Graphs of Edge-Intersecting and Non-Splitting Paths

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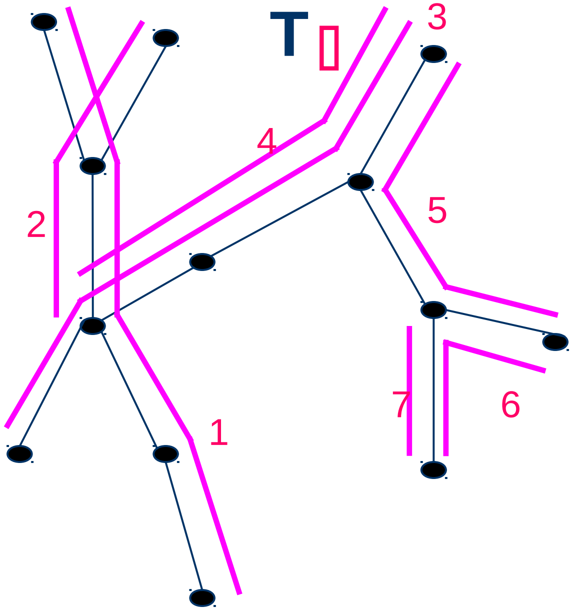
EPT and EPG Graphs

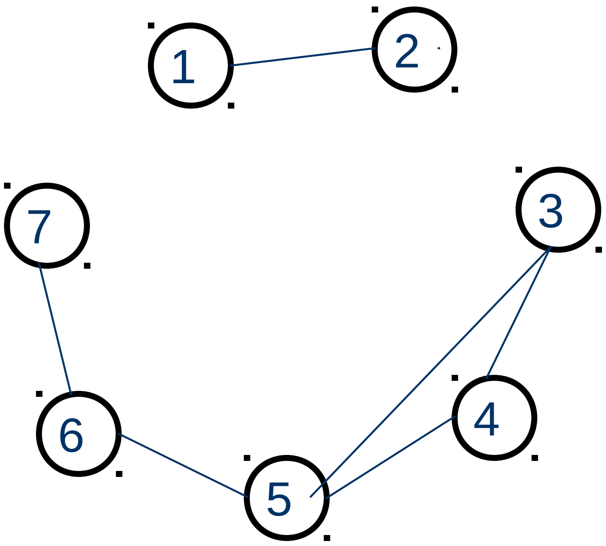
[1] Golumbic, M. C. & Jamison, R. E. (1985), 'The edge intersection graphs of paths in a tree', *Journal of Combinatorial Theory, Series B* **38**(1), 8 - 22.

[2] Golumbic, M. C.; Lipshteyn, M. & Stern, M. (2009), 'Edge intersection graphs of single bend paths on a grid', *Networks* **54**(3), 130-138.

[3] Heldt, D.; Knauer, K. & Ueckerdt, T. (2013), 'Edge-intersection graphs of grid paths: the bend-number', *Discrete Applied Mathematics*.

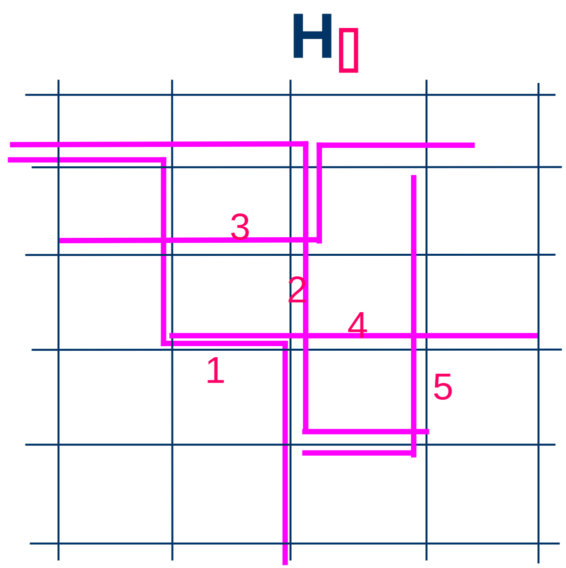
The EPT Graph EPT(□)

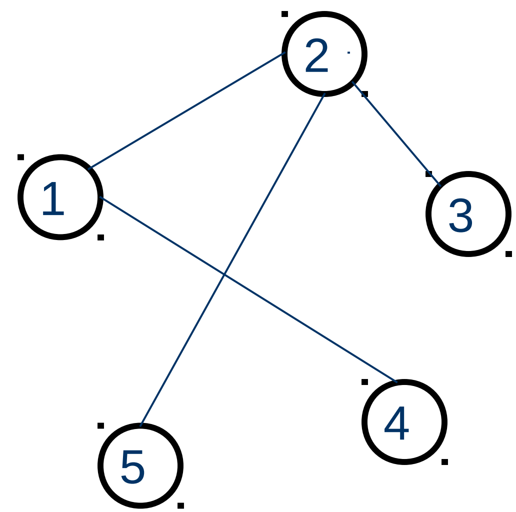




In this talk “intersection” means “edge intersection”

The EPG Graph EPG(□)





- A graph is B_k-EPG if it has a representation with paths of at most k bends.

(This is a B_3-EPG graph)

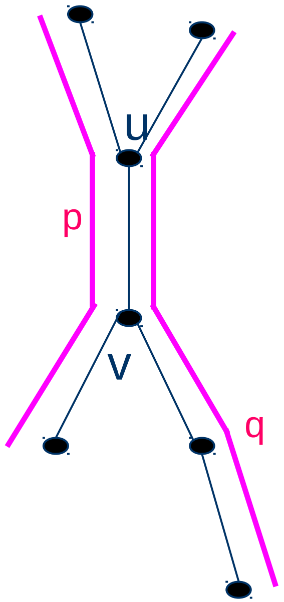
Results

- [2] Every graph is EPG
- [3] $B_1 - EPG \not\subset B_2 - EPG \not\subset$ K

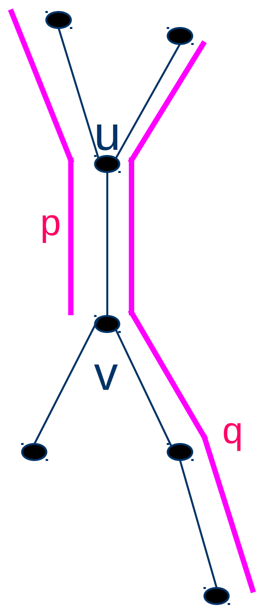
ENPT Graphs

[4] Boyacı, A.; Ekim, T.; Shalom, M. & Zaks, S., Graphs of Edge-Intersecting Non-Splitting Paths in a Tree: Towards Hole Representations, (WG2013)

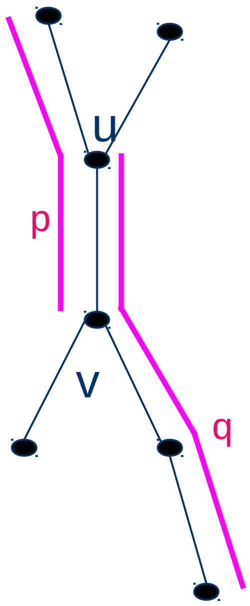
(Edge) Intersecting Paths (on a tree)



$$split(p,q)=\{u,v\}$$

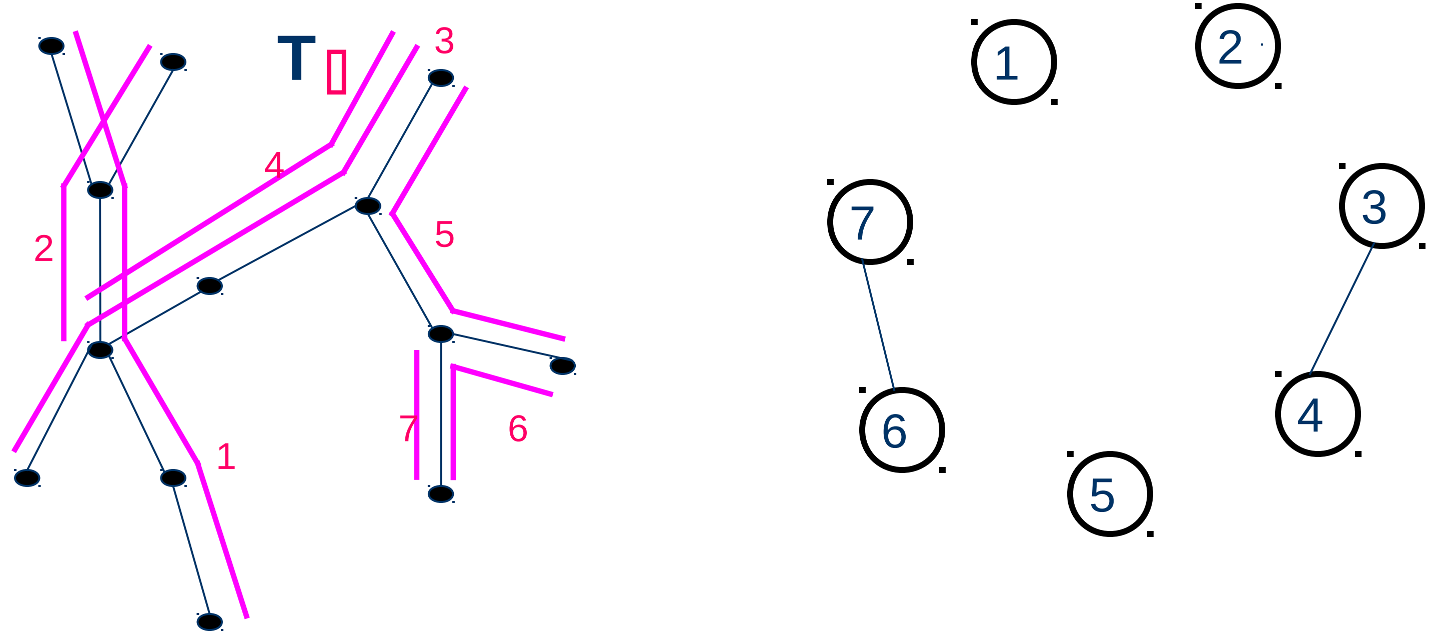


$$split(p,q)=\{u\}$$



$$split(p,q)=\emptyset$$

The ENPT Graph ENPT($\mathcal{P}$)



$$V(ENPT(\mathcal{P})) = V(EPT(\mathcal{P})) = \mathcal{P}$$

$$E(ENPT(\mathcal{P})) \subseteq E(EPT(\mathcal{P}))$$

ENP/ENPG Graphs

Graphs of Edge-Intersecting and Non-Splitting Paths / in a Grid

Our Results

- ENP=ENPG
- Not every graph is ENPG
- $B_{72^{2^{0}-1}} - ENPG \not\subset B_{72^{2^{1}-1}} - ENPG \not\subset \ldots$

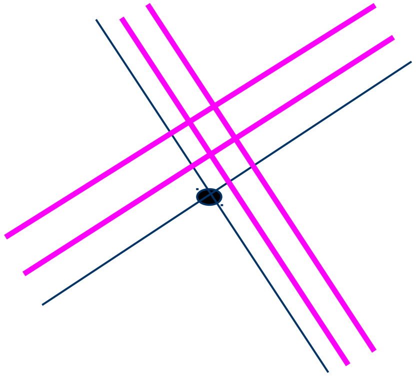
ENP = ENPG

ENP = ENPG

1) Every representation on any host graph H can be embedded in a plane in general position:

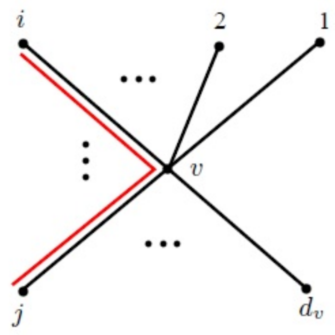
- Edges are embedded to straight line segments
- At most two edges intersect at any given point

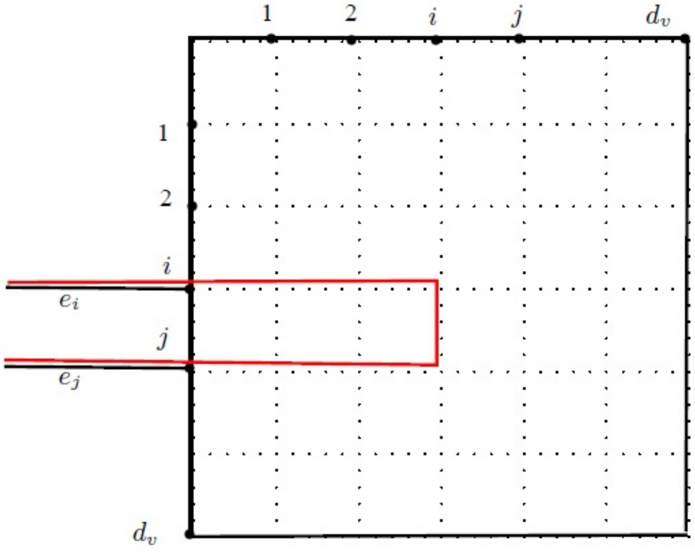
2) H’ is planar.



ENP = ENPG

3) H’’ is planar with maximum degree at most 4.





4) Yanpei et al. (1991) → H’’’ is a Grid.

$$\text{CO-BIPARTITE} \not\subset ENPG$$

Representation of a Clique

- The union of the paths representing a clique is a trail.
- If the trail is open there is an edge that intersects every path.
- If the trail is closed there is a set of at most two edges that intersects every path.

$\text{CO-BIPARTITE} \not\subset ENPG$

- Consider a co-bipartite graph C(K,K',E) with |K|=|K'|=n.
- There are 2^{n^2} such graphs. We now show that the number of possible ENPG representations is at most $(26n)!^3$
- The union of the paths representing a clique is a trail. Moreover, there is a set of at most two edges that intersects every path.
- The intersection of the two trails can be uniquely divided into a set of segments.
- Let S be the set of segments induced by the representations of the cliques K, K'.
- The paths representing two adjacent edges v of K and v' of K' can intersect only in edges of S.

$\text{CO-BIPARTITE} \not\subset ENPG$

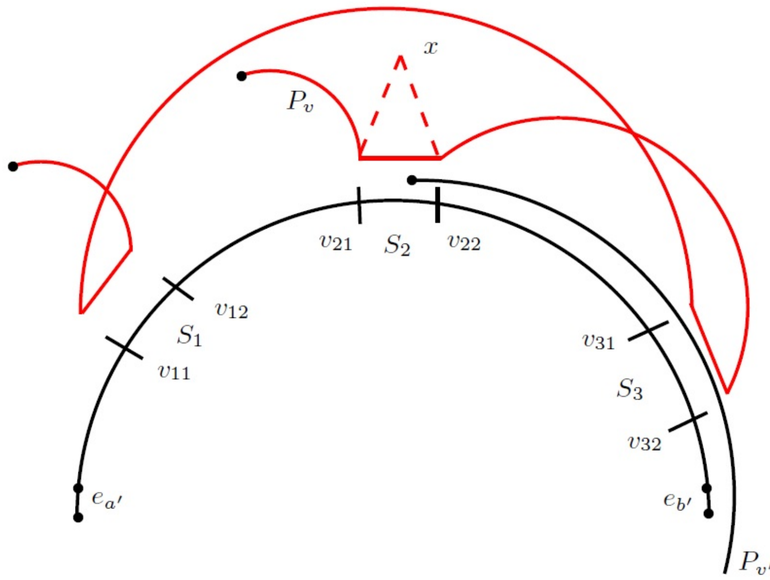
- The graph depends only on the order of the 2 |S| segments endpoints and 4n path endpoints on each trail.
- Lemma: The number of different orderings is at most

$$(4n)!(2n+2|S|)!^2$$

- It remains to bound |S|.
- We show that for every representation there is an equivalent one with |S| <= 12 n.

$\text{CO-BIPARTITE} \not\subset ENPG$

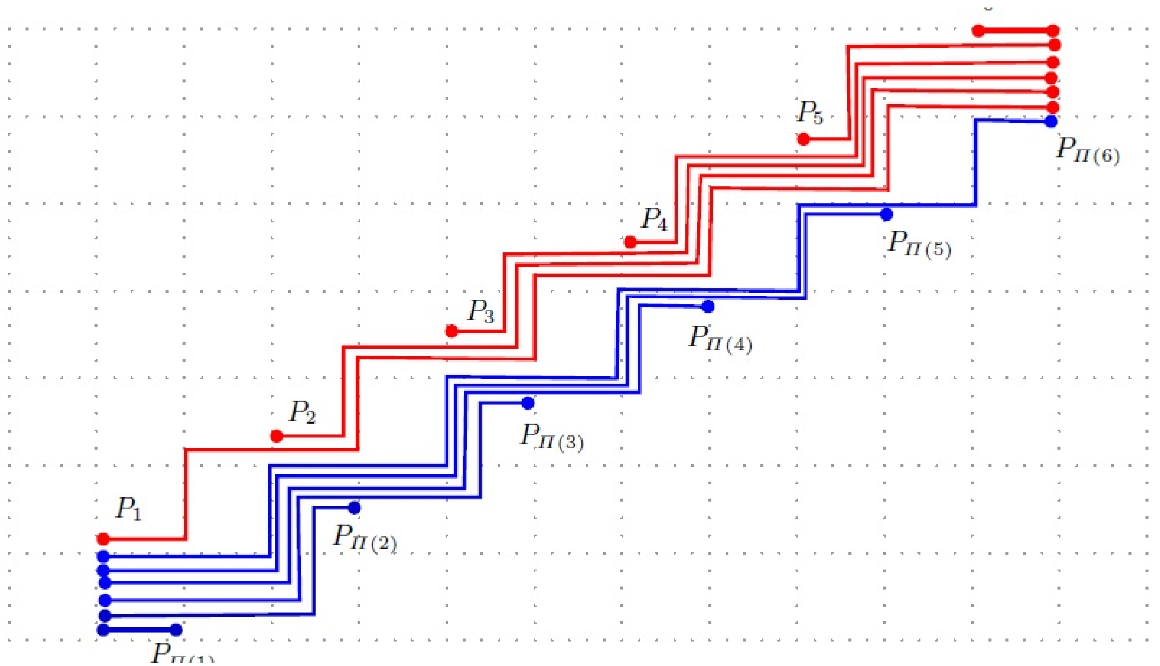
- A segment is *quiet* if it does not contain any path endpoints.
- The number of non-quiet segments is at most 4n.
- We now show that there are at most 4 quiet segments between two consecutive endpoints.



$$B_{72^{2^{0}-1}}-EPG \not\subset B_{72^{2^{1}-1}}-EPG \not\subset \ldots$$

Bend number of a "perfect matching"

Consider the co-bipartite graph PM_n=(K,K',E) where E is a perfect matching. $PM_n \in B_{2(n-1)} - ENPG$



Bend number of a “perfect matching”

We show that for every k and for sufficiently big n

$$PM_n \notin B_k - ENPG$$

- We first observe that |S| <= 3k. (There are at most three paths covering the trail).
- Every edge of the perfect matching is realized in at least one segment.
- For sufficiently big n, there is at least one segment realizing at least 2|S| edges.
- The paths representing the corresponding vertices are either within the segment or going out from different parts of the segments.
- Therefore there are the least two paths from one side that their both endpoints are in the same segments, i.e. “equivalent”.

Bend number of a “perfect matching”

- Consider the vertices corresponding to these paths and their two neighbors in the matching.
- They contain a (not necessarily induced) C_4 .
- This C_4 is part of the corresponding EPG graph.
- We observe that the intersecting paths intersect also when restricted to the segment under consideration.
- Then this C_4 is part of some interval graph. Therefore it has a chord.
- This chord is not in the perfect matching.
- Therefore, the corresponding paths split from each other.
- A contradiction to the “equivalence” of the two paths.

B_1-ENPG (work in progress)

- The Recognition of B_1-ENPG is NP-C even for SPLIT graphs.
- B_1-ENPG CO-BIPARTITE graphs can be recognized in linear time.
- Trees and cycles are B_1-ENPG.
- “at most k bends” is more powerful than “exactly k bends”.
- $B_1 - ENPG \not\subset B_2 - ENPG$

Other Results

- Every tree is B_1-ENPG
- Every cycle is B_1-ENPG
- If a Split graph S(K,S,E) is B_1-ENPG then $\sqrt{|K|} \leq |S| < |K|^2$
- $B_1 - ENPG \not\subset B_2 - ENPG$
- “at most k bends” is more powerful than “exactly k bends”


Thanks


Grazie


Teşekkürler


תודה