

Graphs of Edge-Intersecting and Non-Splitting Paths

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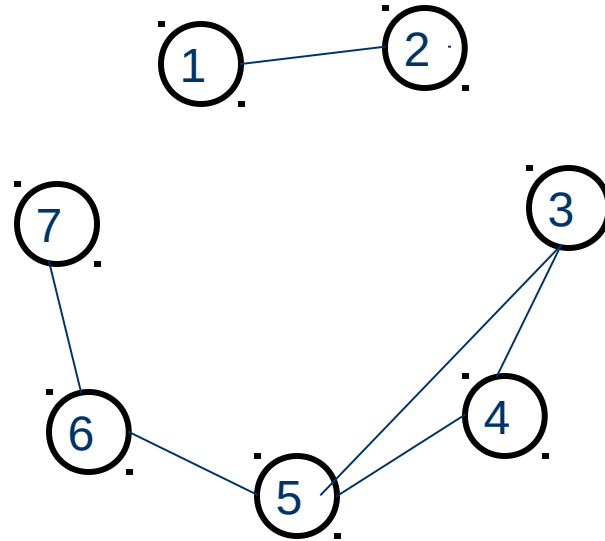
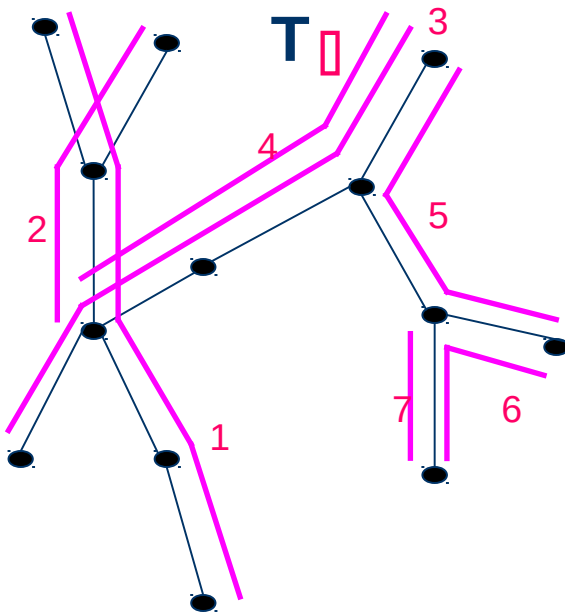
S. Zaks

Technion

EPT and EPG Graphs

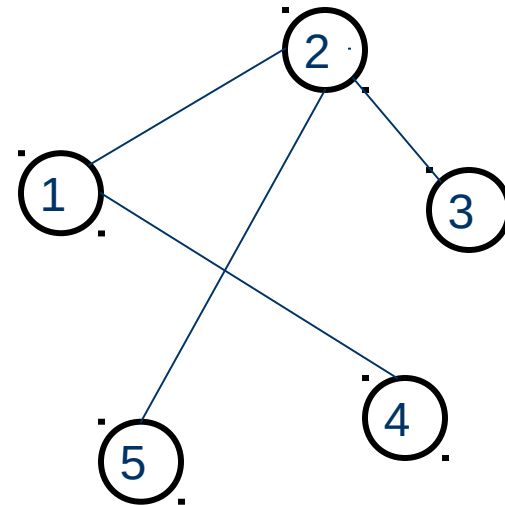
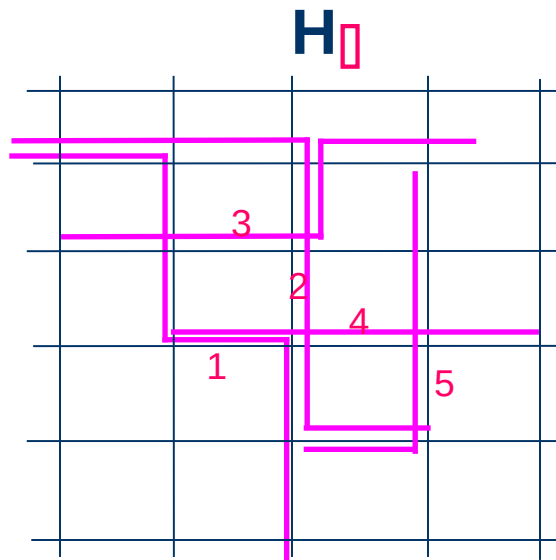
- [1] Golumbic, M. C. & Jamison, R. E. (1985), 'The edge intersection graphs of paths in a tree', *Journal of Combinatorial Theory, Series B* **38**(1), 8 - 22.
- [2] Golumbic, M. C.; Lipshteyn, M. & Stern, M. (2009), 'Edge intersection graphs of single bend paths on a grid', *Networks* **54**(3), 130-138.
- [3] Heldt, D.; Knauer, K. & Ueckerdt, T. (2013), 'Edge-intersection graphs of grid paths: the bend-number', *Discrete Applied Mathematics*.

The EPT Graph $EPT(\mathcal{T})$



In this talk “intersection” means “edge intersection”

The EPG Graph $EPG(\square)$



- A graph is B_k -EPG if it has a representation with paths of at most k bends.
(This is a B_3 -EPG graph)

Results

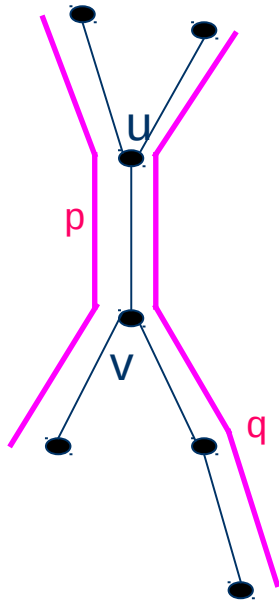
- [2] Every graph is EPG
- [3] $B_1 - EPG \not\subset B_2 - EPG \not\subset K$

A decorative graphic on the left side of the slide, consisting of a light green vertical bar and a dark blue horizontal bar with rounded ends.

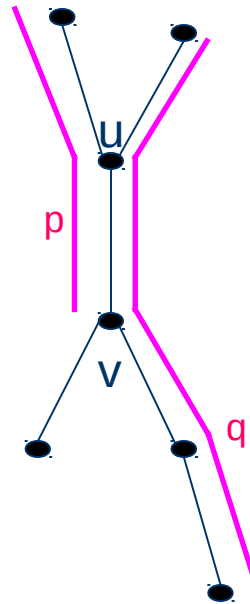
ENPT Graphs

[4] Boyacı, A.; Ekim, T.; Shalom, M. & Zaks, S., Graphs of Edge-Intersecting Non-Splitting Paths in a Tree: Towards Hole Representations, (WG2013)

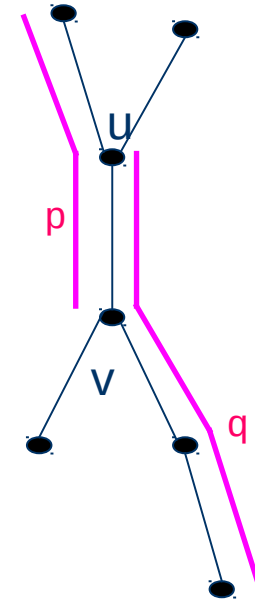
(Edge) Intersecting Paths (on a tree)



$$\text{split}(p, q) = \{u, v\}$$

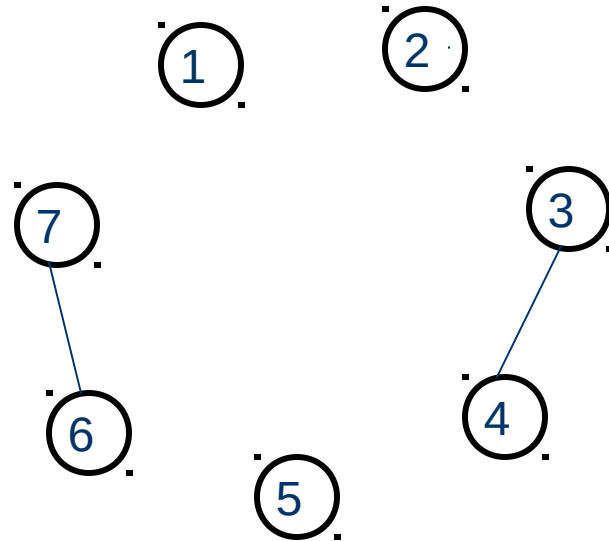
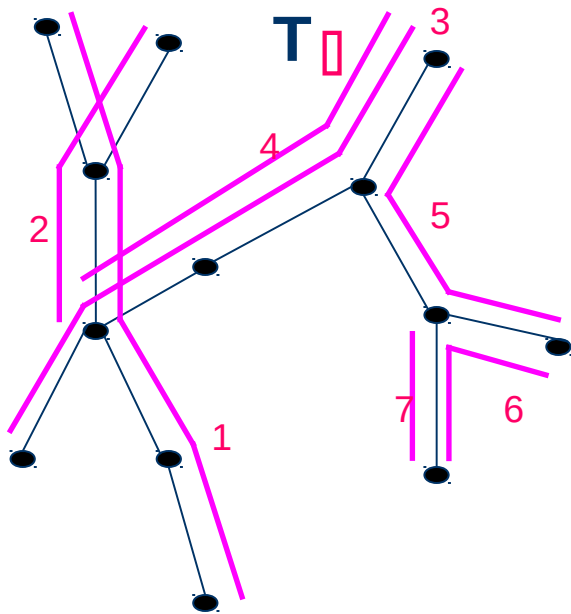


$$\text{split}(p, q) = \{u\}$$



$$\text{split}(p, q) = \emptyset$$

The ENPT Graph $ENPT(P)$



$$V(ENPT(P)) = V(EPT(P)) = P$$

$$E(ENPT(P)) \subseteq E(EPT(P))$$



ENP/ENPG Graphs

Graphs of Edge-Intersecting and Non-Splitting Paths
/ in a Grid

Our Results

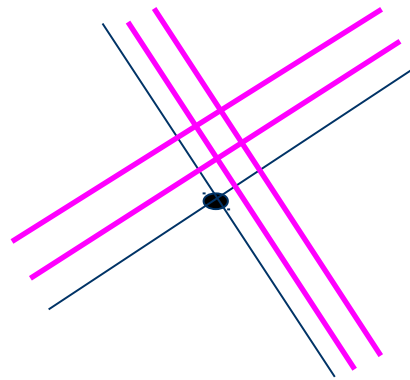
- $ENP = ENPG$
- Not every graph is ENPG
- $B_{72^{2^0-1}} - ENPG \not\subseteq B_{72^{2^1-1}} - ENPG \not\subseteq K$

A decorative graphic on the left side of the slide, consisting of a light green vertical bar and a white rounded rectangle with a green border, partially overlapping a dark blue horizontal bar.

ENP = ENPG

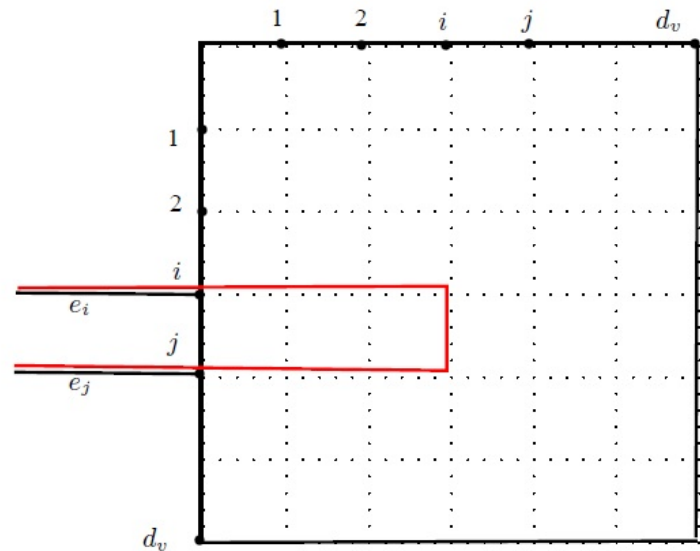
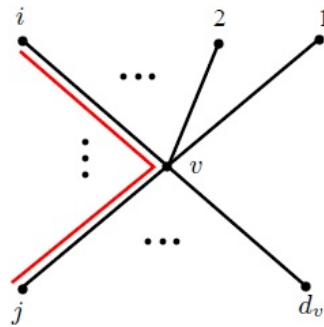
ENP = ENPG

- 1) Every representation on any host graph H can be embedded in a plane in general position:
 - Edges are embedded to straight line segments
 - At most two edges intersect at any given point
- 2) H' is planar.



ENP = ENPG

- 3) H'' is planar with maximum degree at most 4.



- 4) Yanpei et al. (1991) $\rightarrow H'''$ is a Grid.

A decorative graphic on the left side of the slide, consisting of a light green vertical bar and a dark blue horizontal bar with rounded ends.

CO - BIPARTITE $\not\subset$ *ENPG*

Representation of a Clique

- The union of the paths representing a clique is a trail.
- If the trail is open there is an edge that intersects every path.
- If the trail is closed there is a set of at most two edges that intersects every path.

CO - BIPARTITE $\not\subset$ ENPG

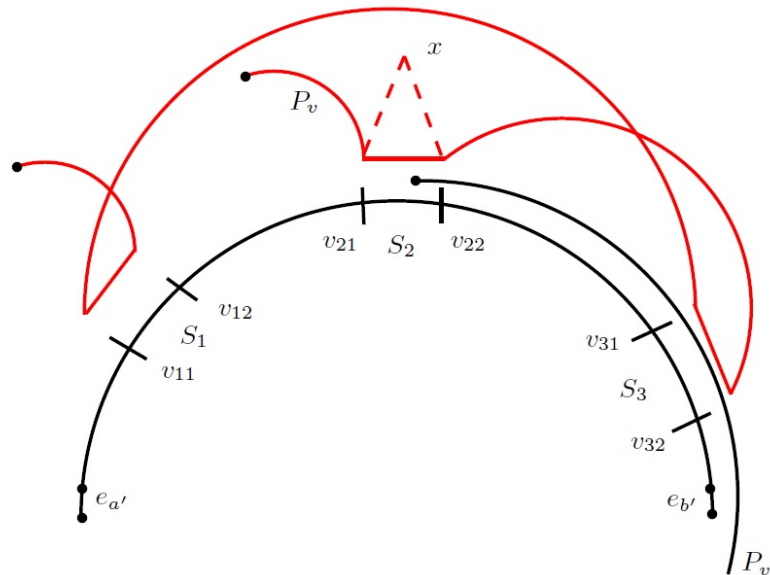
- Consider a co-bipartite graph $C(K, K', E)$ with $|K|=|K'|=n$.
- There are 2^{n^2} such graphs. We now show that the number of possible ENPG representations is at most $(26n)^3$
- The union of the paths representing a clique is a trail. Moreover, there is a set of at most two edges that intersects every path.
- The intersection of the two trails can be uniquely divided into a set of segments.
- Let S be the set of segments induced by the representations of the cliques K, K' .
- The paths representing two adjacent edges v of K and v' of K' can intersect only in edges of S .

CO - BIPARTITE $\not\subset$ ENPG

- The graph depends only on the order of the $2 |S|$ segments endpoints and $4n$ path endpoints on each trail.
- Lemma: The number of different orderings is at most $(4n)!(2n + 2|S|)!^2$
- It remains to bound $|S|$.
- We show that for every representation there is an equivalent one with $|S| \leq 12n$.

CO-BIPARTITE $\not\subset$ ENPG

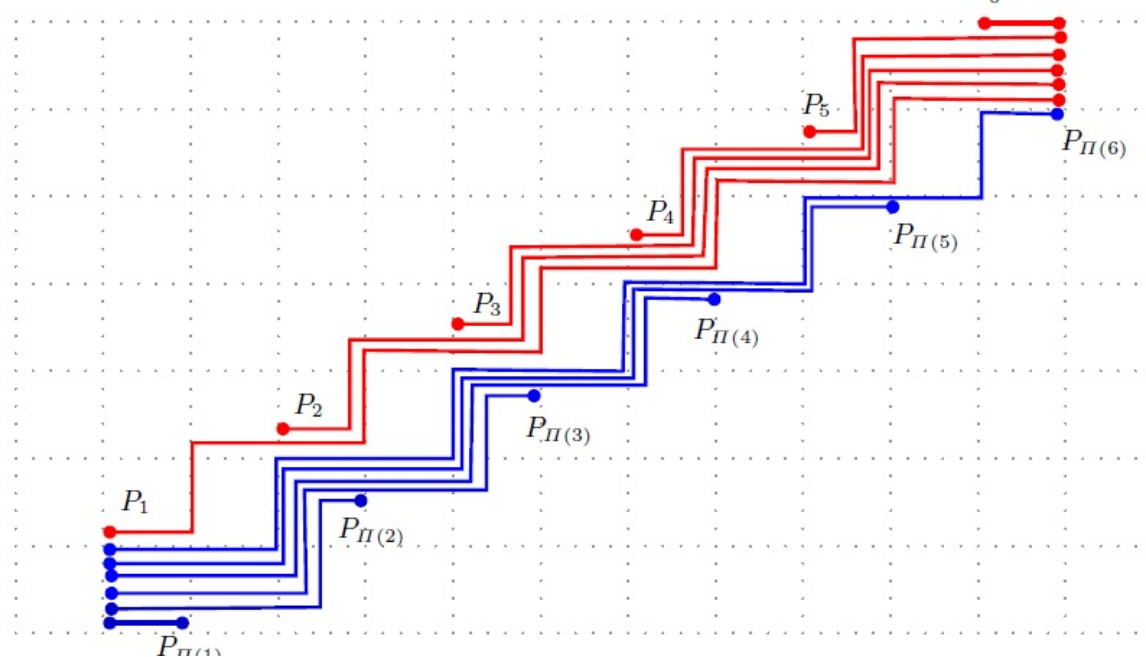
- A segment is *quiet* if it does not contain any path endpoints.
- The number of non-quiet segments is at most $4n$.
- We now show that there are at most 4 quiet segments between two consecutive endpoints.



$$B_{72^{2^0-1}} - EPG \not\subset B_{72^{2^1-1}} - EPG \not\subset K$$

Bend number of a “perfect matching”

Consider the co-bipartite graph $PM_n=(K,K',E)$ where E is a perfect matching. $PM_n \in B_{2(n-1)} - ENPG$



Bend number of a “perfect matching”

We show that for every k and for sufficiently big n

$$PM_n \notin B_k - ENPG$$

- We first observe that $|S| \leq 3k$. (There are at most three paths covering the trail).
- Every edge of the perfect matching is realized in at least one segment.
- For sufficiently big n , there is at least one segment realizing at least $2|S|$ edges.
- The paths representing the corresponding vertices are either within the segment or going out from different parts of the segments.
- Therefore there are the least two paths from one side that their both endpoints are in the same segments, i.e. “equivalent”.

Bend number of a “perfect matching”

- Consider the vertices corresponding to these paths and their two neighbors in the matching.
- They contain a (not necessarily induced) C_4 .
- This C_4 is part of the corresponding EPG graph.
- We observe that the intersecting paths intersect also when restricted to the segment under consideration.
- Then this C_4 is part of some interval graph. Therefore it has a chord.
- This chord is not in the perfect matching.
- Therefore, the corresponding paths split from each other.
- A contradiction to the “equivalence” of the two paths.

B_1 -ENPG (work in progress)

- The Recognition of B_1 -ENPG is NP-C even for SPLIT graphs.
- B_1 -ENPG CO-BIPARTITE graphs can be recognized in linear time.
- Trees and cycles are B_1 -ENPG.
- “at most k bends” is more powerful than “exactly k bends”.
- $B_1 - ENPG \not\subset B_2 - ENPG$

Other Results

- Every tree is B_1 -ENPG
- Every cycle is B_1 -ENPG
- If a Split graph $S(K,S,E)$ is B_1 -ENPG then $\sqrt{|K|} \leq |S| < |K|^2$
- B_1 -ENPG $\not\subset$ B_2 -ENPG
- “at most k bends” is more powerful than “exactly k bends”



Thanks

A stylized graphic where the word "Thanks" is written in a bold, purple, blocky font. The letter 'h' is replaced by a brown hand with fingers spread, as if giving a thumbs up. There are three green lines radiating from the top of the hand. The text is set against a light blue, slightly tilted rectangular background.



Grazie

The word "Grazie" is written in a blue, cursive script. Above the text is a green oval containing a line drawing of a rose branch with two pink roses and several green leaves.



Teşekkürler

The word "Teşekkürler" is written in a blue, serif font. It is centered within a pink rectangular box that has a decorative border of small yellow floral motifs.



תודה

The Hebrew word "תודה" (Toda) is written in white, bold, block letters. It is centered within a red rectangular box with a slight 3D effect.