WORKSHOP ON *NONLINEAR PDES AND APPLICATIONS*
ON THE OCCASION OF THE RETIREMENT OF
PROFESSOR MARIA CESARINA SALVATORI

Perugia, February 3-5, 2019
Meeting Room A3 – Dipartimento di Matematica e Informatica
Università degli Studi di Perugia

Abstracts

A complex function theory useful in Mellin analysis. Applications

Carlo Bardaro
Università degli Studi di Perugia
Monday, 3rd February – 12:15-12:45

The theory of Mellin transform, and its consequent Mellin analysis, is now a wide field of investigations, despite formal equivalence with the Fourier transform, through a change of variable and a function. That this equivalence is only formal can be deduced by several aspects. One of the main point of divergence between the two theories is the classical Paley–Wiener theorem which describes precisely the structure of the class of band–limited functions. In order to describe such a structure for Mellin band–limited functions, a new complex function theory appears very suitable. Indeed, using the classical complex analysis one has to characterize the space of the Mellin band–limited functions through the analyticity of a function defined over the Riemann surface S of the complex logarithm and employing analytical branches. For many reasons, this approach is not fully convenient, especially from a computational point of view. Therefore, taking inspiration from the geometrical model of S as an helicoidal surface, we introduced a notion of analyticity, called polar–analyticity, that enables one to describe in a simple way analytic functions over S. Starting with this concept, we develop a complex function theory. Then we apply this theory to certain interpolation formulae, like the exponential sampling formula, the exponential Boas' differentiation formula and certain quadrature formulae of Gaussian type.

Solution existence results for nonlocal differential problems

Irene Benedetti
Università degli Studi di Perugia
Monday, 3rd February – 16:30-17:00

The aim of this talk is to present some recent existence results for nonlocal differential problems based on the use of Lyapunov functions in order to consider quite general growth conditions on the nonlinear term. This approach provides a unifying method for studying models describing reaction–diffusion processes in several frameworks. We will consider nonlocal initial conditions such as the Cauchy multipoint and the mean value conditions, and we can handle nonlinearity of integral type which accounts for nonlocal diffusion behaviours.

Problems involving second order semilinear differential inclusions

Tiziana Cardinali
Università degli Studi di Perugia

Monday, 3rd February – 14:30-15:00

In this talk I speak on some recent results respectively achieved in two papers in collaboration with *Serena Gentili* and *Eleonora De Angelis*. In particular, I present the existence of solutions for controllability problems and for stabilization problems driven by wave equations. In order to obtain these objectives, we rewrite the problems examined in problems involving non–autonomous second order semilinear differential inclusions and we prove the existence of mild solutions for these abstract problems.

In the last part I also describe the progress of this research obtained together with *Silvia Duricchi*.

A quasilinear elliptic problem: some existence results

Anna Maria Candela
Università degli Studi di Bari

Wednesday, 5th February – 9:15-9:45

In the last years we have investigated the existence of solutions of the quasilinear elliptic equation

$$(P) \qquad -\operatorname{div}(A(x,u)|\nabla u|^{p-2}\nabla u) + \frac{1}{p}A_t(x,u)|\nabla u|^p \;=\; g(x,u) \qquad \text{in } \Omega,$$

with $p>1$, either $\Omega=\mathbb{R}^N$ or Ω open bounded domain in $\mathbb{R}^N$ and $u=0$ on $\partial\Omega$ ($N\geq 3$), where $A(x,t)$, $A_t(x,t)=\frac{\partial A}{\partial t}(x,t)$ and $g(x,t)$ are Carathéodory functions on $\Omega\times\mathbb{R}$.

Taking $G(x,t)=\int_0^t g(x,s)ds$, suitable assumptions on $A(x,t)$ and $g(x,t)$ set off the variational structure of (P) and its related functional is

$$\mathcal{J}(u) \;=\; \frac{1}{p}\int_\Omega A(x,u)|\nabla u|^p dx - \int_\Omega G(x,u)dx,$$

which is C^1 but not verifies the classical Palais–Smale condition on the Banach space $X=W_0^{1,p}(\Omega)\cap L^\infty(\Omega)$ equipped with the intersection norm $\|\cdot\|_X$.

Anyway, following an approach which exploits the interaction between $\|\cdot\|_X$ and the standard norm on $W_0^{1,p}(\Omega)$, we apply suitable generalizations of classical variational theorems to $\mathcal{J}$ in X so to prove the existence of weak solutions of (P) by comparing the growth of $A(x,t)|\xi|^p$ with that one of $G(x,t)$.

These results are part of joint works with *Genni Fragnelli*, *Dimitri Mugnai*, *Giuliana Palmieri* and *Addolorata Salvatore*.

Bose–Einstein systems in dimension two

Daniele Cassani
Università degli Studi dell'Insubria

Wednesday, 5th February – 11:45-12:15

We prove, using variational methods, the existence in dimension two of positive vector ground states solutions for the Bose–Einstein type systems

$$\begin{cases} -\Delta u + \lambda_1 u = \mu_1 u(e^{u^2} - 1) + \beta v\,(e^{uv} - 1) \ \text{ in } \Omega\ , \\ -\Delta v + \lambda_2 v = \mu_2 v(e^{v^2} - 1) + \beta u\,(e^{uv} - 1) \ \text{ in } \Omega\ , \\ u, v \in H_0^1(\Omega) \end{cases}$$

where Ω is a bounded smooth domain, $\lambda_1, \lambda_2 > -\Lambda_1$ (the first eigenvalue of $(-\Delta, H_0^1(\Omega))$, $\mu_1, \mu_2 > 0$ and β is either positive (small or large) or negative (small). The nonlinear interaction between two Bose fluids is assumed to be of critical exponential type in the sense of J. Moser. For 'small' solutions the system is asymptotically equivalent to the corresponding one in higher dimensions with power–like nonlinearities.

Fractional Harmonic Wavelets

Carlo Cattani
Università degli Studi della Tuscia

Monday, 3rd February – 11:15-11:45

In this presentation, fractional operators based on wavelets will be considered to explore both the localization–dilation properties of wavelets and the non–integer order of the differential integral operators. Operational matrix will be explicitly given. The sinc–fractional derivative is extended to the Hilbert space based on Shannon wavelets.

Atomic decompositions, two stars theorems, and distances for the Bourgain–Brezis–Mironescu space and other big spaces

Luigi D'Onofrio
Università degli Studi di Napoli *Parthenope*

Tuesday, 4th February – 14:30-15:00

Given a Banach space E with a supremum–type norm induced by a collection of operators, we prove that E is a dual space and provide an atomic decomposition of its predual. We apply this result, and some results obtained previously by one of the authors, to the function space $\mathcal{B}$ introduced recently by *Bourgain, Brezis* and *Mironescu*. This yields an atomic decomposition of the predual $\mathcal{B}_*$, the biduality result that $\mathcal{B}_0^* = \mathcal{B}_*$ and a formula for the distance from an element $f \in \mathcal{B}$ to $\mathcal{B}_0$. This is a joint project with *L. Greco, K.M. Perfect, C. Sbordone* and *R. Schiattarella*.

Elliptic problems with gradient terms

Roberta Filippucci
Università degli Studi di Perugia

Tuesday, 4th February – 9:45-10:15

We present an existence result for nonnegative solutions of a Dirichlet problem on a bounded smooth domain of $\mathbb{R}^N$ for a p–Laplacian elliptic equation with a convection term.

Our proof uses a priori bounds, obtained by a blow up type technique.

Fractional Kirchhoff problems involving singular terms and critical nonlinearities

Alessio Fiscella
University of Campinas

Tuesday, 4th February – 15:00-15:30

In this talk, we discuss about recent results for Kirchhoff problems driven by nonlocal fractional operators, involving singular terms and critical nonlinearities. Our variational problems present some difficulties due to the bi–nonlocal nature of the elliptic part, the lack of compactness at critical level and the nondifferentiability of the related functional. For this, in order to state multiplicity results, we introduce different proof techniques based on approximation and minimization arguments. After retracing the historical path, we conclude the talk presenting some interesting open questions and new future directions. The results presented in this talk are written in collaboration with *P.K. Mishra*.

Controllability for a cascade system

Genni Fragnelli
Università degli Studi di Bari

Monday, 3rd February – 17:00-17:30

We consider a linear degenerate parabolic system in one space dimension that describes the interaction between two different species. The two species depend on the location $x \in (0,1)$, on time $t \in (0,T)$ and on age $a \in (0,A)$. Here $T > 0$ is a fixed time horizon, while A is the maximal age of life for the two species.

We will study the controllability for this model via Carleman estimates and observability inequalities for the associated adjoint problem.

A review on fractional p–Laplacian problems

Antonio Iannizzotto
Università degli Studi di Cagliari

Tuesday, 4th February – 12.15-12:45

We consider a Dirichlet type problem of the following type:

$$\begin{cases} (-\Delta)_p^s u = f(x,u) & \text{in } \Omega \\ u = 0 & \text{in } \Omega^c, \end{cases}$$

where $p \geq 2$, $s \in (0,1)$, $\Omega \subset \mathbb{R}^N$ $(N > ps)$ is a bounded and smooth domain, and the leading operator is the fractional p–Laplacian defined by

$$(-\Delta)_p^s u(x) = \lim_{\varepsilon \to 0^+} \int_{B_\varepsilon(x)^c} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{N+ps}}\, dy.$$

We will present a review of old and new result on existence, multiplicity, interior and boundary regularity of solutions, equivalence of minimizers of the energy functional in Sobolev and Hölder topology, and a study of constant sign and nodal solutions. The talk is based on several papers in collaboration with different researchers.

References

[1] A. Iannizzotto, S. Liu, K. Perera, M. Squassina, Existence results for fractional p-Laplacian problems via Morse theory, *Adv. Calc. Var.* **9** (2016) 101–125.

[2] A. Iannizzotto, S. Mosconi, M. Squassina, Global Hölder regularity for the fractional p–Laplacian, *Rev. Mat. Iberoam.* **32** (2016) 1353–1392.

[3] A. Iannizzotto, S. Mosconi, M. Squassina, Fine boundary regularity for the degenerate fractional p–Laplacian, (preprint).

[4] A. Iannizzotto, S. Mosconi, M. Squassina, Sobolev versus Hölder minimizers for the fractional p–Laplacian, *Nonlinear Anal.* **191** (2020).

[5] S. Frassu, A. Iannizzotto, Extremal constant sign solutions and nodal solutions for the fractional p–Laplacian, (preprint).

On properties of solutions to a class of Keller–Segel problems

Monica Marras

Università degli Studi di Cagliari

Wednesday, 5th February – 11:15-11:45

We discuss qualitative properties as blow–up phenomena to solutions of some classes of parabolic problems. In particular, we consider a problem of Keller–Segel type under Neumann boundary conditions in a smooth and bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 3$, and we show a criterion which ensures that, under suitable conditions on the data, the solution, which blows up in finite time in the $L^\infty(\Omega)$–norm, it also blows up in the $L^p(\Omega)$–norm with suitable exponents p.

Moreover, we consider unbounded solutions of a chemotaxis system with nonlinear diffusion. Under appropriate assumptions on data, a safe interval of existence of the solution is derived with a lower bound of the lifespan.

Multivalued minimax relationships

Anna Martellotti

Università degli Studi di Perugia

Monday, 3rd February – 16:00-16:30

Since its appearance in the Fifties, several generalizations of the celebrated Fan's minimax inequality appeared in the literature. The main directions of these extensions were aimed either to relaxing the compactness assumptions, or to weaken the convexity conditions. In this second framework one meets two different directions: on one side convexity is replaced by quasi convexity particularly when the most important scope is relaxing compactness, while convex–like maps are introduced if the goal is to abandon linearity assumptions on the domain. Roughly speaking, from the technical point of view, one may say that quasi–convex minimax statements are generally based upon some form of coincidence condition, such as the *Finite Intersection Property*, or more commonly the so called KKM Property, while the convex–like minimax results do usually make use of some form of the *Hahn Banach Separation Theorem*.

Up to this point minimax relationships were basically given for single valued maps. The investigation presented in this talk is devoted to multivalued minimax relationships, both inequalities and equalities. To achieve such forms one has primarily to establish suitable forms of surrogate convexity for multimaps. Therefore we shall present the appropriate definition of quasi convexity and convex–like multimaps, as well as the multivalued adaptations of their more recent surrogates, the so called *transfer formulations*. We shall also propose

some relaxations of the compactness and we shall present multivalued minimax relationship both of *hit* and *inclusion* type.

The talk is based on joint work with *I. Benedetti*.

Isometric invariant solutions for elliptic problems on homogeneous Hadamard manifolds

Giovanni Molica Bisci
Università degli Studi di Urbino *Carlo Bo*

Tuesday, 4th February – 11:15-11:45

In this talk we discuss the initial part of a joint research project with *Patrizia Pucci* devoted to the study of nonlinear elliptic problems on homogeneous Hadamard manifolds. Motivated by a wide interest in the current literature, we present some existence and multiplicity results on elliptic problems settled on non–compact manifolds. Certain open problems will be briefly presented.

Heir–equations and their properties with applications

Maria Clara Nucci
Università degli Studi di Perugia

Monday, 3rd February – 10:15-10:45

Heir–equations were determined by iterating the nonclassical symmetry method. Apart from inheriting the same Lie symmetry algebra of the original partial differential equation, and thus yielding more (and different) symmetry solutions than expected, the heir–equations are connected to conditional Lie–Bäcklund, and generalized conditional symmetries; moreover they solve the inverse problem, namely a special solution corresponds to the nonclassical symmetries. Applications to reaction–diffusion equations will be shown.

Hölder regularity for nonlocal double phase equations

Giampiero Palatucci
Università degli Studi di Parma

Wednesday, 5th February – 9:45-10:15

We present some regularity estimates for viscosity solutions to a class of possible degenerate and singular integro-differential equations whose leading operator switches between two different types of fractional elliptic phases, according to the zero set of a modulating coefficient $a = a(\cdot,\cdot)$. The model case is driven by the following nonlocal double phase operator,

$$\int \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{n+sp}}\,\mathrm{d}y + \int a(x,y)\frac{|u(x)-u(y)|^{q-2}(u(x)-u(y))}{|x-y|^{n+tq}}\,\mathrm{d}y,$$

where $q \geq p$ and $a(\cdot,\cdot) \geqq 0$. Our results do also apply for inhomogeneous equations, for very general classes of measurable kernels. By simply assuming the boundedness of the modulating coefficient, we are able to prove that the solutions are Hölder continuous, whereas similar sharp results for the classical local case do require a to be Hölder continuous. To our knowledge, this is the first (regularity) result for nonlocal double phase problems. Work in collaboration with *C. De Filippis*, Available at DOI: 10.1016/j.jde.2019.01.017

References

[1] M. Colombo, G. Mingione, Regularity for Double Phase Variational Problems, *Arch. Rational Mech. Anal.* **215** (2015), 443–496.

[2] M. Colombo, G. Mingione, Bounded Minimisers of Double Phase Variational Integrals, *Arch. Rational Mech. Anal.* **218** (2015), 219–273.

[3] C. De Filippis, G. Palatucci, Hölder regularity for nonlocal double phase equations, *J. Differential Equations* **267** (2019), 547–586.

[4] G. Palatucci, The Dirichlet problem for the p–fractional Laplace equation, *Nonlinear Anal.* **177** (2018), 699–732.

A time–dependent obstacle problem in linearised elasticity

Paolo Piersanti

Karl–Franzens–Universität Graz, Austria

Tuesday, 4th February – 16:00-16:30

In this talk an existence theorem for a time–dependent obstacle problem modelling the displacement of a general linearly elastic body subjected to a planar obstacle is presented.

The problem statement and the definition of the concept of solution of its will be recovered as a result of a rigorous analysis, whose departure point is a natural penalised hyperbolic model. The elasticity functional setting requires the adoption of more sophisticated analysis tools as well as the contrivance of new techniques.

The presented work is based on the following recent results:

1. Piersanti, P. An existence and uniqueness theorem for the dynamics of flexural shells, *Math. Mech. Solids*, in press.
2. Shen, X., Piersanti, L. and Piersanti, P. Numerical simulations for the dynamics of flexural shells, *Math. Mech. Solids*, in press.
3. Piersanti, P. A time–dependent obstacle problem in linearised elasticity, *Nonlinear Anal.*, in press.

Qualitative properties for solutions to some classes of parabolic systems

Stella Piro Vernier

Università degli Studi di Cagliari

Wednesday, 5th February – 10:15-10:45

We consider different classes of parabolic systems of Keller–Segel type defined in a convex bounded and smooth domain Ω of R^N, for $N \in \{2, 3\}$, under homogeneous Neumann boundary conditions. By introducing suitable energy functions in terms of the solution, sufficient conditions on the data are assumed to have bounded solutions of the system and exponential decay of such energies. The method can be extended to systems of Attraction–Repulsion Keller–Segel type.

P–Einstein–type structures

Marco Rigoli

Università degli Studi di Milano Statale

Tuesday, 4th February – 9:15-9:45

On Riemannian manifolds we introduce what we call an Einstein type structure or a P–Einstein–type structure. Motivations for the first directly come from geometry, for instance

Ricci (harmonic) solitons, for the second for instance from Einstein field equations. This enables us to introduce some new curvature tensors that we call ϕ–curvatures and study their geometry intimately related to the above structures. The results are joint work with *Andrea Anselli* and *Giulio Colombo*.

On some evolution processes and related models

Paola Rubbioni
Università degli Studi di Perugia

Monday, 3rd February – 15:00-15:30

In this talk, we first present a spatial nonlocal diffusion model with feedback controls driven by a parametric differential equation. The existence of controlled dynamics to the considered problems – a periodic problem, a multipoint boundary problem, a mean value problem – is proved by transforming the parametric differential equation describing the process in an ordinary differential inclusion and by applying techniques of multivalued analysis and of degree theory for condensing operators in Hilbert spaces.

Then, we show either the controllability of an impulsive integro–differential process and the existence of solutions to a model of population dynamics with memory effects, both subject to instantaneous external actions. In many fields of applied sciences the past of the system influences its evolution, leading to models with formalized system's dependence on the past. The action of external forces on the model – represented by impulse functions – causes sudden abrupt changes on the state of the system, so that the solution trajectories are not continuous. In biology the impulse functions, called "*regulation functions*", are used for example in the study of population dynamics to keep the population in a prescribed range. We treat these models by reinterpreting them as impulsive Cauchy problems driven respectively by a semilinear differential inclusion with distributed delay and by a semilinear differential equation with functional delay, using an extension–with–memory technique.

This is a joint project with *L.Malaguti* and *T.Cardinali*.

Helmholtz and Gausson Solitary Waves in Nonlinear Elastodynamics

Giuseppe Saccomandi
Università degli Studi di Perugia

Monday, 3rd February – 9:15-9:45

A nonlinear elastodynamic system is investigated which is descriptive of transverse wave propagation in an isotropic, incompressible, hyperelastic material subject to body forces associated with a nonlinear substrate potential. Notably, by introducing an appropriate ansatz several exact solutions are provided. These solutions are cnoidal, Helmholtz–type and gausson–type solitary waves. The linear stability of these solutions is studied under the assumption that the speed of propagation of the wave is small enough compared to the speed at which transverse waves travel in the linear regime and in the absence of external actions.

Non-local to local transition for ground states of fractional equations

Simone Secchi
Università degli Studi di Milano–Bicocca

Tuesday, 4th February – 16:30-17:00

I will review some recent results about the convergence as $s \to 1^-$ of ground states to the fractional equation

$$(-\Delta)^s u + V(x)u = f(x,u) \quad \text{in } \Omega,$$

where Ω is an open subset of $\mathbb{R}^N$. This research is in collaboration with *Bartosz Bieganowski*.

Nonlocal problems with lack of compactness

Raffaella Servadei
Università degli Studi di Urbino *Carlo Bo*

Tuesday, 4th February – 10:15-10:45

Motivated by the interest shown in the literature for nonlocal operators of elliptic type, aim of this talk is to present some recent existence and multiplicity results for fractional nonlocal nonlinear problems, obtained using variational and topological methods. These results extend the validity of some theorems known in the classical Laplacian setting to the nonlocal framework of the fractional Laplacian.

Bending, unbending and eversion wrinkles

Luigi Vergori
Università degli Studi di Perugia

Monday, 3rd February – 11:45-12:15

The topic of this talk is arguably one of the most common modes of deformation found in Nature, science and engineering, namely the large elastic bending of curved structures, as well as its inverse, unbending, which can be brought beyond complete straightening to turn into eversion. We show that the mathematical solution to these problems always exists and is unique when the solid is modelled as a homogeneous, isotropic, incompressible hyperelastic material with a strain–energy satisfying the strong ellipticity condition. We also provide explicit asymptotic solutions for thin sectors. When the deformations are severe enough, the compressed side of the elastic material may buckle and wrinkles will then develop. We analyse in detail the onset of this instability for the Mooney–Rivlin strain energy, which covers the cases of the neo–Hookean model in exact non–linear elasticity and of third–order elastic materials in weakly non–linear elasticity. In particular the associated theoretical and numerical treatment allows us to predict the number and wavelength of the wrinkles.

Some recent results in Approximation Theory and concrete applications

Gianluca Vinti
Università degli Studi di Perugia

Monday, 3rd February – 9:45-10:15

In this talk I will present some recent approximation results that make use of sampling type operators, whose implementation has emerged to be fundamental for the reconstruction and the enhancement of digital images. I will show how these results, related to digital image processing, open the way for several concrete applications in various fields.

The wave equation with acoustic boundary conditions

Enzo Vitillaro
Università degli Studi di Perugia

Tuesday, 4th February – 11:45-12:15

The aim of the talk is to give some recent results, obtained in a collaboration (still in progress) with *Delio Mugnolo*, on the wave equation with acoustic boundary conditions, which has been subject of a large literature starting from the original analysis of *Beale* and *Rosencrans* in the 70's and continuing up today, where the effect of nonlinear perturbations of any type is studied. In particular, after summarizing well-posedness and regularity results in the associated energy space, we shall show that for a bounded domain some physically inexplicable stationary solutions make the problem not asymptotically stable, even with an effective damping, while it is asymptotically stable provided that the initial data are restricted to a 1-codimensional subspace, which is invariant under the flow. This mathematical result leads to a physical re-thinking of the derivation of the model itself in theoretical acoustic. In particular, starting from Newton's Second Law, we shall show that the PDEs appearing in it need to be integrated with an integral condition which is exactly the one found in the stability analysis of it, a fact never observed in the existing literature.