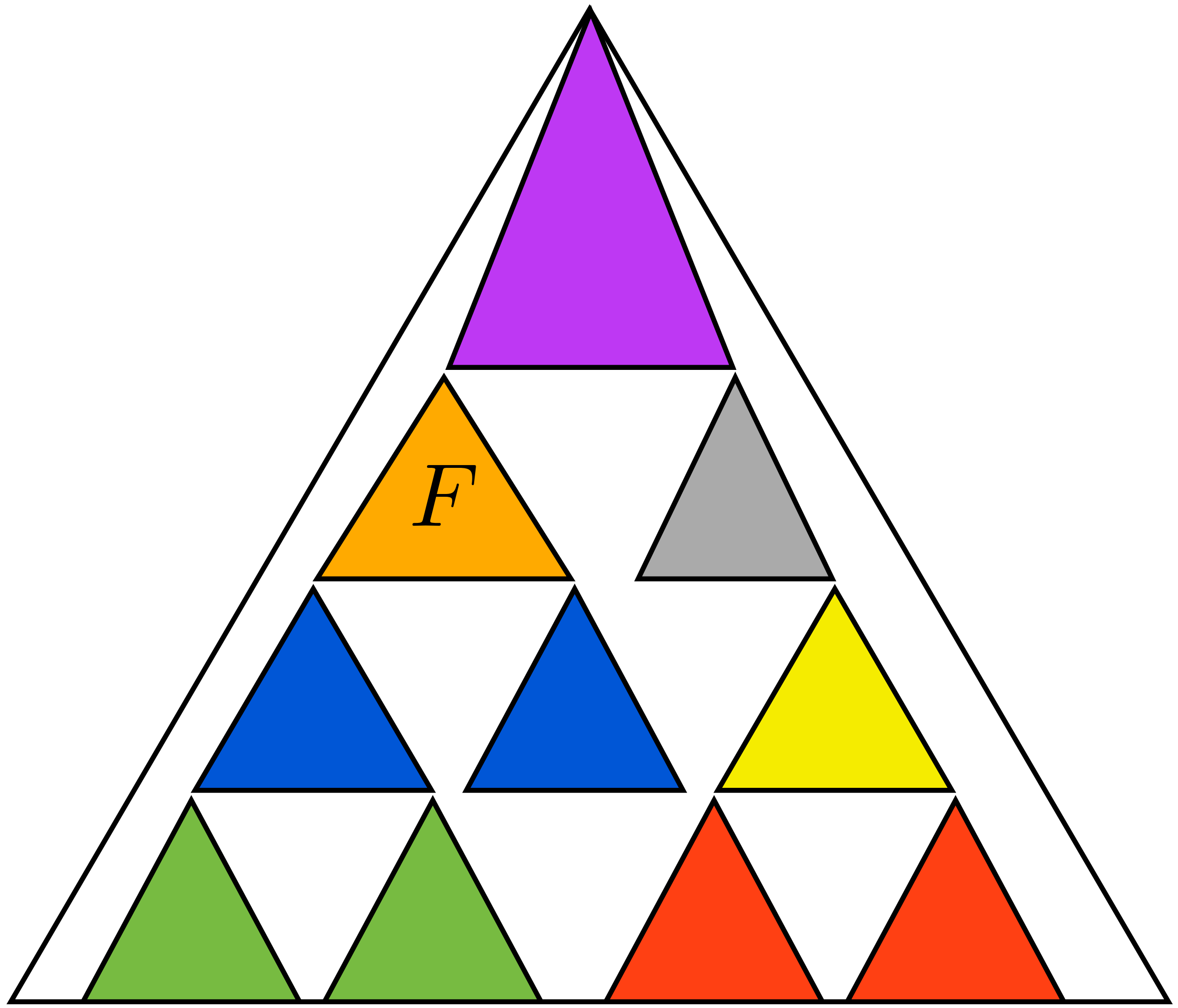

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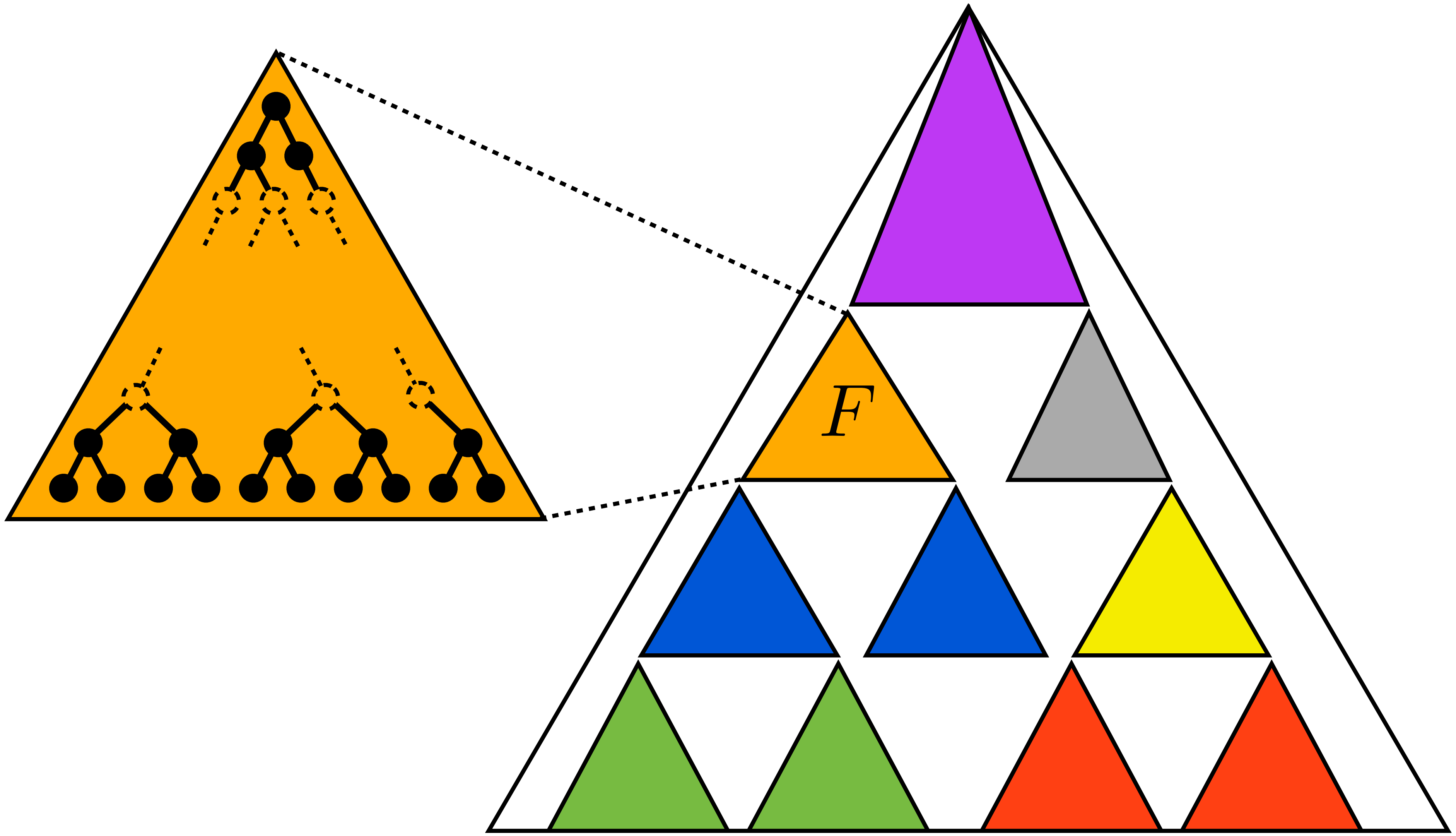

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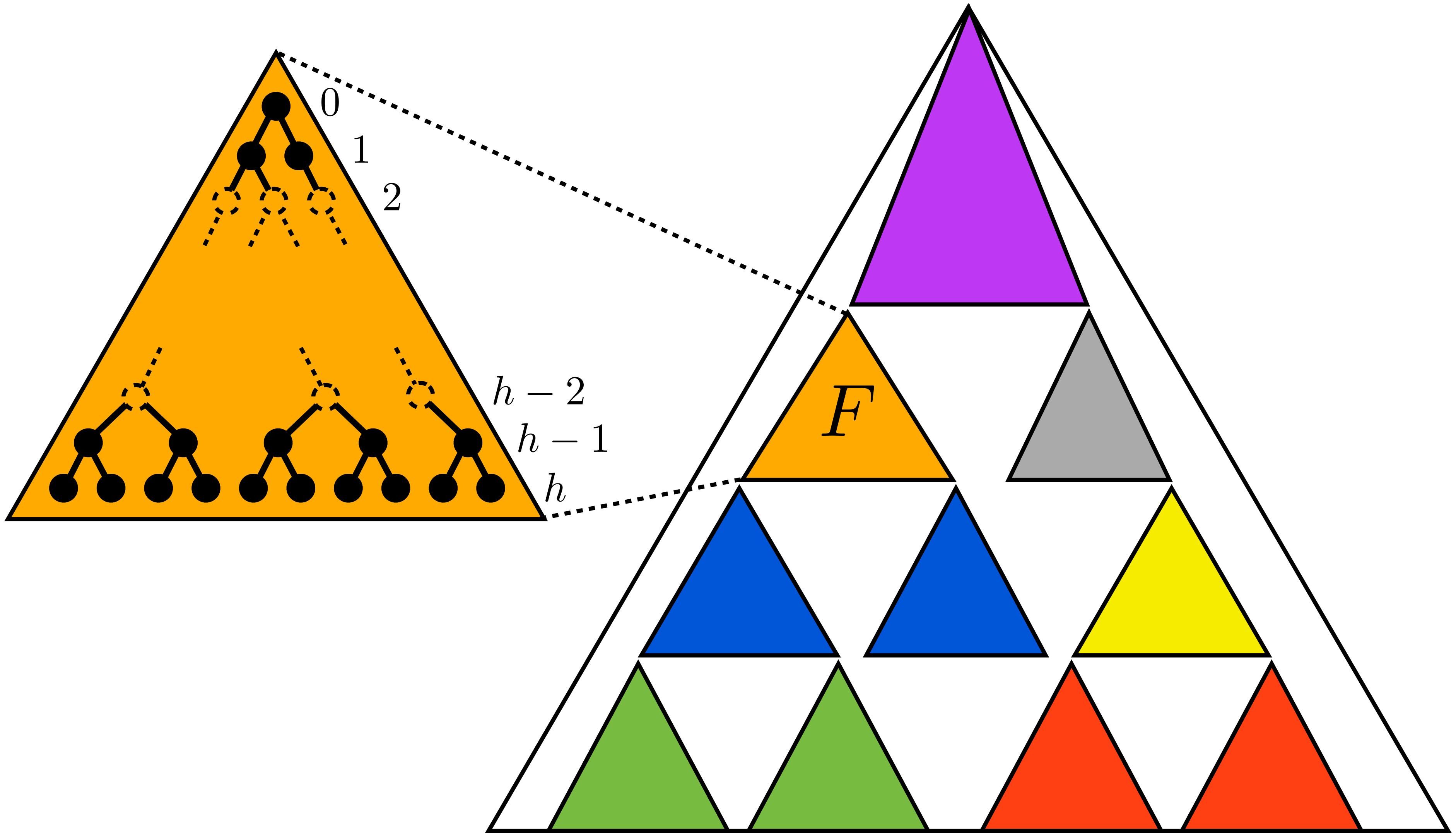

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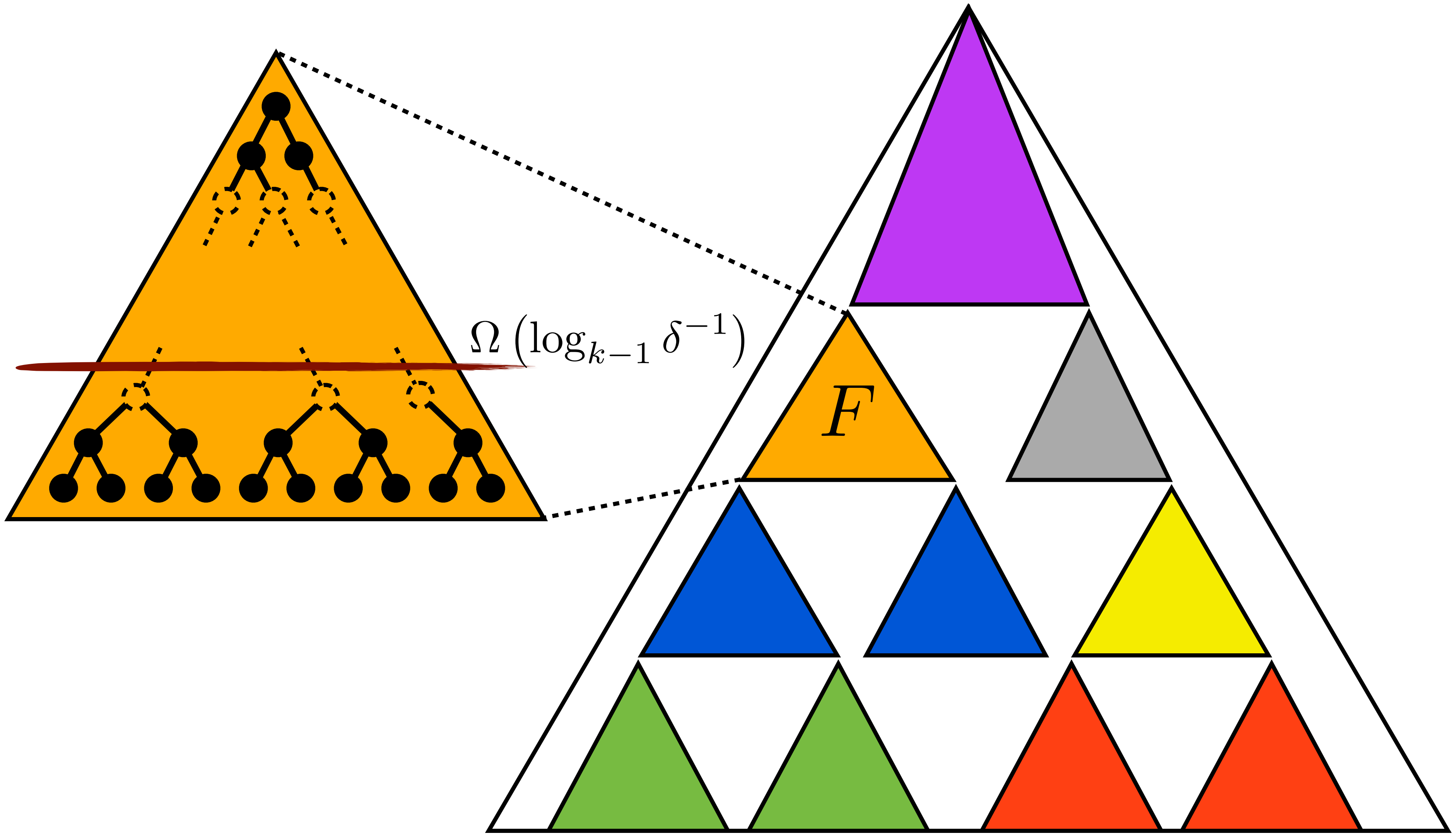

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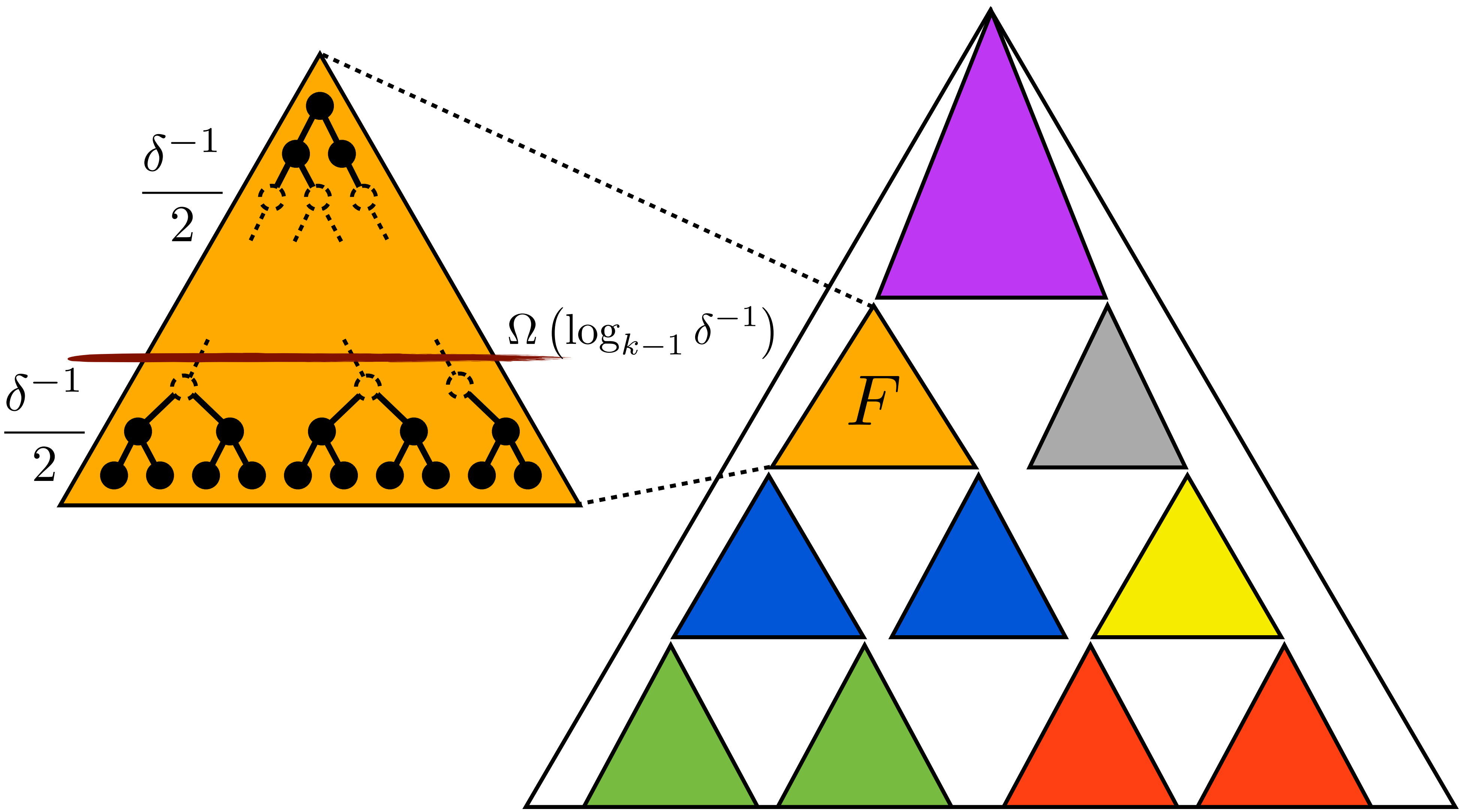

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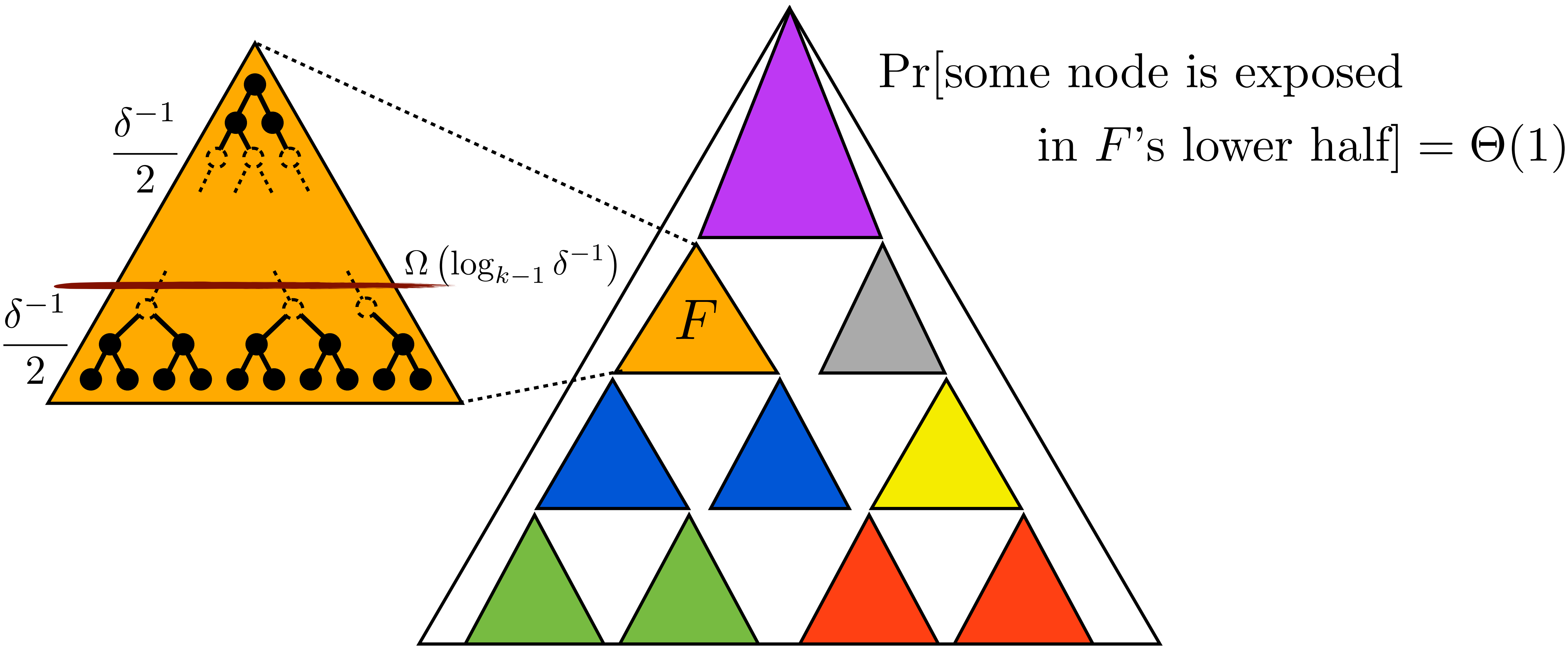

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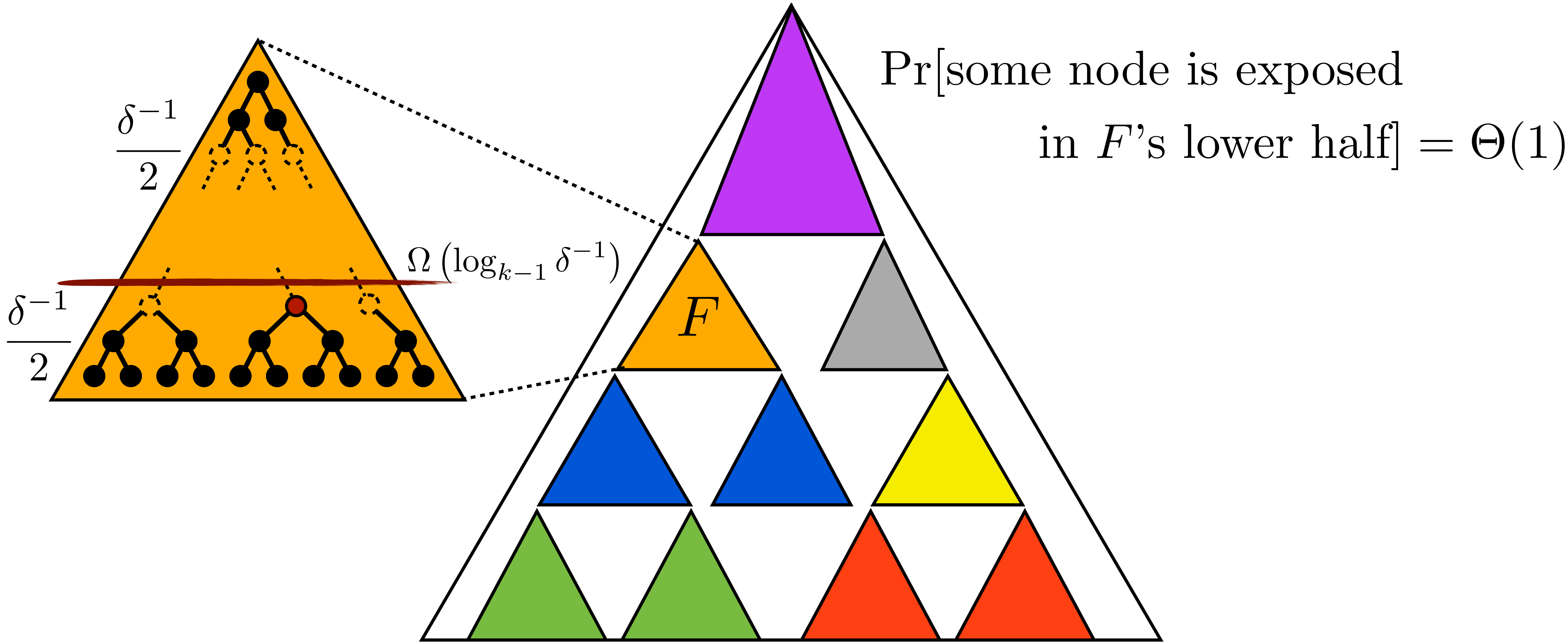

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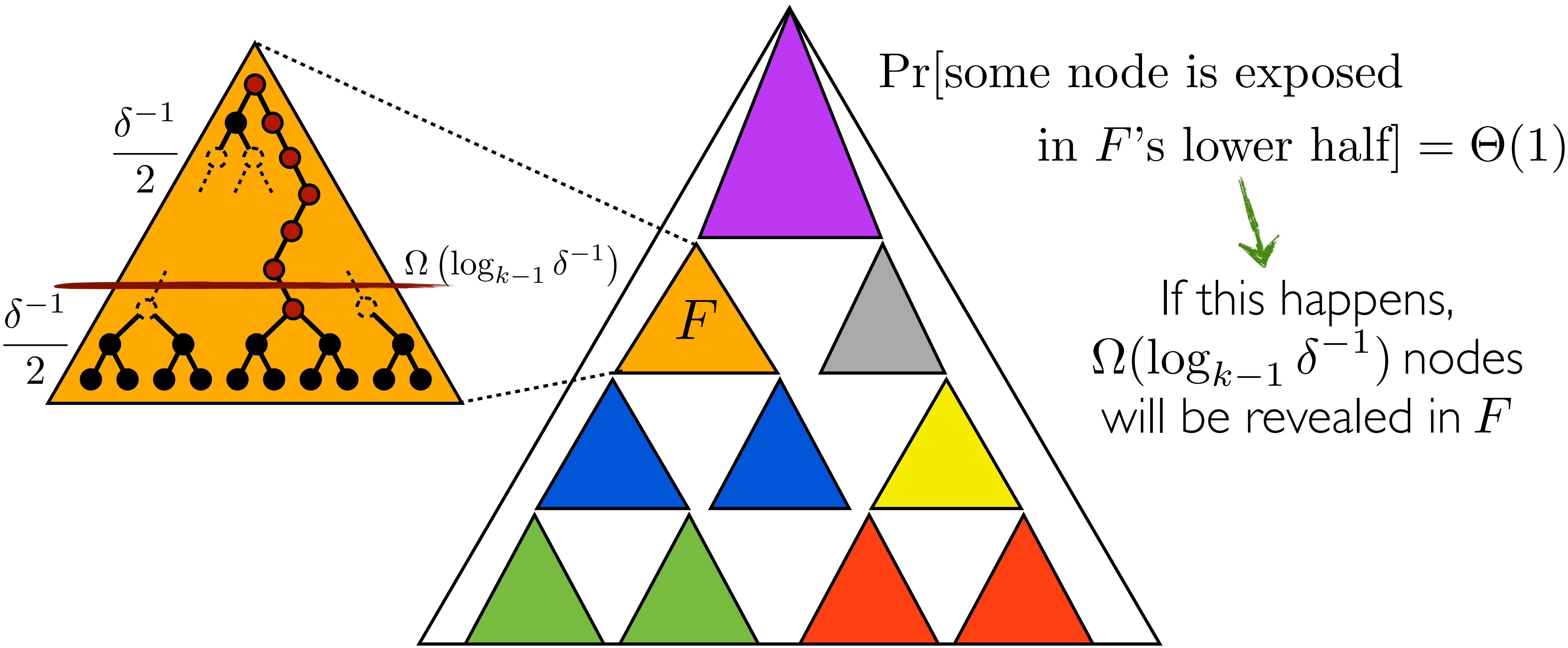

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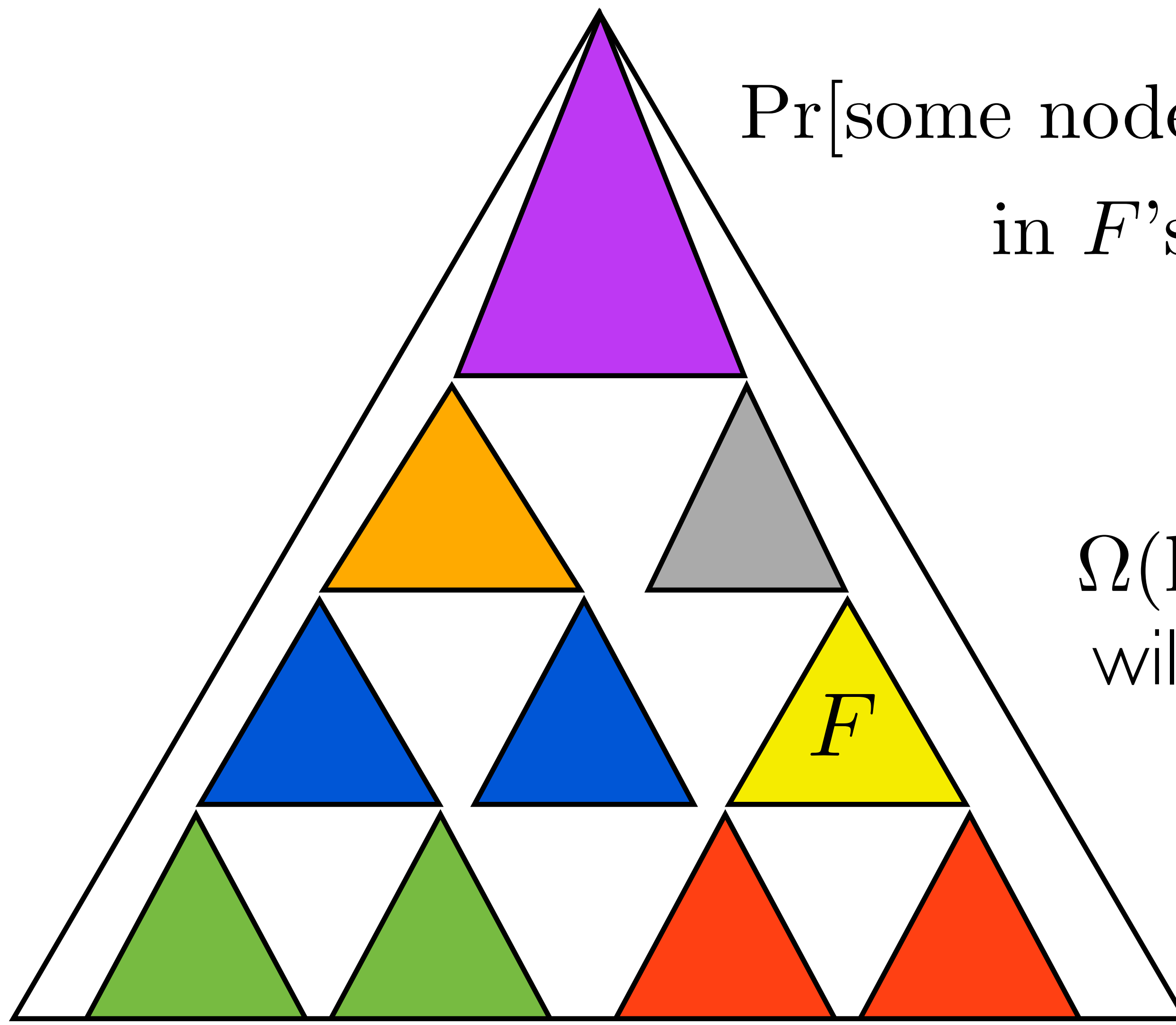

$\Pr[\text{some node is exposed in } F\text{'s lower half}] = \Theta(1)$

If this happens,
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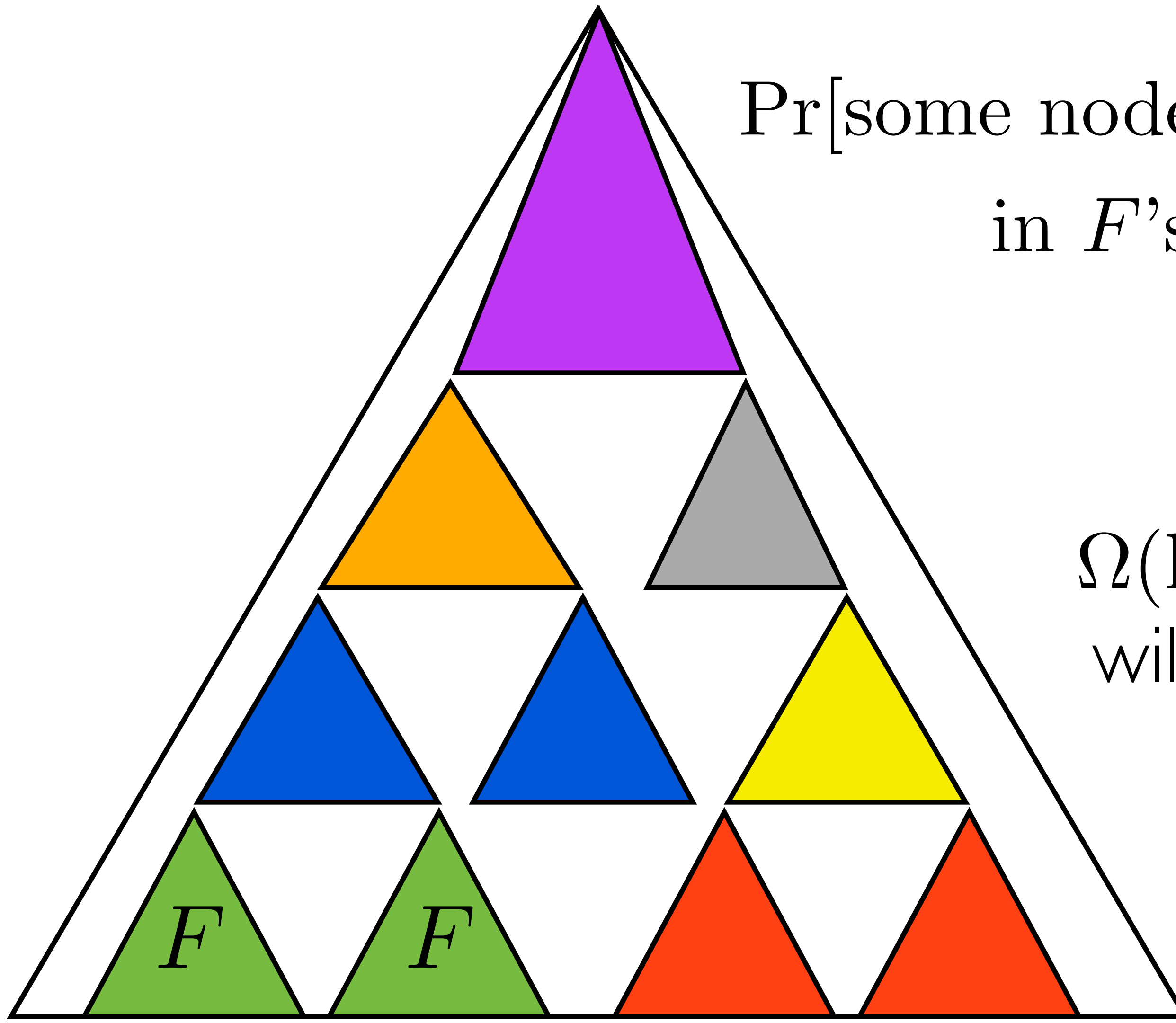

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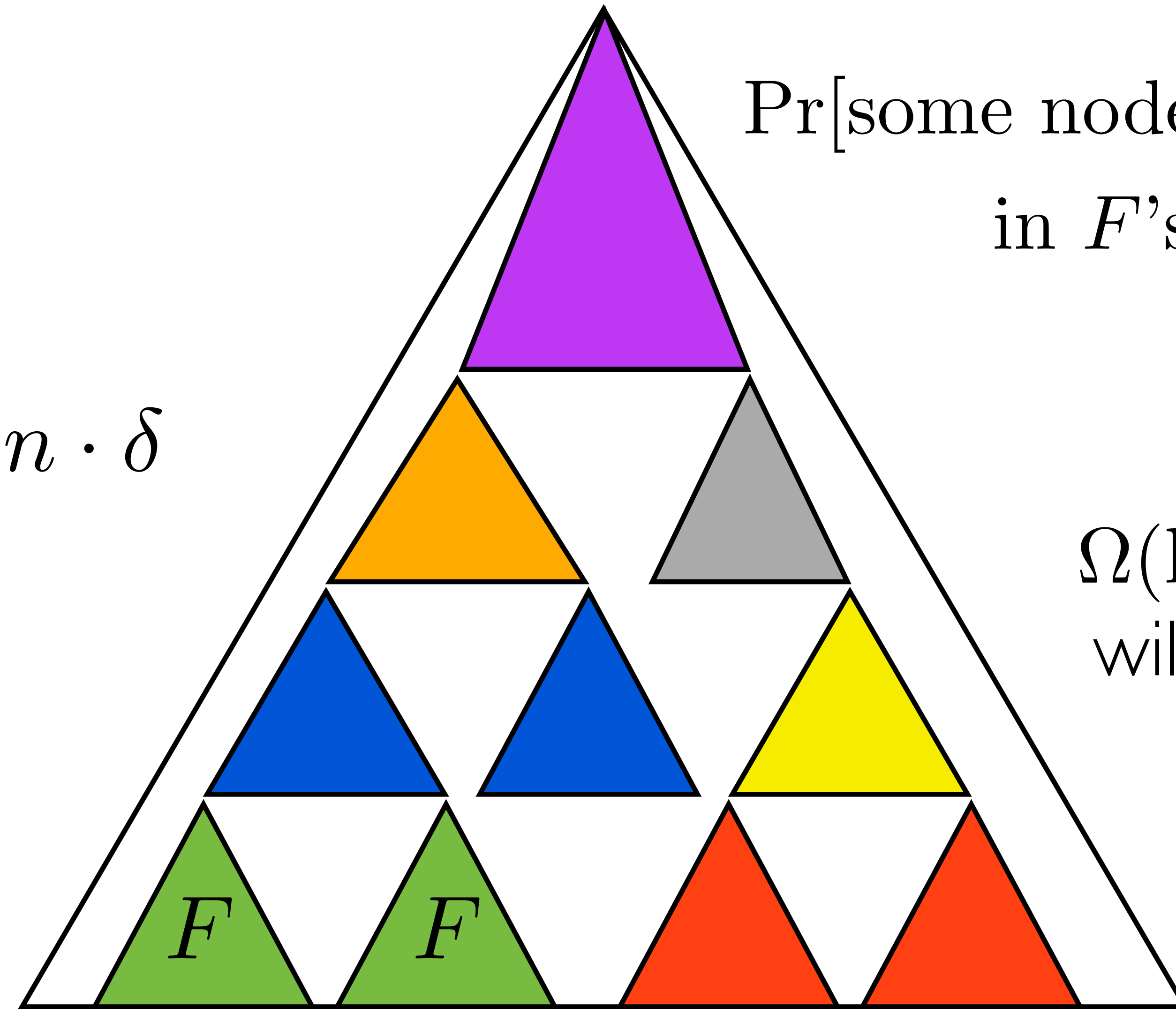

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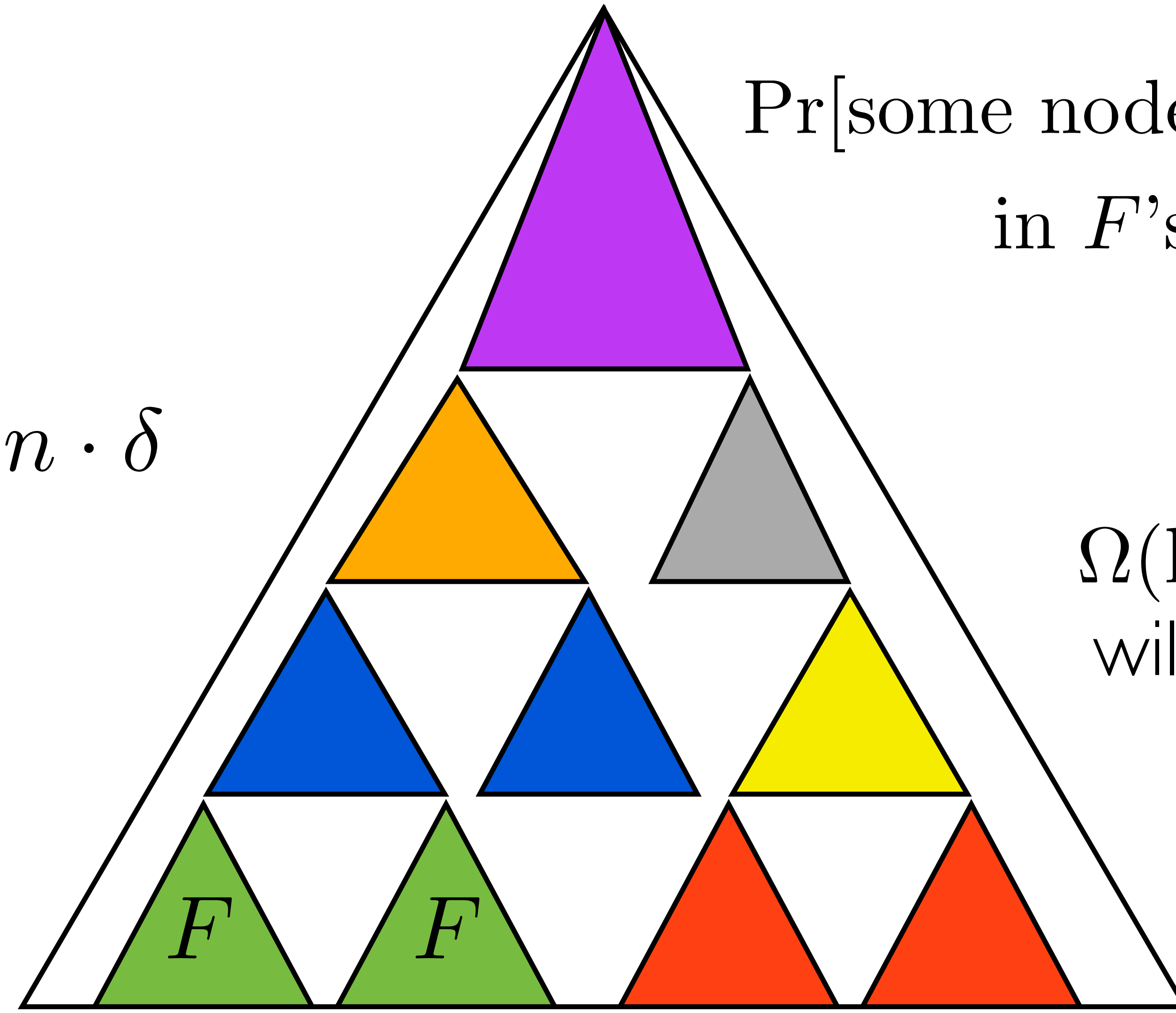

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$$\Pr[\Omega(n \cdot \delta \cdot \log_{k-1} \delta^{-1}) \text{ nodes will be revealed}] = 1 - o(1)$$

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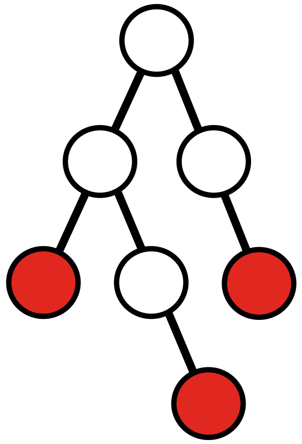

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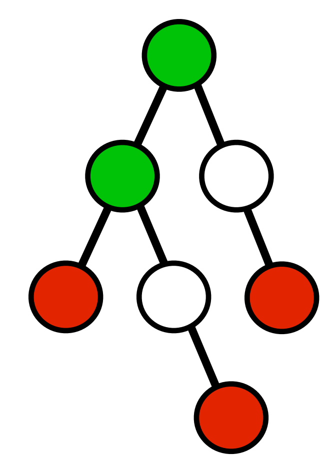

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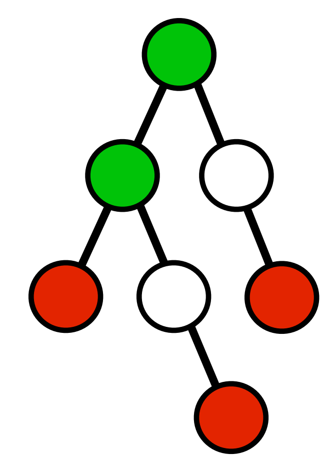

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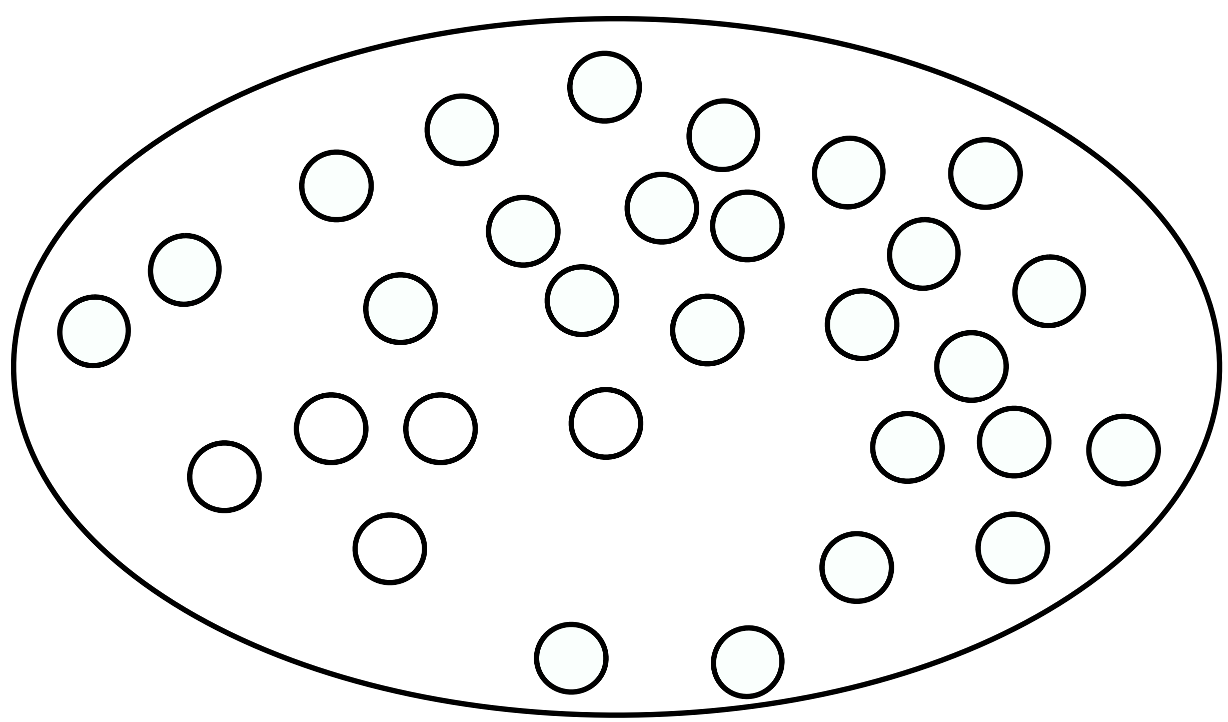

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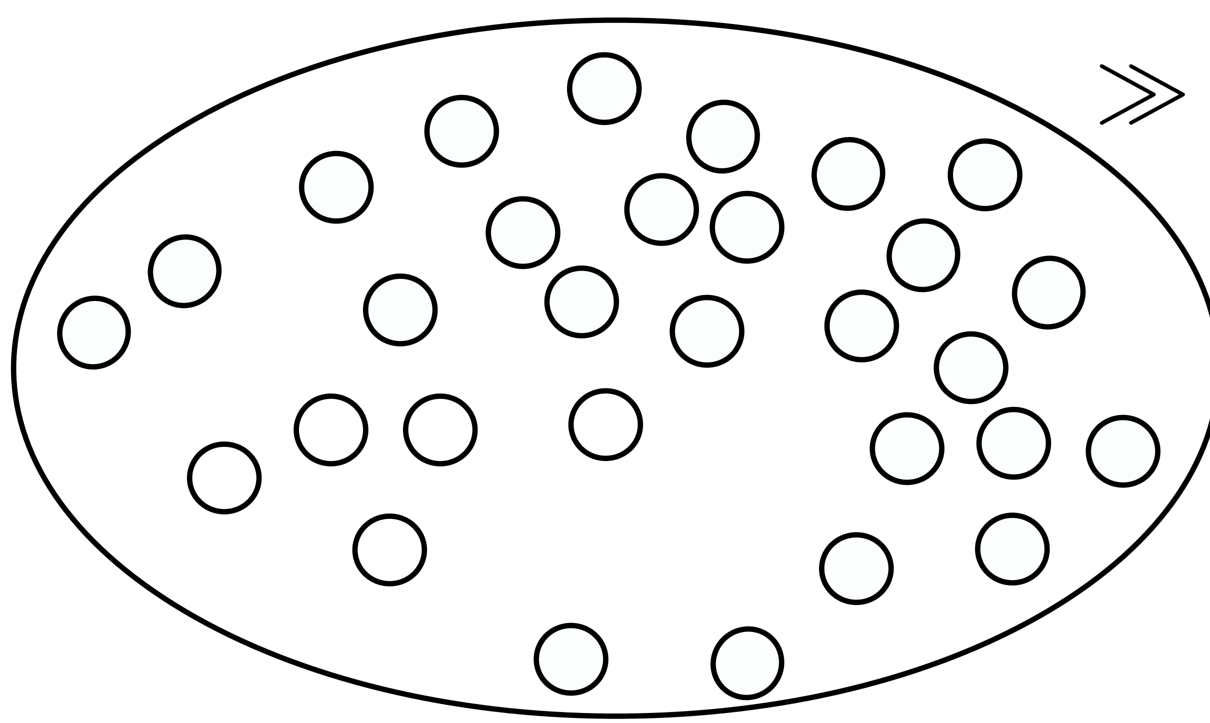
$\gg n \cdot \delta \cdot \log_{k-1} \delta^{-1}$



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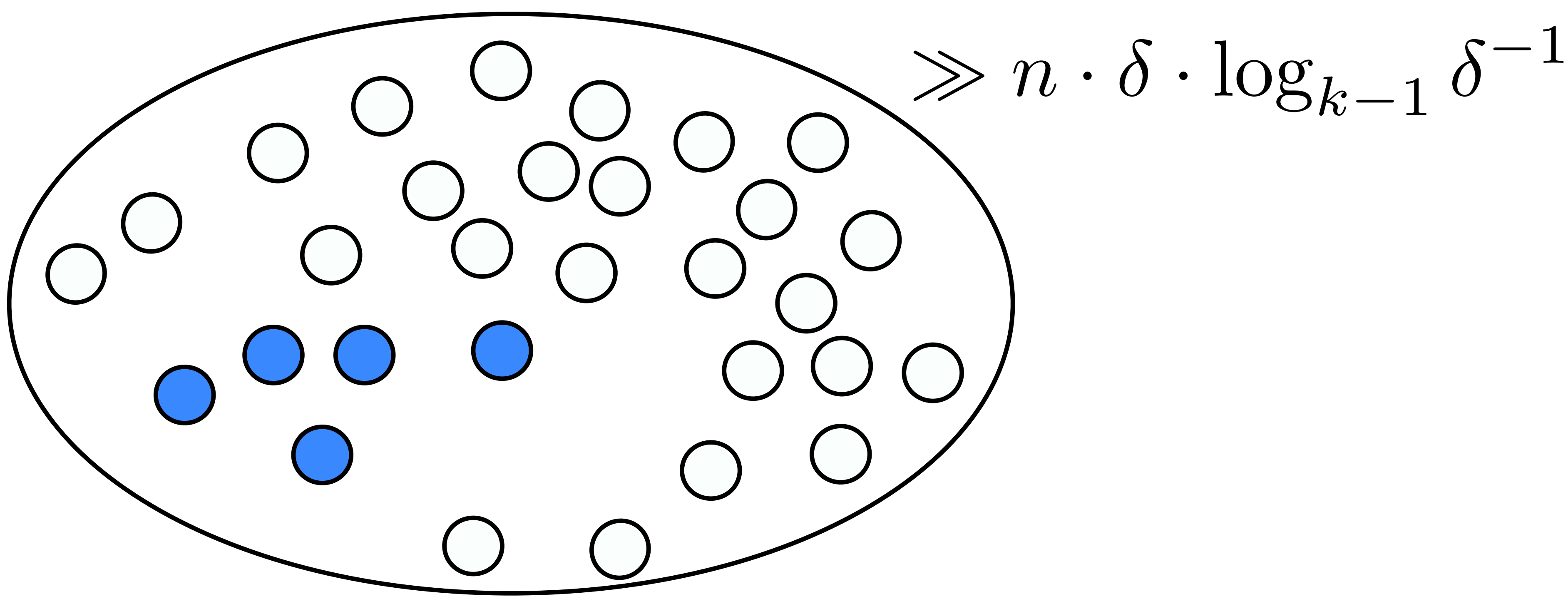

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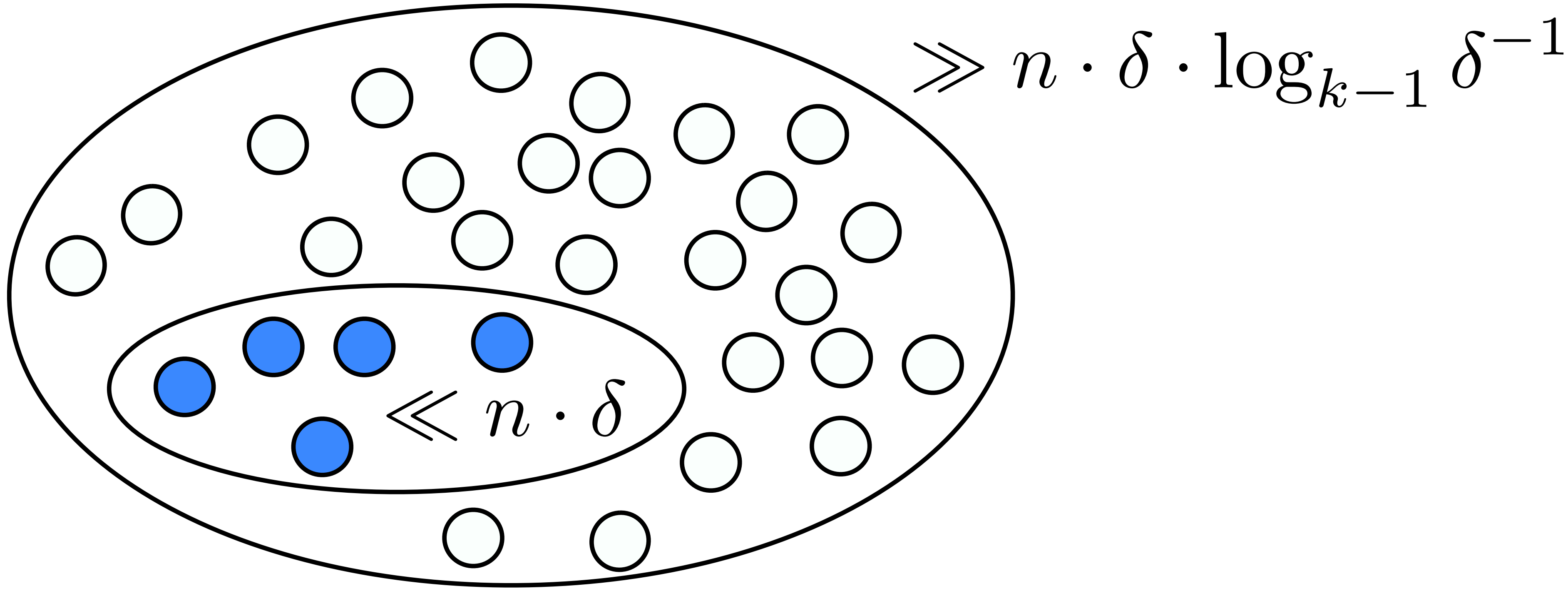

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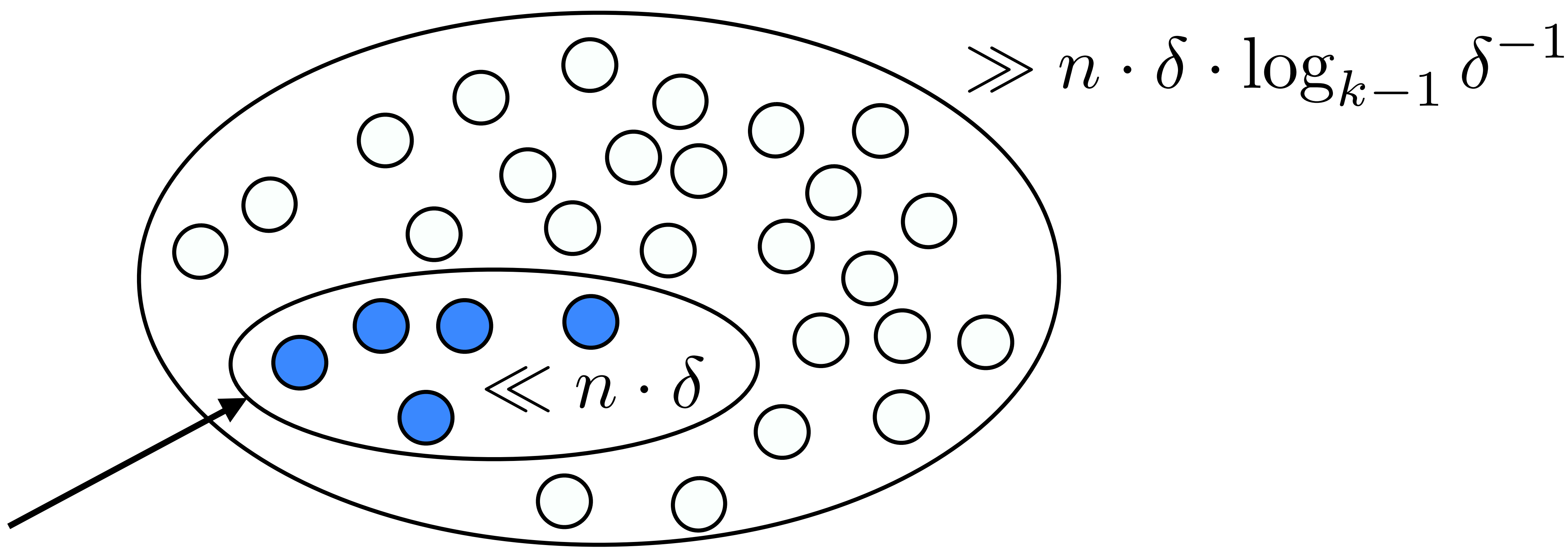

A $\dfrac{1}{\log_{k-1} \delta^{-1}}$ fraction of the set.

Single-Child Fraction

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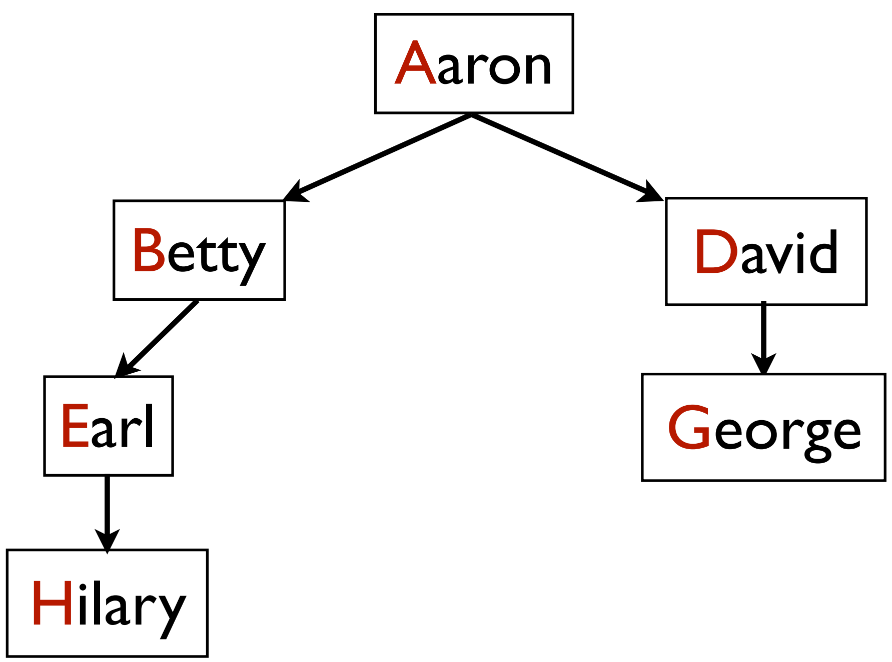
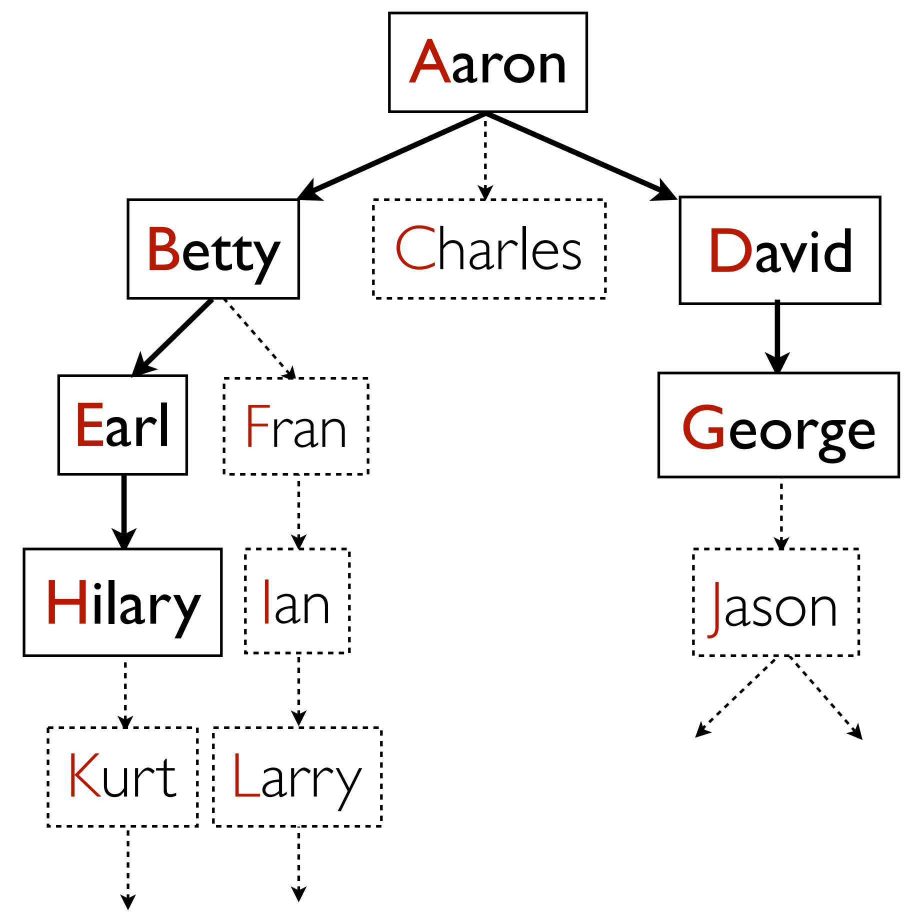
The high single-child fraction can be explained by assuming just a degree bound on the unknown tree

Number of Signers

Revealed Tree

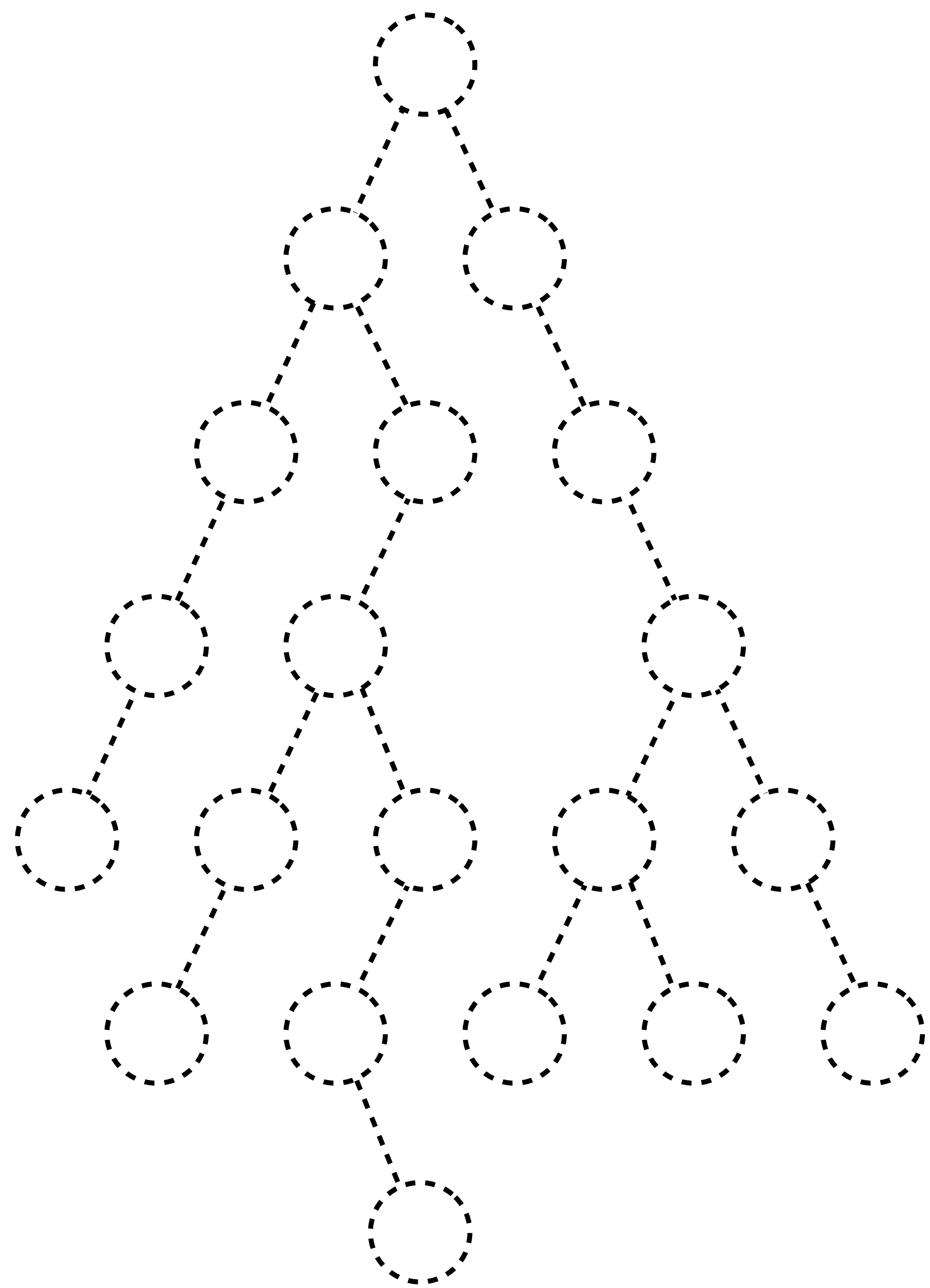


Unknown Tree



How to guess the size of the unknown tree?

Unknown Tree Exposure



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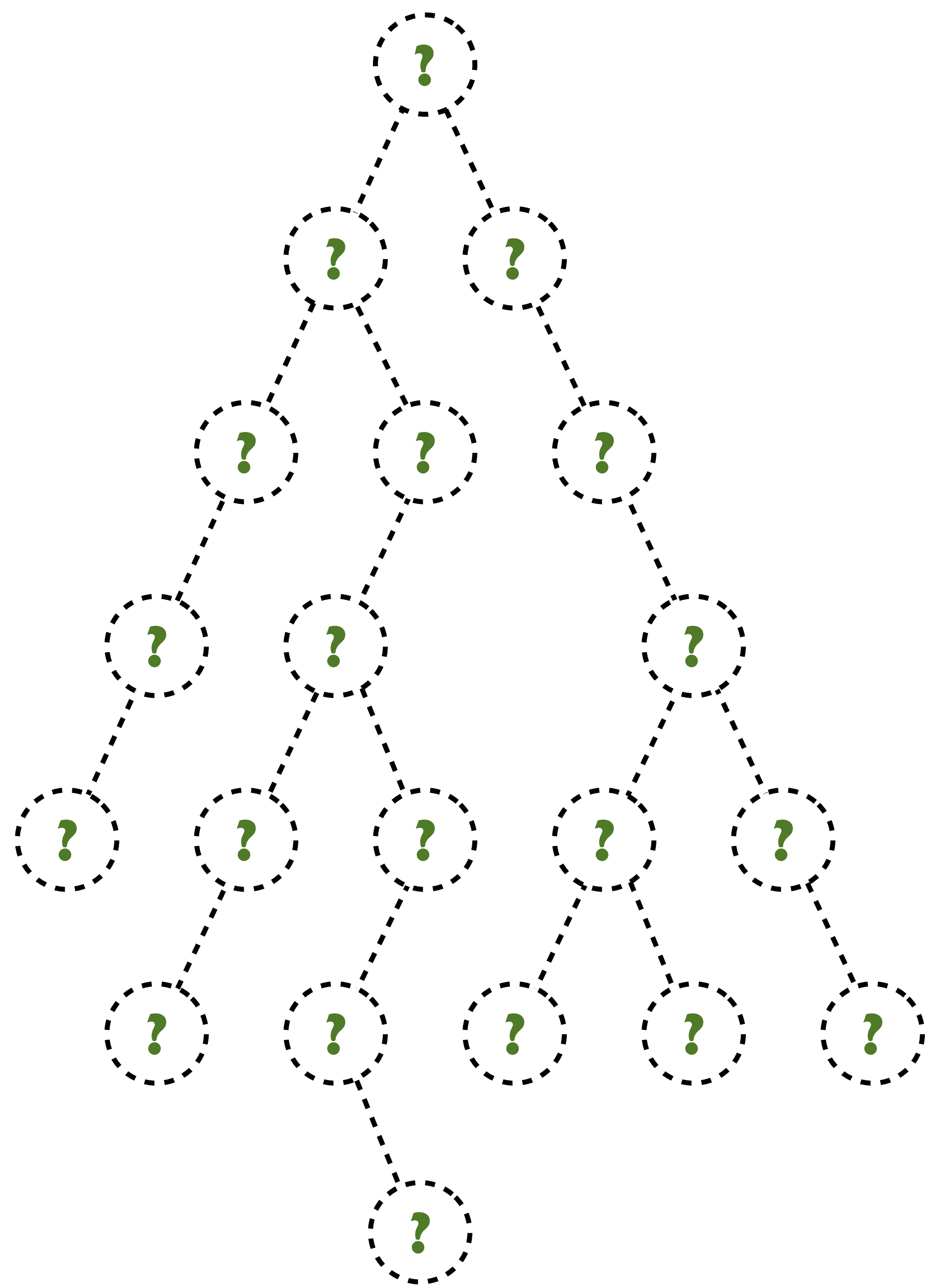

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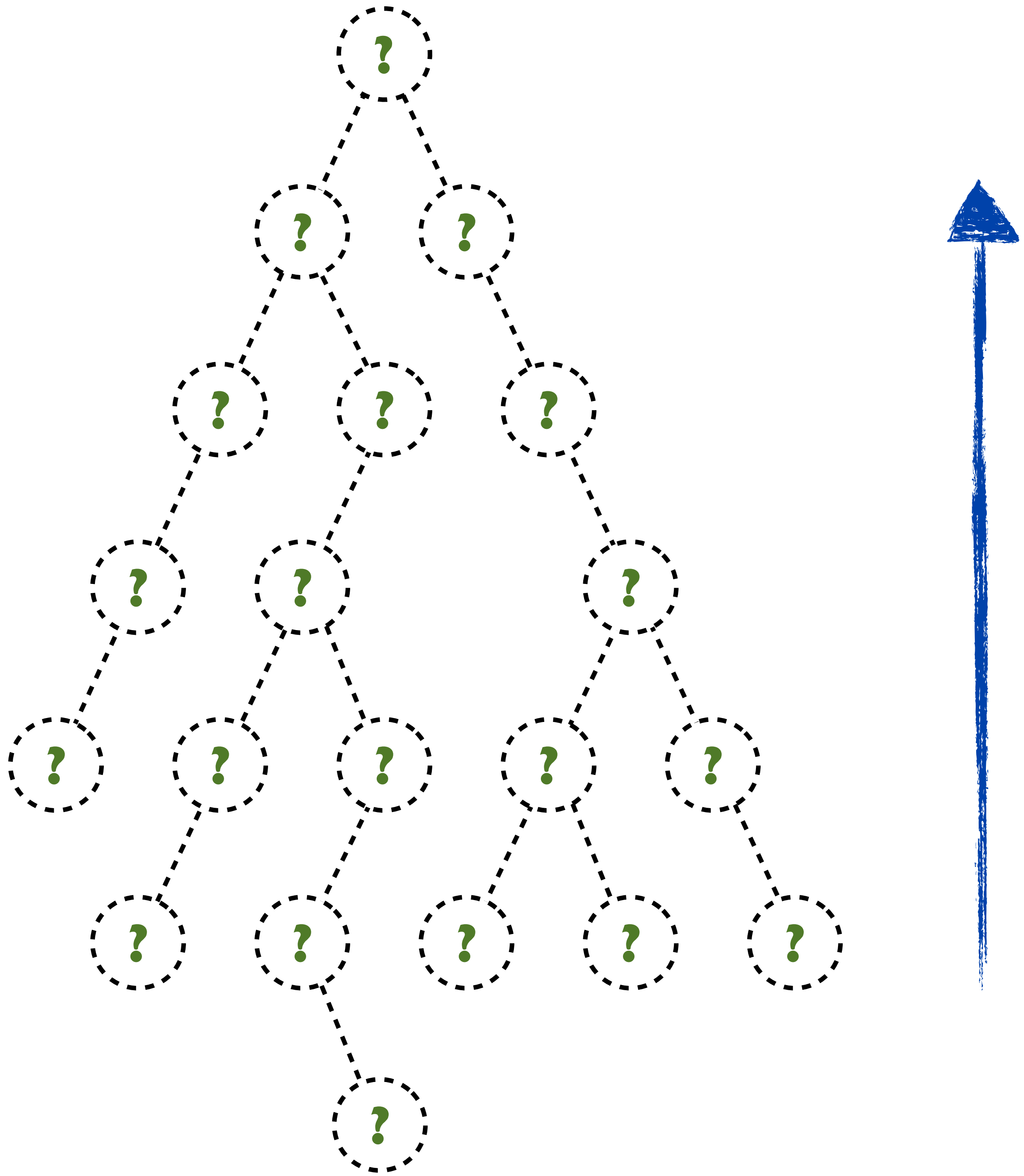
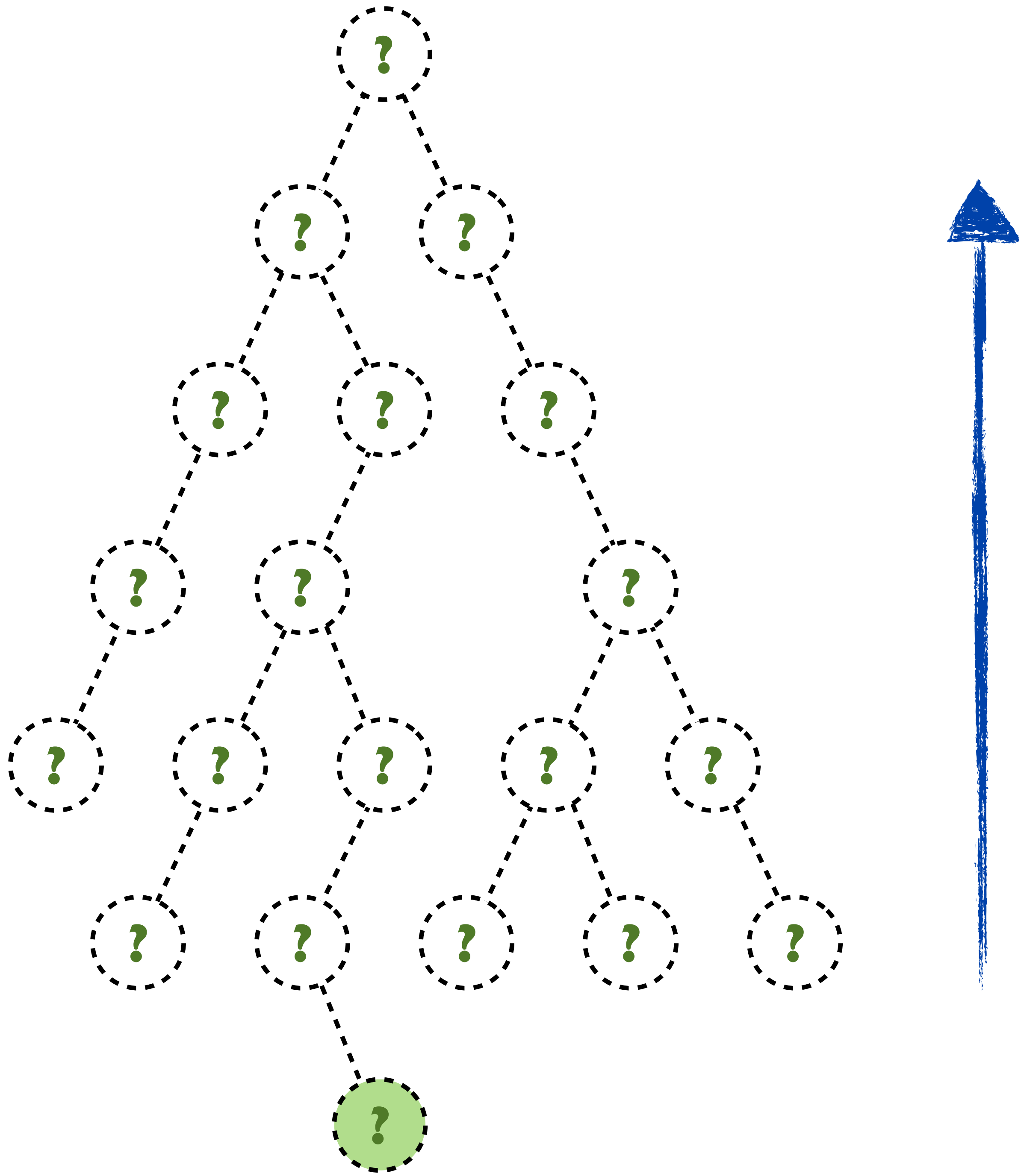


Unknown Tree Exposure



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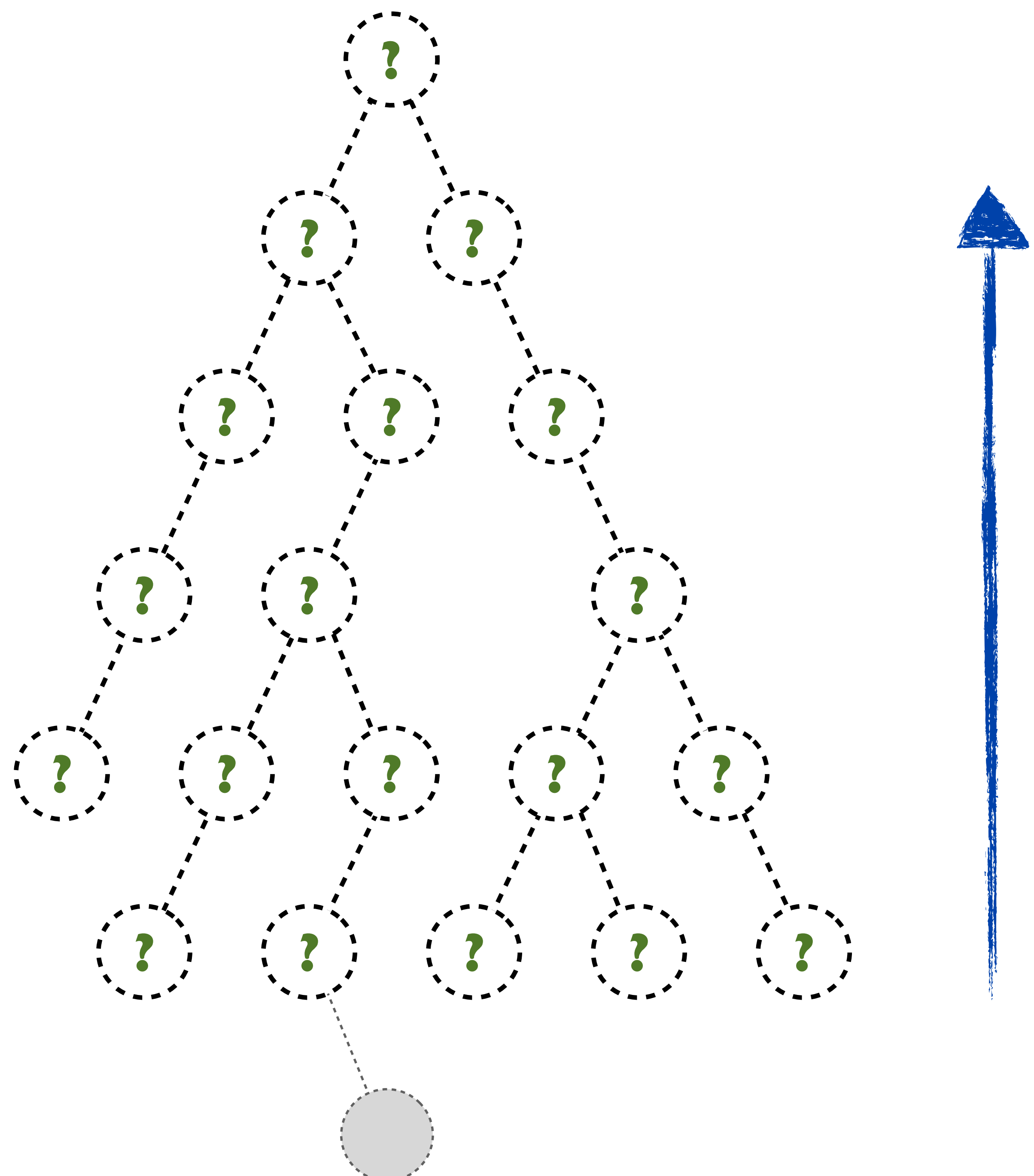

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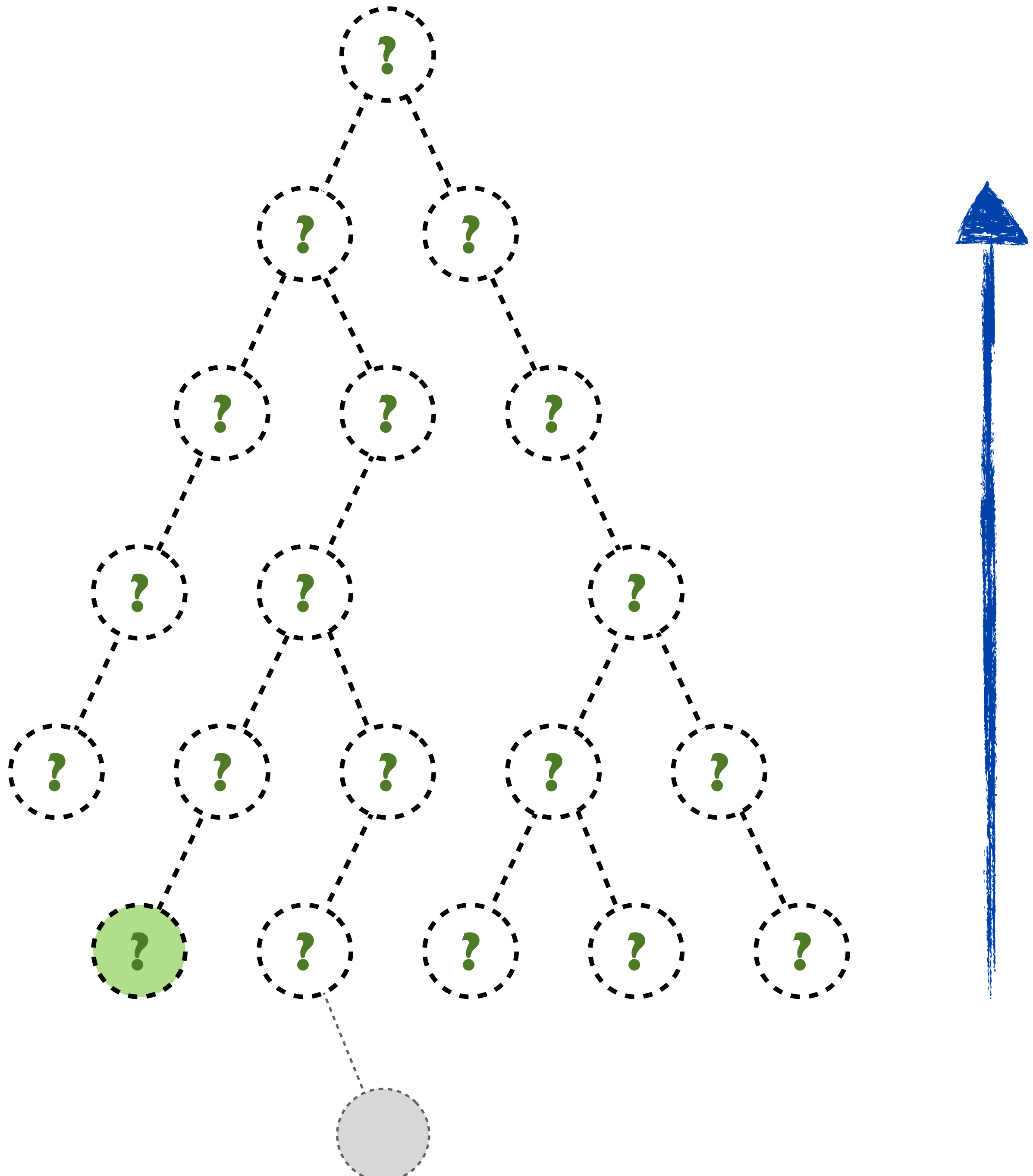

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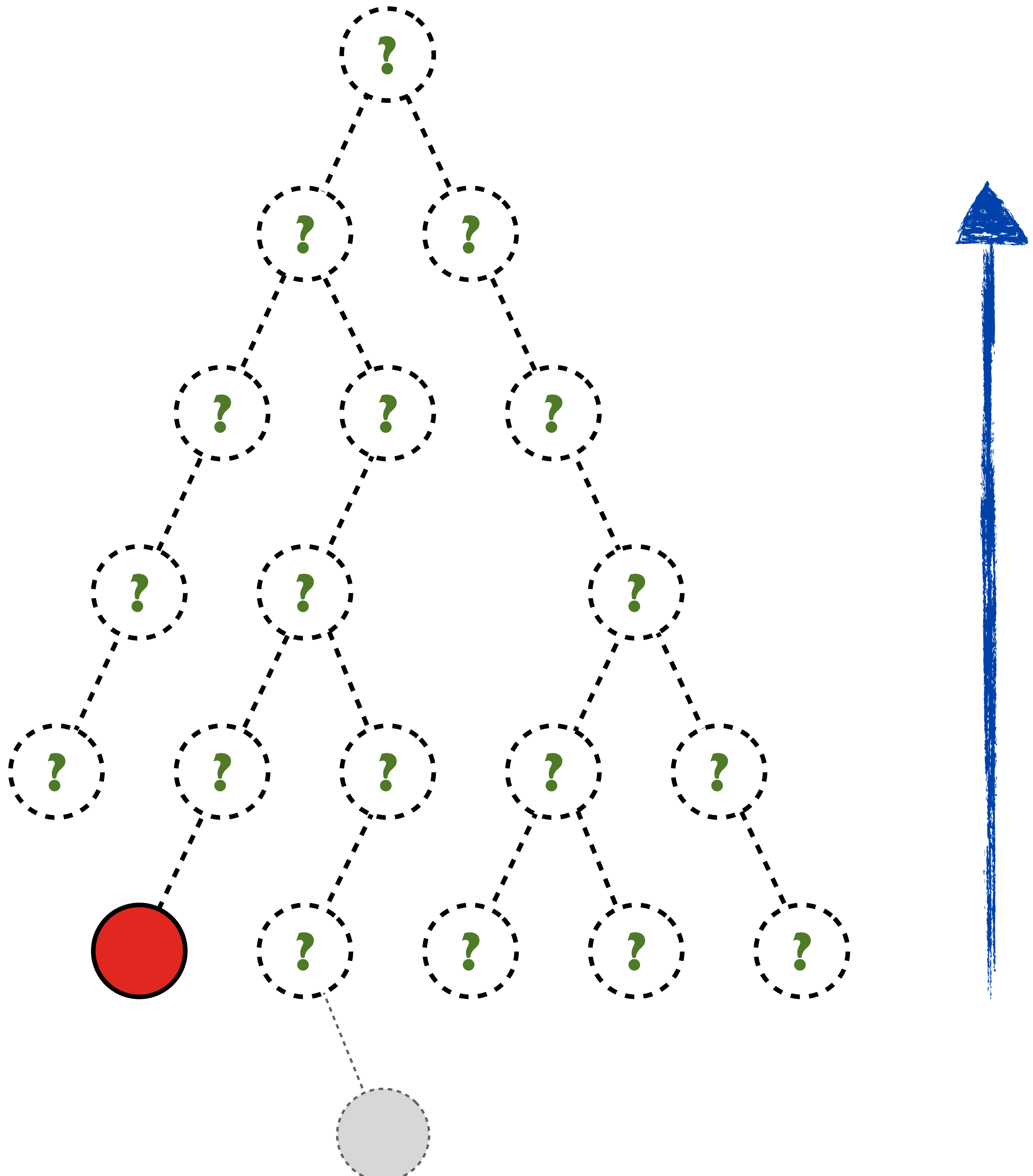

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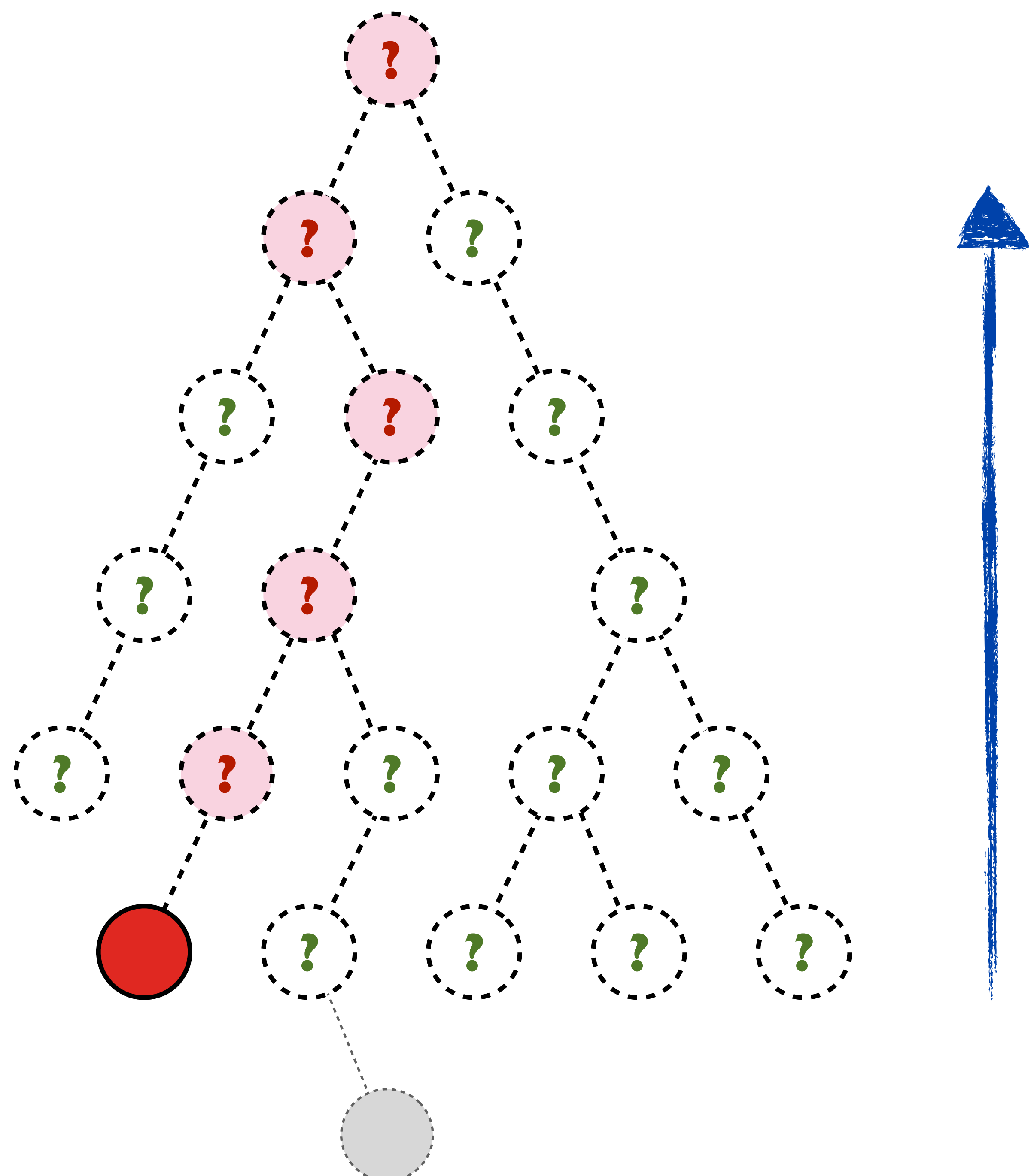

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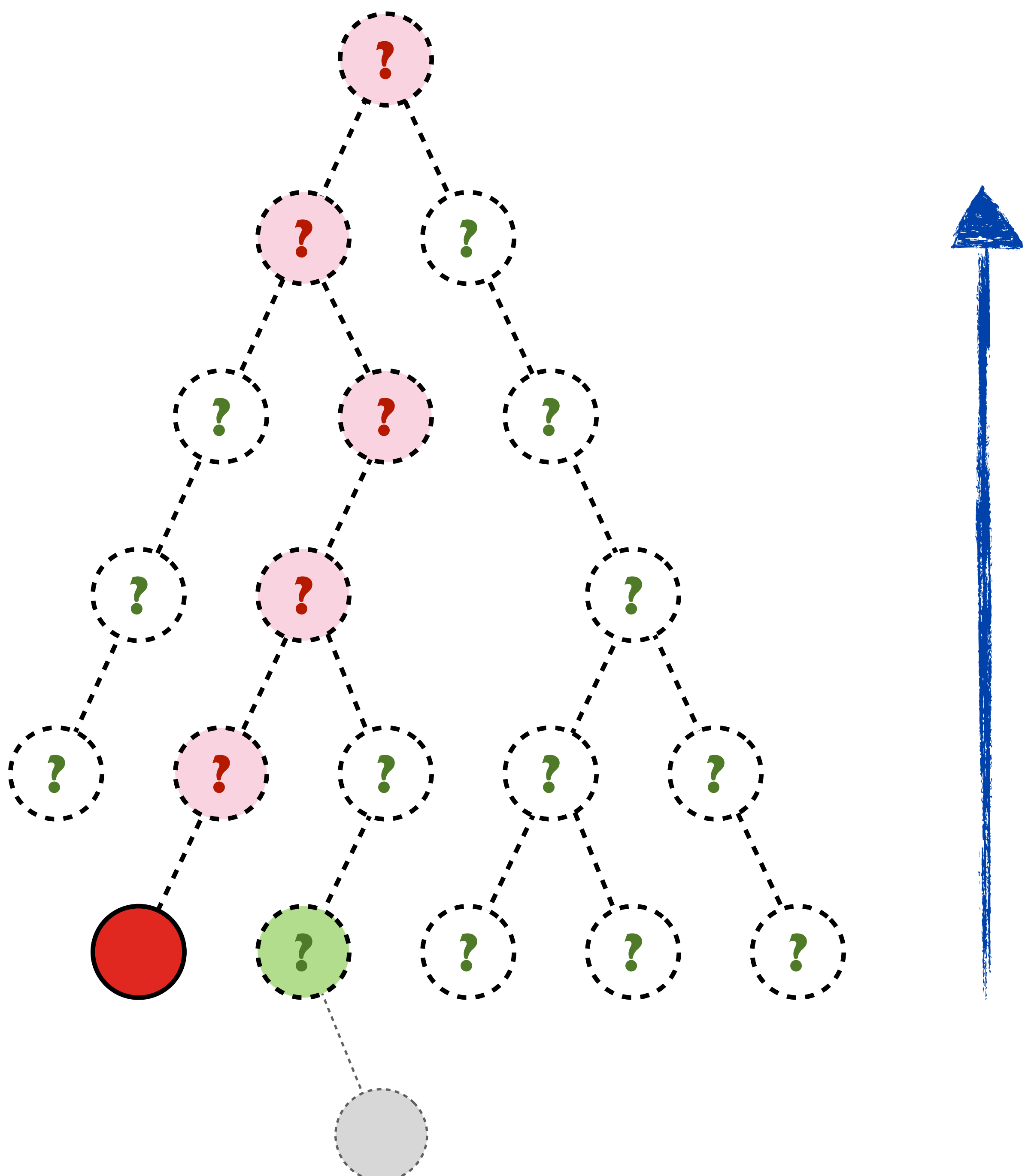

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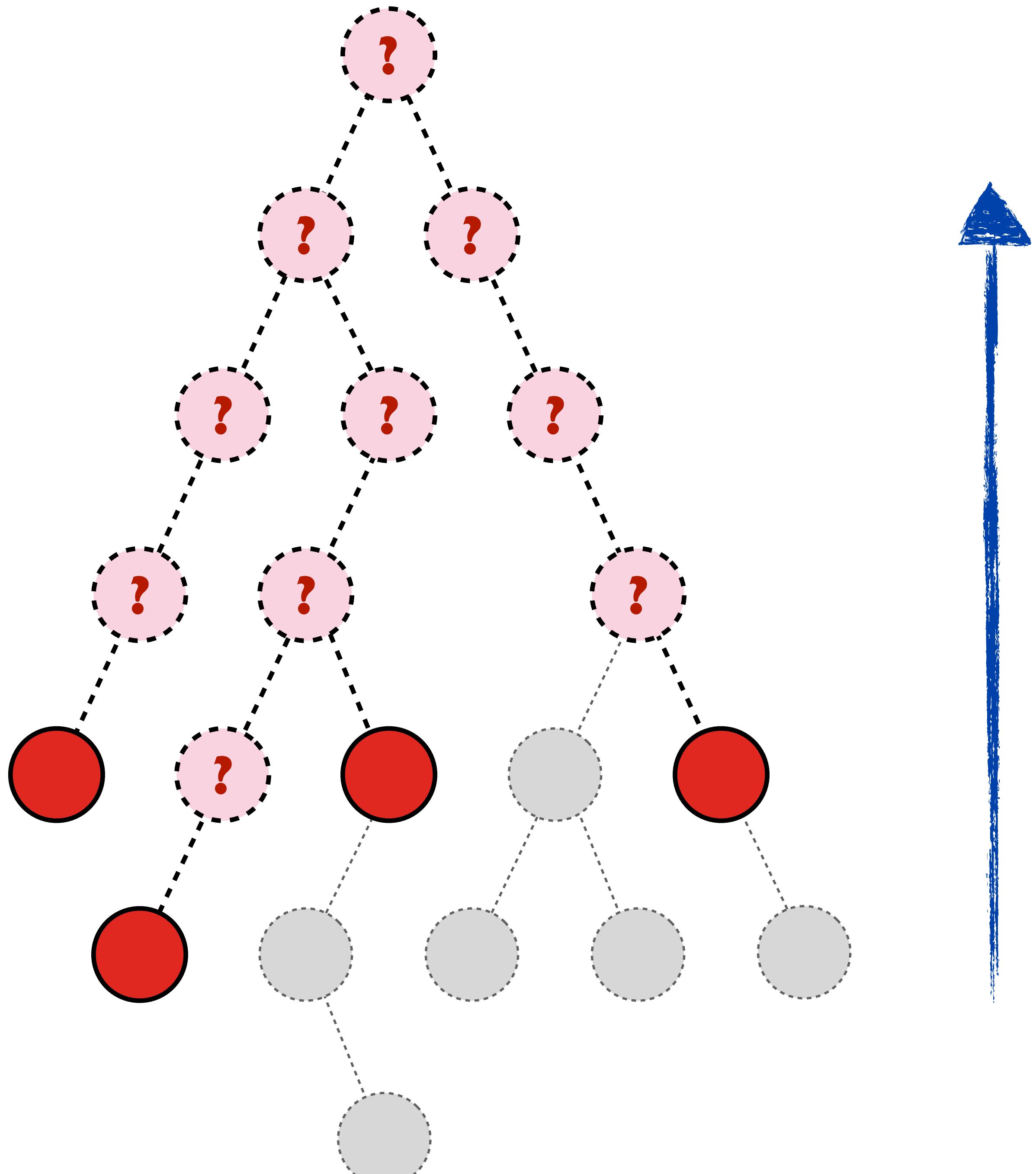
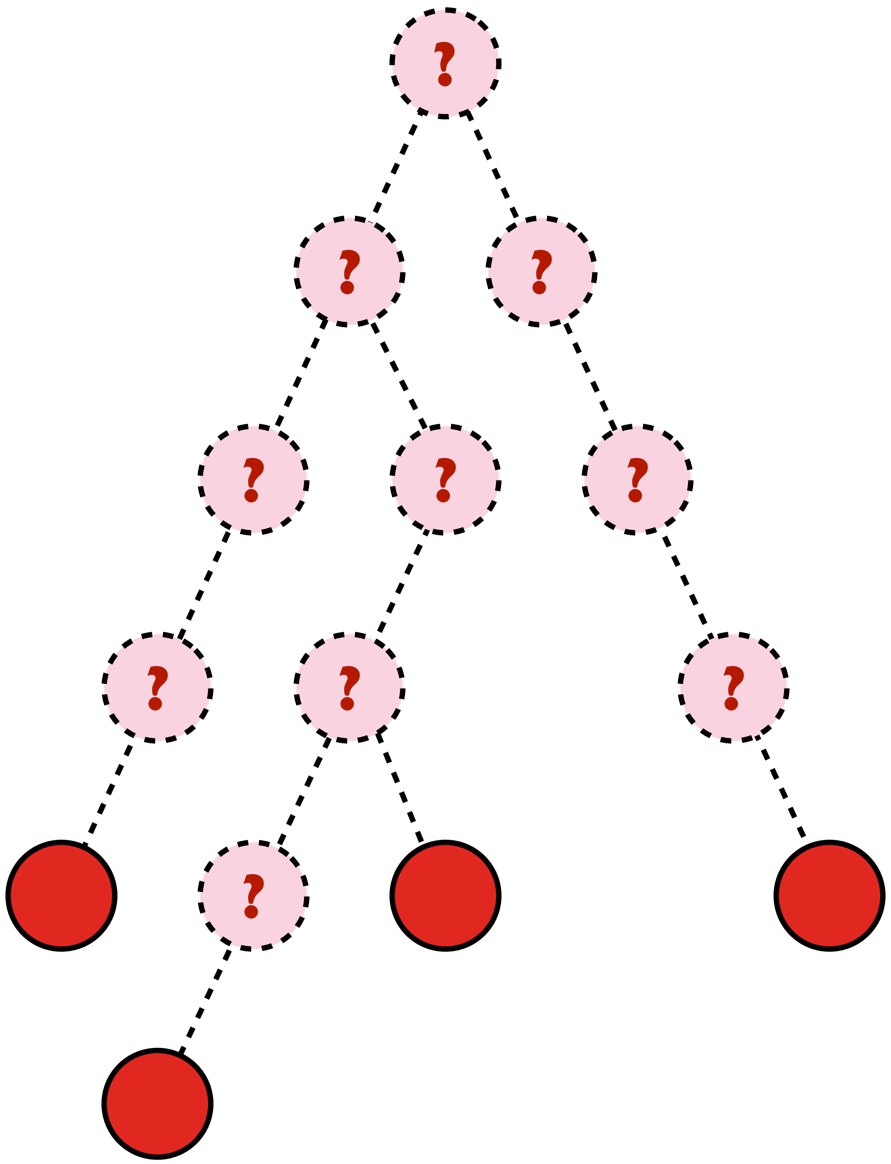
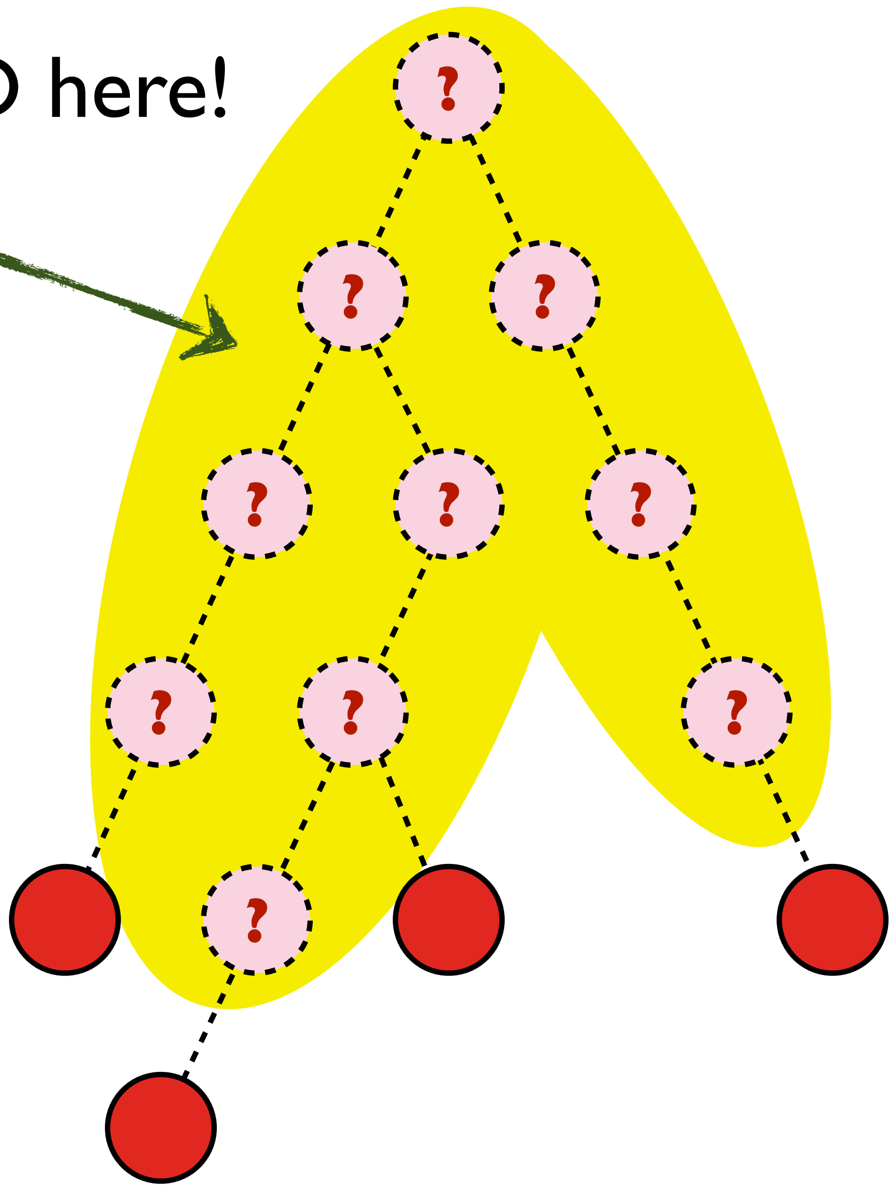


Revealed Tree



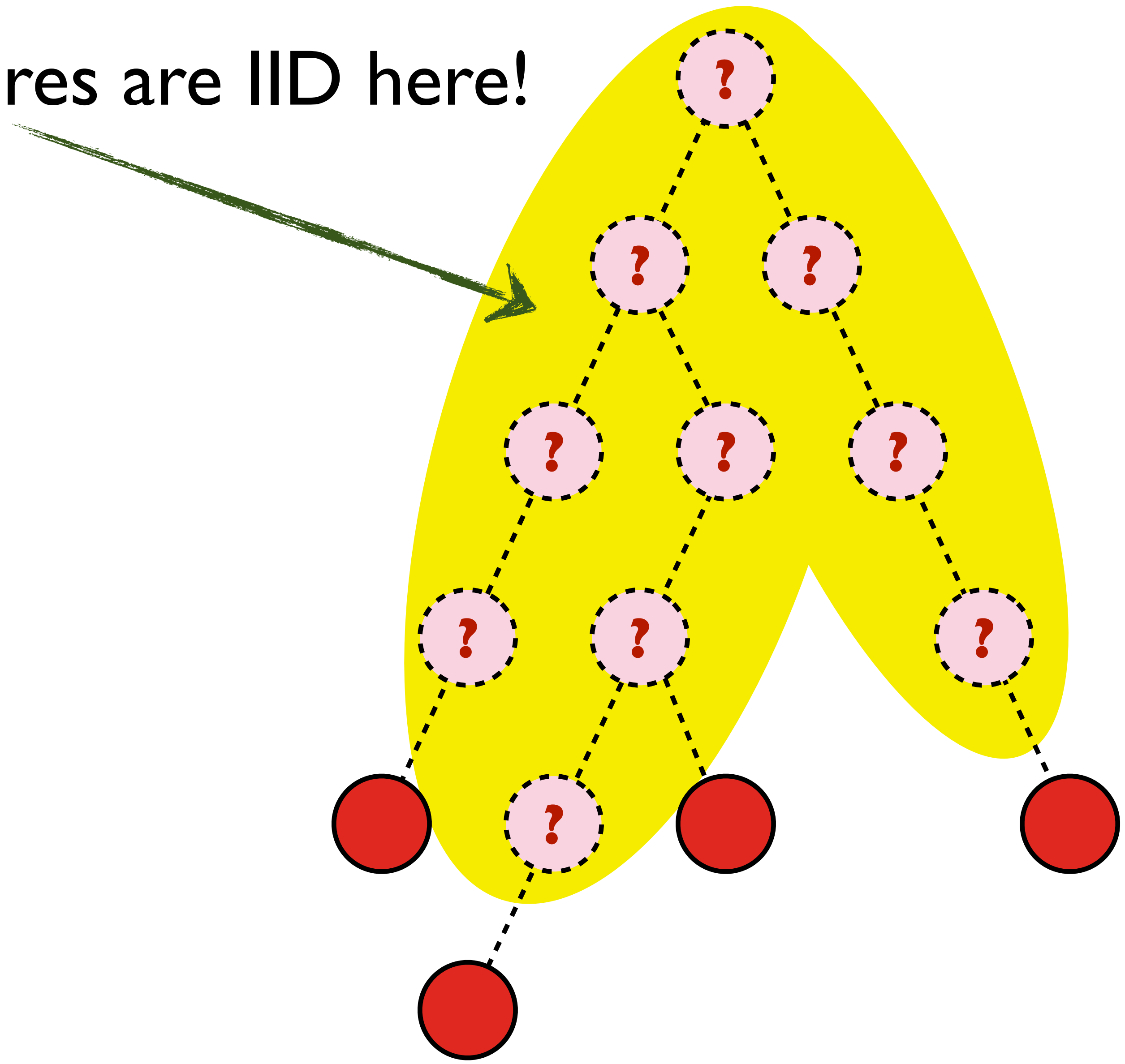
Revealed Tree

Nodes exposures are IID here!



Size Estimation

Nodes exposures are IID here!



1. Estimate δ

Size Estimation

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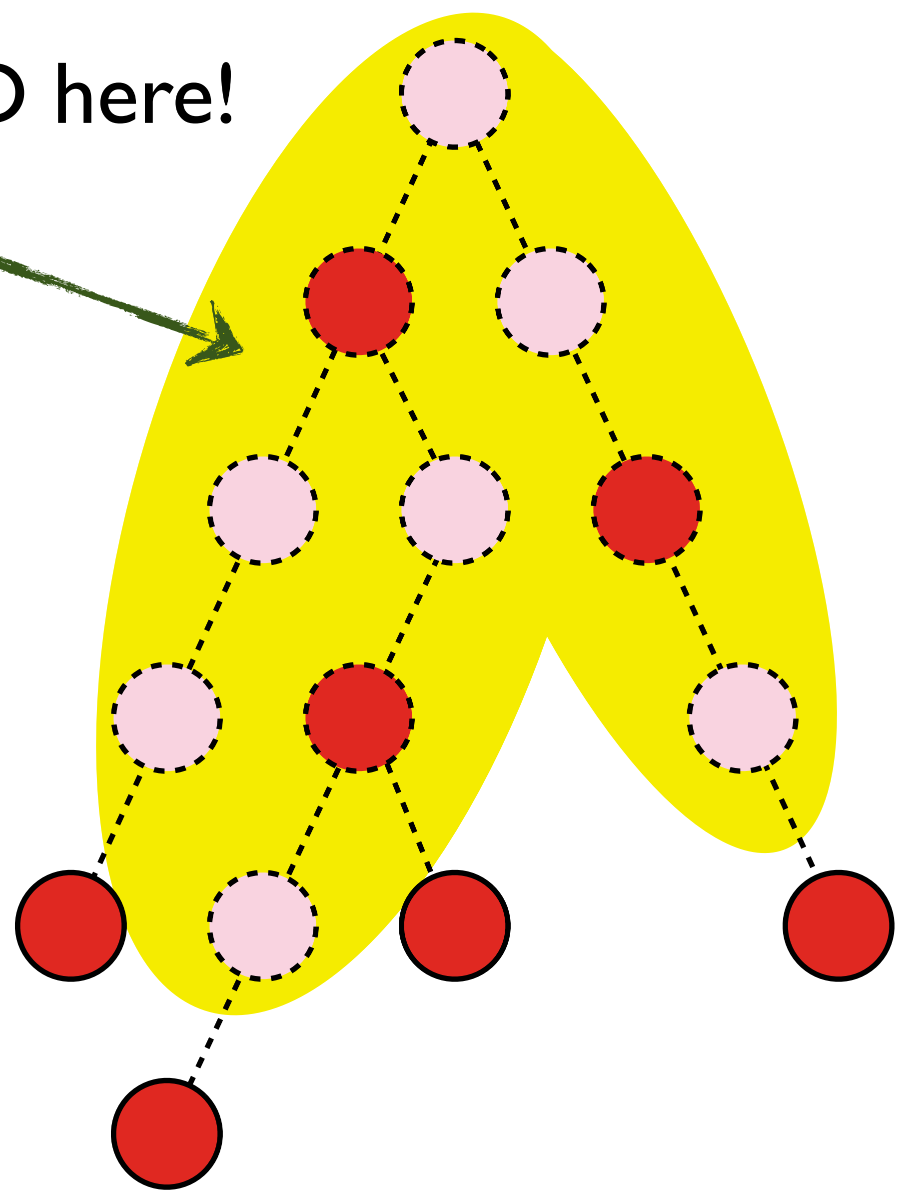

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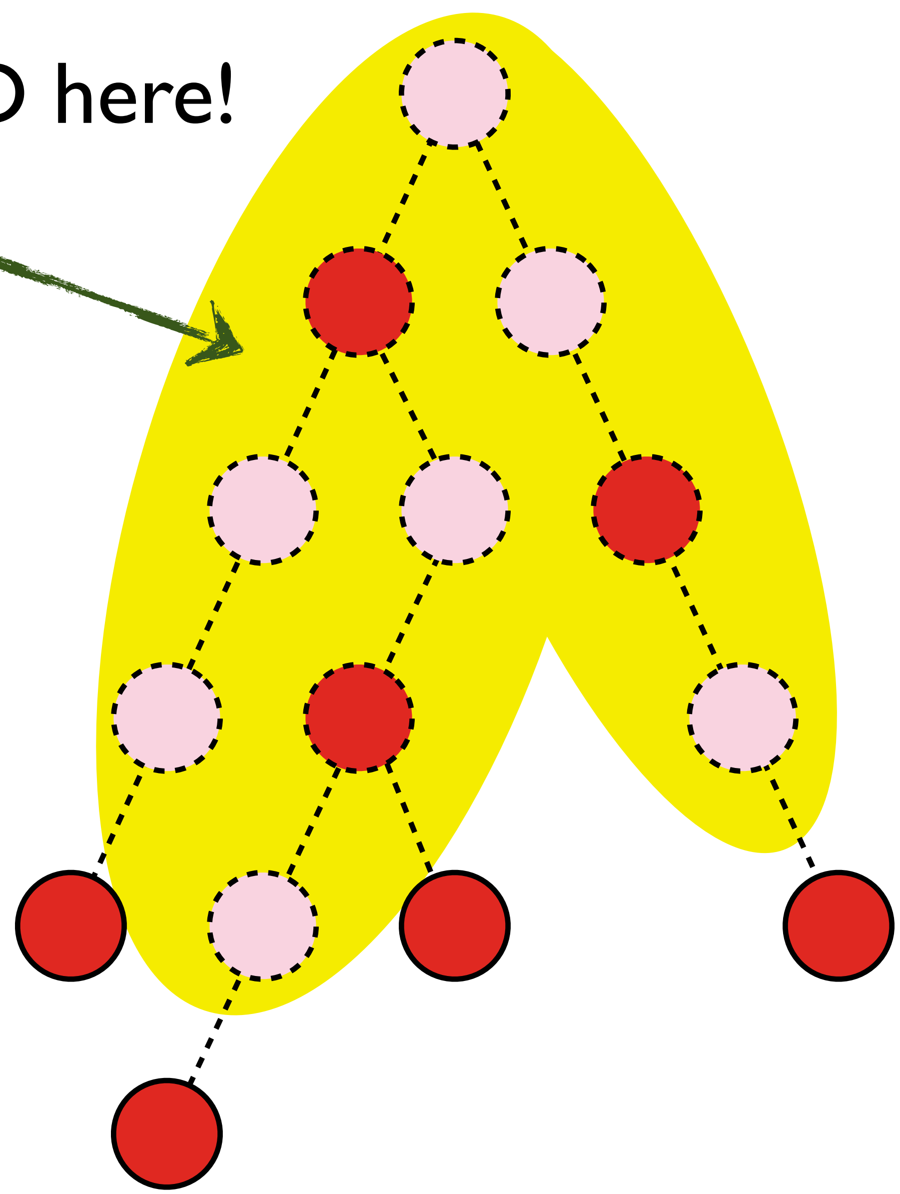

$$\delta \simeq \frac{3}{10}$$

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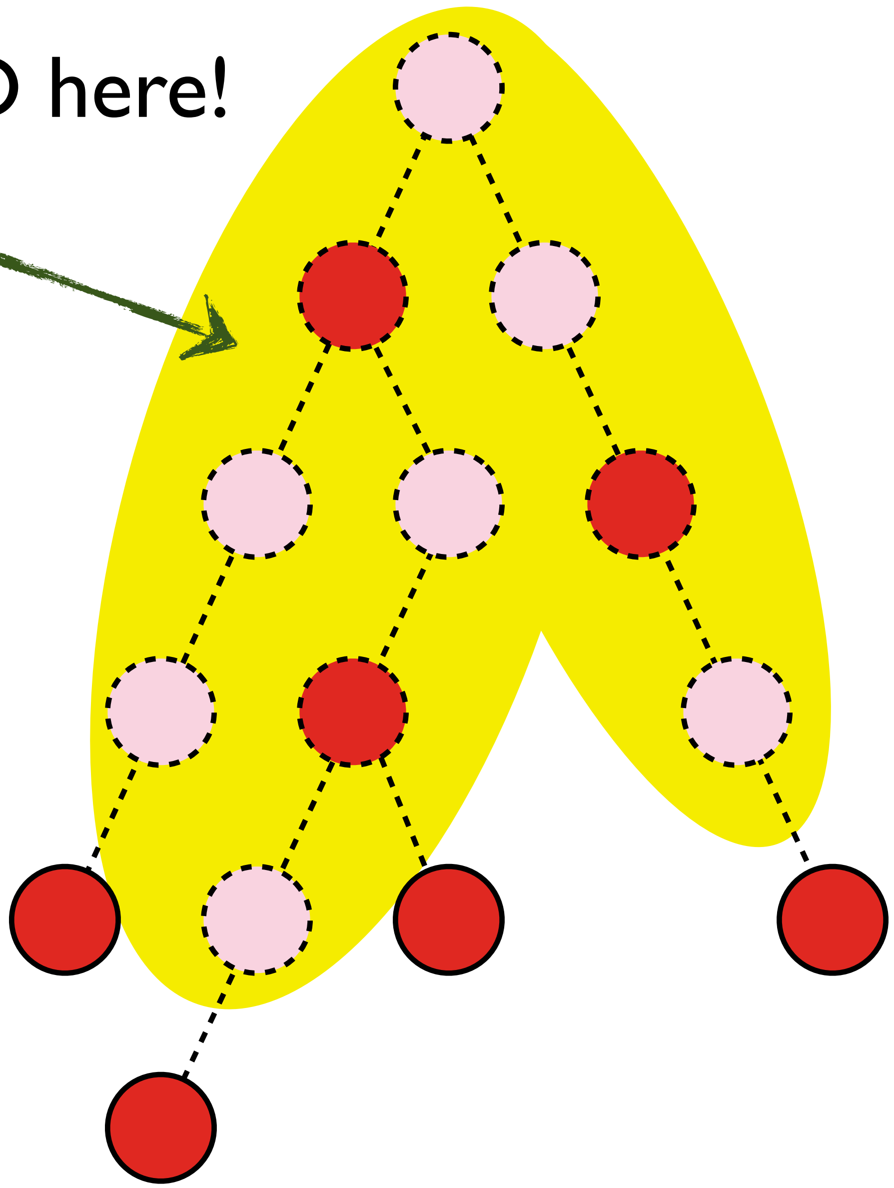

$\delta \simeq \frac{3}{10}$

2. Estimate $n \cdot \delta$ using the number of exposed nodes in the revealed tree

Size Estimation

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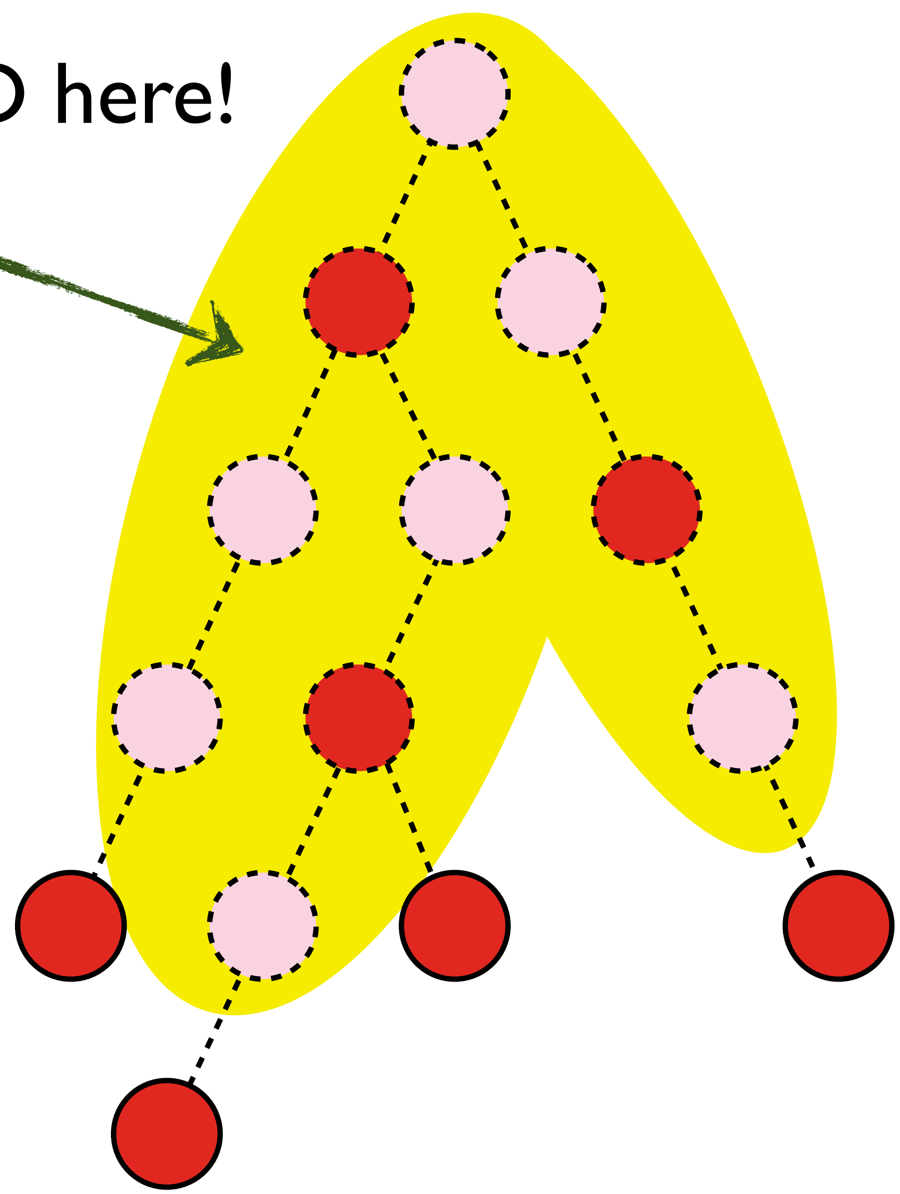

$$\delta \simeq \frac{3}{10}$$

$$n \cdot \delta \simeq 7$$

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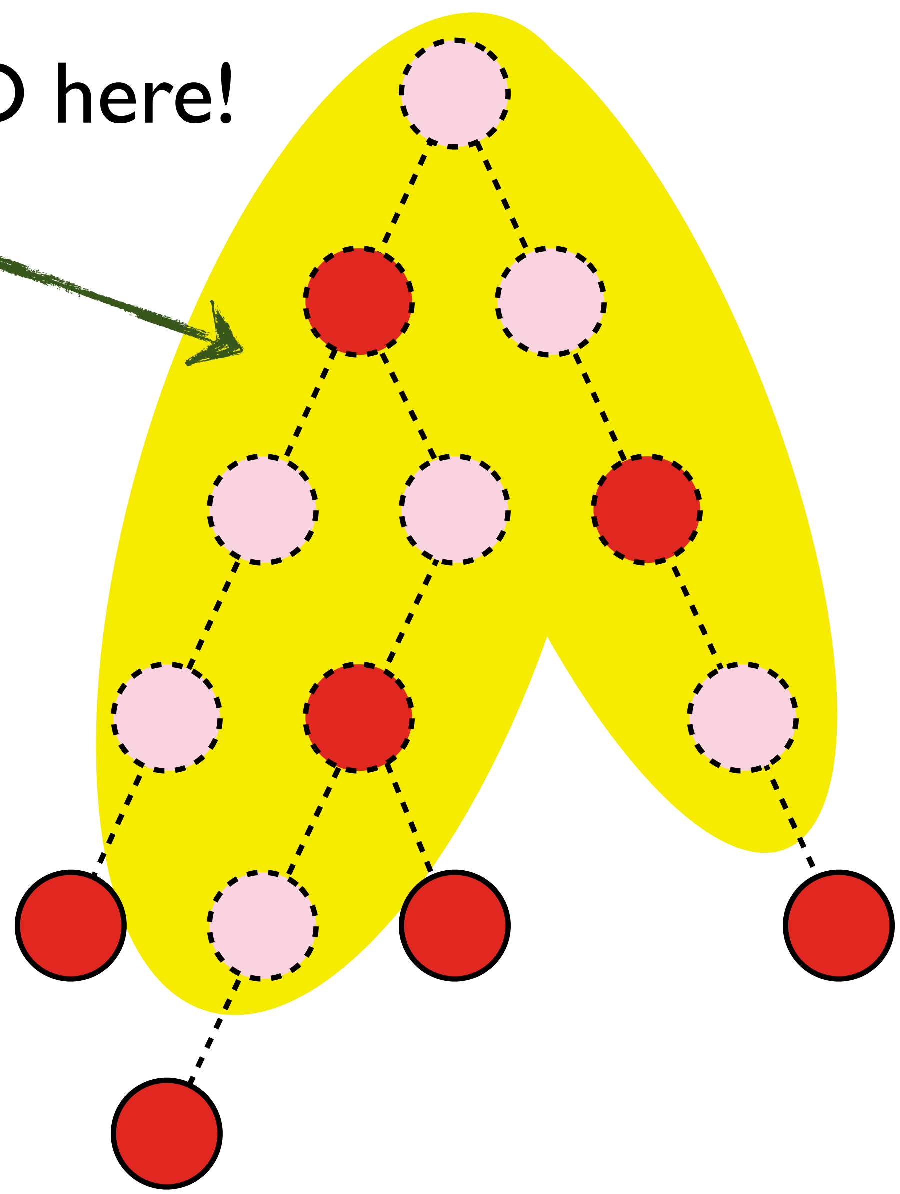

$$\delta \simeq \frac{3}{10}$$

$$n \cdot \delta \simeq 7$$

$$n \simeq 23.\bar{3}$$

3. Take the ratio

Size Estimation

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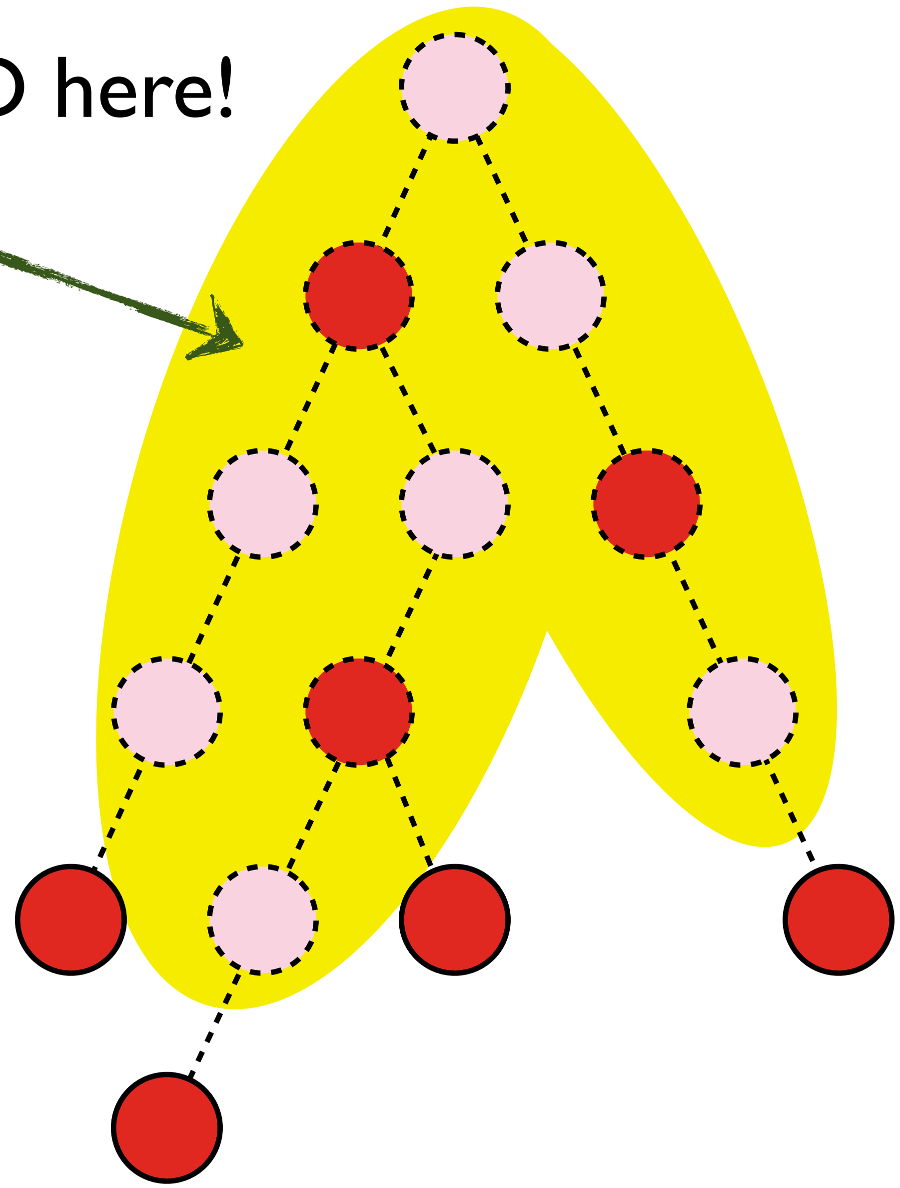

$$\delta \simeq \frac{3}{10}$$

$$n \cdot \delta \simeq 7$$

$$n \simeq 23.\bar{3}$$

What can go wrong?

Size Estimation

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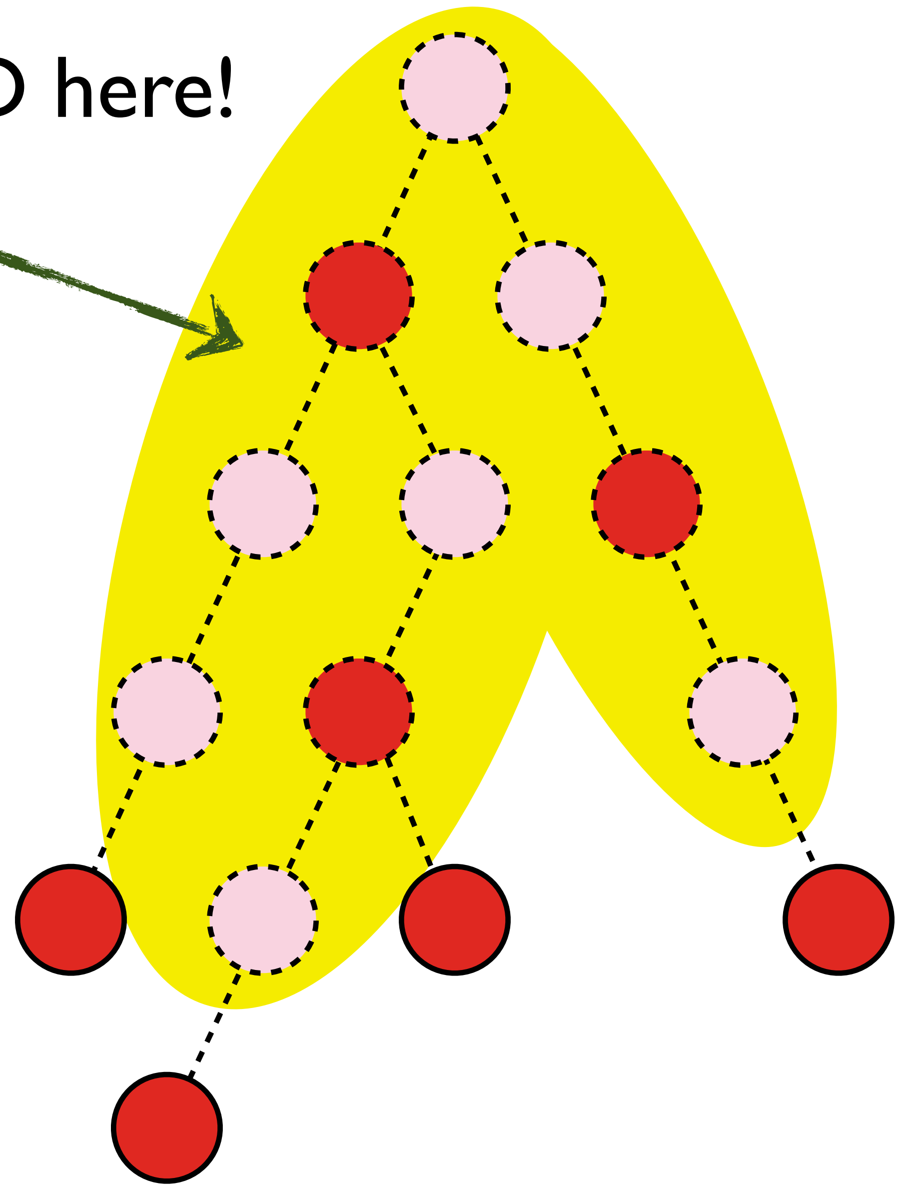

$$\delta \simeq \frac{3}{10}$$

$$n \cdot \delta \simeq 7$$

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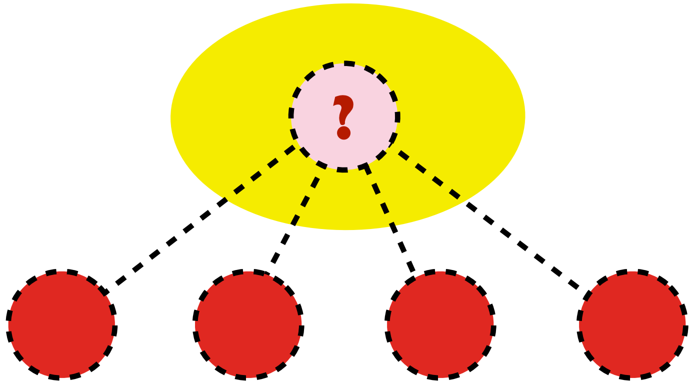
The "yellow area" could contain too few nodes for the estimation of δ to be successful.

Size Estimation



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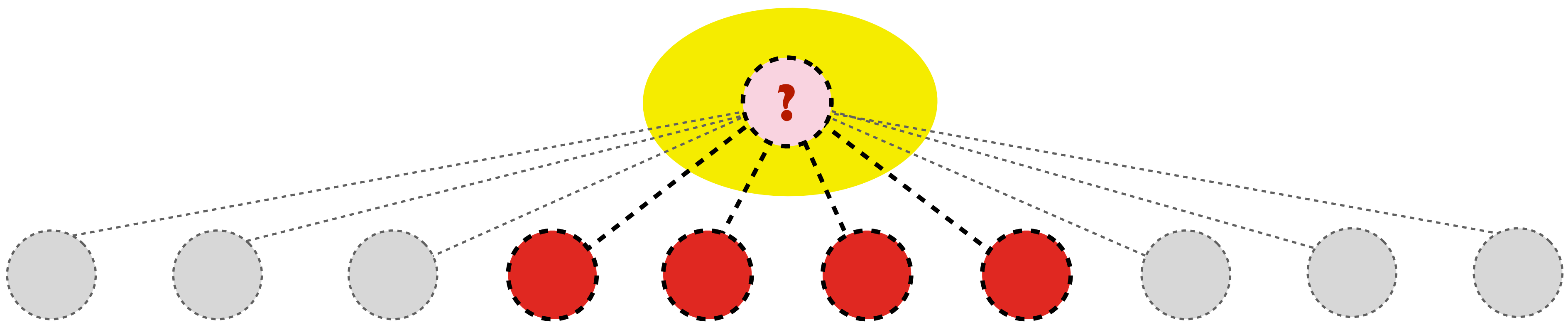

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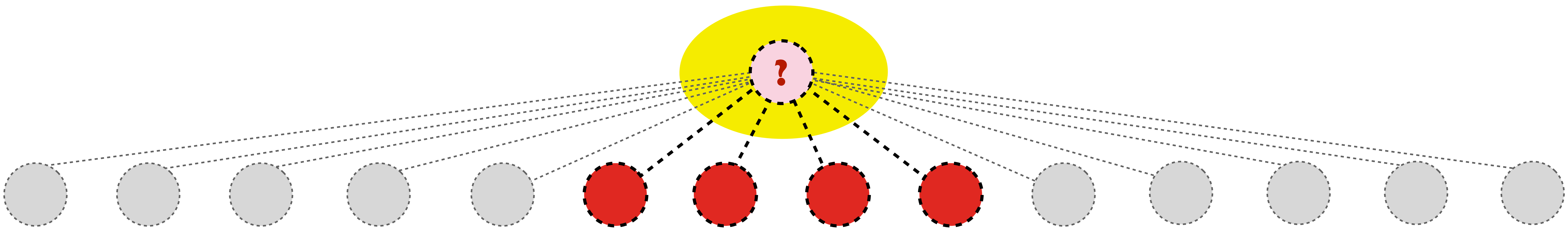

The "yellow area" could contain too few nodes for the estimation of δ to be successful.

Theorem

- The previous algorithm can guess the size with high probability if

$$n > \tilde{\Omega}\left(\max\left(\delta^{-2}, \delta^{-1} \cdot k\right)\right)$$

k is the maximum number of children in the unknown tree,
δ is the exposing probability.

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$k < \tilde{O}\left(\sqrt{n}\right)$ $\delta > \tilde{\Omega}\left(\sqrt{\frac{1}{n}}\right)$ satisfy the requirement

Theorem

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$$n > \tilde{\Omega}\left(\max\left(\delta^{-2}, \delta^{-1} \cdot k\right)\right)$$

  k is the maximum number of children in the unknown tree,
  δ is the exposing probability.

- No algorithm can do it if n is smaller.

IRAQ Tree Size

- We refined our asymptotic theorem for the IRAQ revealed tree (18k nodes)
- Assuming the tree-revealing model,
  we estimate that
  the number of signers of the IRAQ petition
  is within a factor of 2 of 173k
  with probability $\geq 95\%$

Conclusion

Reconstruction Problems

- Answers depend on:
  - the random model that drives the process;
  - the number of traces we have;
  - the random algorithm that reveals pieces of information by “cutting” them out of a model run.

Random Model

- It is important to check whether the random model is close to reality;

- without this step, the guesses might be “close” to the model’s runs, but very far from reality.

This step is possibly the hardest one to do.

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- without this step, the guesses might be “close” to the model’s runs, but very far from reality.

This step is possibly the hardest one to do.

Checking whether the guesses
are close to reality is sometimes
the only viable approach.

Number of Traces

- If we want to get a significant understanding of some partially-known process,

- we first and foremost need to verify whether the questions we are asking can be significantly answered by the amount of data we have.

This step can be carried out
(theorems and/or simulations)

Trace Generation

- The number of “traces” needed depends strongly on which pieces of information of a model’s run are revealed (as well as on the random model).

- In some cases, it might be possible to get more informative traces by a deeper mining of the data.

Complexity of Guessing

- **If not enough traces are available, we should simplify the questions** we ask about the unknown process.

- Otherwise, our guesses might be completely off and insignificant.

Complexity of Guessing

- How many traces do we need for other guessing tasks?
- Very, very, rich area of problems.

Thanks!