TWO-DAY MEETING ON PDES

Perugia, February 23-24, 2017
Meeting Room B3 – Dipartimento di Matematica e Informatica
Università degli Studi di Perugia

Abstracts

Soluzioni periodiche e nonlocali per processi diffusivi

Irene Benedetti
Università degli Studi di Perugia

Thursday, 23rd February – 18:00-18:40

Un noto modo di studiare le equazioni alle derivate parziali è quello di trasformarle in equazioni ordinarie in spazi astratti. Quando si utilizzano metodi topologici per risolvere questo tipo di equazioni differenziali, rileggendo le soluzioni come punti fissi di opportuni operatori, vengono usualmente introdotte delle ipotesi di compattezza sui dati del problema che sono generalmente difficili da verificare in spazi infinito dimensionali. In questo intervento verranno presentate alcune tecniche che indeboliscono le ipotesi di compattezza classiche presenti in letteratura. Questo permette di studiare svariati processi diffusivi di tipo integro–differenziale con condizioni al bordo periodiche o nonlocali.

Semilinear integro–differential inclusions

Tiziana Cardinali
Università degli Studi di Perugia

Thursday, 23rd February – 10:20-11:00

We study a reaction–diffusion nonlocal problem involving a distributed time delay, via the existence of mild solutions for a semilinear integro–differential inclusion subject to a nonlocal initial condition. In particular, we obtain that the nonlocal mild solution set is nonempty and compact. Moreover, we present a result on the existence of mild solutions for an impulsive Cauchy problem with feedback controls involving integro–partial differential equations. This is joint work with *Paola Rubbioni*.

Critical and supercritical Hamiltonian systems of Schrödinger equations in dimension two

Daniele Cassani
Università degli Studi dell'Insubria

Friday, 24th February – 12-12:40

We consider in the whole plane the Hamiltonian coupling of Schrödinger equations where the nonlinearities have critical or even supercritical growth in the sense of Moser. In the critical case, we prove that the (nonempty) set of ground state solutions is compact up to translations. Moreover, ground states are uniformly bounded and uniformly decaying at

infinity. Then we prove that actually the ground state is positive and radially symmetric. We apply those results to prove the existence of semiclassical ground states solutions to singularly perturbed systems. Namely, in presence of an external potential which is bounded away from zero, we prove the existence of minimal energy solutions which concentrate around the closest local minima of the potential with some precise asymptotic rate. In the supercritical case we prove the existence of higher (though finite) energy solutions in a suitable Lorentz space framework.

Ground States for Diffusion Dominated Free Energies with Logarithmic Interaction

Daniele Castorina
Università degli Studi di Roma *Tor Vergata*

Friday, 24th February – 12-12:40

Replacing linear diffusion by a degenerate diffusion of porous medium type is known to regularize the classical two–dimensional parabolic–elliptic Keller–Segel model [*V. Calvez* and *J. A. Carrillo*, J. Math. Pures Appl. (9), 86 (2006), pp. 155175]. The implications of nonlinear diffusion are that solutions exist globally and are uniformly bounded in time. We analyze the stationary case showing the existence of a unique, up to translation, global minimizer of the associated free energy. Furthermore, we prove that this global minimizer is a radially decreasing compactly supported continuous density function which is smooth inside its support, and it is characterized as the unique compactly supported stationary state of the evolution model. This unique profile is the clear candidate to describe the long time asymptotics of the diffusion dominated classical Keller–Segel model for general initial data.

This is a joint work with *J.A. Carrillo* (Imperial College, London) and *Bruno Volzone* (Napoli Parthenope).

Relation between Lusin (N) condition and Det = det

Luigi D'Onofrio
Università degli Studi di Napoli *Parthenope*

Thursday, 23rd February – 17:10-17:50

I will present a joint work with *Stanislav Hencl*, *Jan Maly* and *Roberta Schiattarella* in which we show that for a continuous mapping, the validity of the Lusin (N) condition implies that the distributional Jacobian equals to the pointwise Jacobian.

Nonexistence results for elliptic and parabolic problems

Roberta Filippucci
Università degli Studi di Perugia

Thursday, 23rd February – 16:10-16:50

In this talk we discuss first some Liouville type results for elliptic inequalities and systems of elliptic inequalities involving both the p–Laplacian operator and the mean curvature operator and in which a gradient factor is included in the right hand side. In the second part of the talk we investigate some Fujita type results for parabolic inequalities with gradient terms.

Multiplicity results for critical fractional Kirchhoff type problems

Alessio Fiscella
University of Campinas

Thursday, 23rd February – 15:20-16:00

In this talk we discuss about Kirchhoff type problems involving critical Sobolev nonlinearities and driven by nonlocal fractional operators. By combining variational methods, topological techniques and truncation arguments, we show the existence of infinitely many solutions. In particular, very recently, we covered the delicate *degenerate* case, that is when the Kirchhoff coefficient is zero at zero. This talk is based on a joint work with *B. Zhang* and *S. Liang*.

Null controllability in degenerate and singular parabolic problems

Genni Fragnelli
Università degli Studi di Bari

Thursday, 23rd February – 11:10-11:50

In this talk we will present the problem of null controllability for parabolic systems and we will focus on some recent results for degenerate and singular problems coming from real–world models, such as Biology, Climatology, Medicine.

L'equazione frazionaria del calore: esistenza e convessità per le soluzioni di segno variabile

Antonio Iannizzotto
Università degli Studi di Cagliari

Friday, 24th February – 11:10-11:50

Consideriamo l'analogo naturale dell'equazione del calore nel contesto non-locale, con una diffusione spaziale regolata dal laplaciano frazionario. Molti contributi su questo problema, incluso un recente lavoro di *Barrios* et al. (ARMA 2014), limitano lo studio alle soluzioni di segno costante. Il nostro risultato fornisce una soluzione canonica, unica, per l'equazione frazionaria del calore in presenza di un dato iniziale di segno variabile a crescita moderata, ed è basato su nuove stime asintotiche sul nucleo del calore frazionario. Inoltre dimostriamo che il flusso del calore frazionario conserva la convessità del dato iniziale (e qualcosa di più). Lavoro in collaborazione con *A. Greco*.

Maximum principles at infinity and the Ahlfors–Khas'minskii duality

Luciano Mari
Scuola Normale Superiore di Pisa

Friday, 24th February – 16:10-16:50

Maximum principles at infinity (or "almost maximum principles") are a powerful tool to investigate the geometry of Riemannian manifolds. Among them, we stress the Ekeland, the Omori–Yau principles and their weak versions, in the sense of Pigola–Rigoli–Setti. These last have nice probabilistic counterparts in terms of stochastic and martingale completeness, which in turn are related to potential theory and parabolicity. The validity of such principles is usually granted via suitable exhaustion functions called Evans–Khas'minskii potentials. In this talk, I discuss an underlying duality that allows to discover new relations between the

principles. Indeed, duality holds for a broad class of fully–nonlinear operators of geometric interest. Our methods use the approach to nonlinear PDEs pioneered by Krylov ('95) and Harvey–Lawson ('09 –), and involve the study of viscosity "almost solutions" of obstacle type problems.

This is joint work with *Leandro F. Pessoa*.

Behaviour in time of solutions to a class of Keller–Segel systems

Monica Marras
Università degli Studi di Cagliari

Friday, 24th February – 14:30-15:10

We consider a class of parabolic–parabolic Keller–Segel type system in a bounded domain in $\mathbb{R}^N$, with $N = 2, 3$, under different boundary conditions, with time dependent coefficients, a nonlinear cross diffusion and a positive source term. The solutions may blow up in finite time T. We discuss different methods to derive explicit estimates for the blow up time.

Supercritical periodic problems

Giovanni Molica Bisci
Università degli Studi Mediterranea di Reggio Calabria

Friday, 24th February – 9:30-10:10

This talk is about a joint work with *V. Ambrosio* and *J. Mawhin*. We discuss the existence and multiplicity of periodic solutions for a class of parametric nonlocal equations with critical and supercritical growth. It is well known that these equations can be realized as local degenerate elliptic problems in a half-cylinder together with a nonlinear Neumann boundary condition, through the extension technique in periodic setting. Exploiting this fact, and by combining the Moser iteration scheme in the nonlocal framework with an abstract multiplicity result valid for differentiable functionals due to Ricceri, we show that the problem under consideration admits at least three periodic solutions with the property that their Sobolev norms are bounded by a suitable constant. Finally, we provide a concrete estimate of the range of these parameters by using some properties of the fractional calculus on a specific family of test functions. This estimate turns out to be deeply related to the geometry of the domain.

Equazioni di Hartree frazionarie

Dimitri Mugnai
Università degli Studi di Perugia

Friday, 24th February – 18:00-18:40

Presenteremo alcuni recenti risultati su equazioni di Hartree frazionarie con varie forme di non linearità.

Improved Sobolev embeddings, profile decomposition, and global compactness for fractional Sobolev spaces

Giampiero Palatucci
Università degli Studi di Parma

Friday, 24th February – 10:20-11:00

We will present an improved Sobolev inequality for fractional Sobolev spaces $\dot{H}^s$ involving Morrey norms ([2]). This refinement allows to derive an alternative, perhaps more transparent, proof of the profile decomposition in $\dot{H}^s$ obtained by *P. Gérard* ([1]), using the abstract approach of dislocations spaces developed in *Tintarev* and *Fieseler* ([5]). As an application, we extend the global compactness result by *M. Struwe* ([4]) to any fractional Sobolev spaces $\dot{H}^s(\Omega)$, for $0 < s < N/2$ and $\Omega \subset \mathbb{R}^N$ a bounded domain with smooth boundary.

The talk is based on a series of papers in collaboration with *Adriano Pisante* of the University Sapienza in Rome.

References

[1] P. Gérard: Description du défaut de compacité de l'injection de Sobolev. *ESAIM: Control, Optimisation and Calculus of Variations* **3** (1998), 213–233.
[2] G. Palatucci, A. Pisante: Improved Sobolev embeddings, profile decomposition, and concentration-compactness for fractional Sobolev spaces. *Calc. Var. Partial Differential Equations* **50** (2014), no. 3-4, 799–829.
[3] G. Palatucci, A. Pisante: A Global Compactness type result for Palais-Smale sequences in fractional Sobolev spaces. *Nonlinear Anal.* **117** (2015), no. 3-4, 1–7.
[4] M. Struwe: A global compactness result for elliptic boundary value problems involving limiting nonlinearities. *Math. Z.* **187** (1984), 511–517.
[5] K. Tintarev, K.-H. Fieseler: *Concentration compactness. Functional-analytic grounds and applications.* Imperial College Press, London, 2007.

Asymptotic stability for nonlinear damped Kirchhoff systems involving the fractional p–Laplacian operator

Sara Saldi
Università degli Studi di Firenze

Friday, 24th February – 15:20-16:00

In this talk we present some results concerning the question of global and local asymptotic stability for nonlinear damped Kirchhoff systems, with homogeneous Dirichlet boundary conditions, under fairly natural assumptions on the external force, the distributed damping, the perturbation term and the dissipative term, when the initial data are in a special region. Particular attention is devoted to the asymptotic behavior of the solutions in the linear case. This talk is based on a joint work with *P. Pucci*, where we extend in several directions recent theorems and cover also the so–called degenerate case, that is when the Kirchhoff function M is zero at zero.

Alcuni risultati recenti su equazioni non lineari governate da operatori frazionari

Simone Secchi
Università degli Studi di Milano–Bicocca

Thursday, 23rd February – 14:30-15:10

Esporrò alcuni risultati recenti, ottenuti anche in collaborazione con *Silvia Cingolani* del Politecnico di Bari, su equazioni differenziali non lineari governate da operatori di tipo frazionario. Fra questi loperatore di Bessel e loperatore di Hartree semirelativistico.

Variational methods for nonlocal fractional problems

Raffaella Servadei
Università degli Studi di Urbino *Carlo Bo*

Thursday, 23rd February – 9:30-10:10

Motivated by the interest shown in the literature for nonlocal operators of elliptic type, in some recent papers we studied nonlocal fractional equations modeled by

$$\begin{cases} (-\Delta)^s u - \lambda u = f(x,u) & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^n \setminus \Omega\,, \end{cases}$$

where $s \in (0,1)$ is fixed, $(-\Delta)^s$ is the fractional Laplace operator, λ is a real parameter, $\Omega \subset \mathbb{R}^n$, $n > 2s$, is an open bounded set and the nonlinearity f satisfies natural growth assumptions.

Aim of this talk is to present some recent existence and multiplicity results for this kind of nonlocal fractional problems, obtained using variational and topological methods, which extend the validity of some theorems known in the classical Laplacian setting to the nonlocal framework.

On the wave equation with hyperbolic dynamical boundary conditions, interior and boundary damping and source

Enzo Vitillaro
Università degli Studi di Perugia

Thursday, 23rd February – 12-12:40

The talk will deals with local well–posedness, regularity, global existence and blow–up results for the wave equation with hyperbolic dynamical boundary conditions, interior and boundary damping and sources. The typical problem studied is

$$\begin{cases} u_{tt} - \Delta u + P(x,u_t) = f(x,u) & \text{in } (0,\infty)\times\Omega, \\ u = 0 & \text{on } (0,\infty)\times\Gamma_0, \\ u_{tt} + \partial_\nu u - \Delta_\Gamma u + Q(x,u_t) = g(x,u) & \text{on } (0,\infty)\times\Gamma_1, \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x) & \text{in } \overline{\Omega}, \end{cases}$$

where Ω is a bounded open C^1 subset of $\mathbb{R}^N$ $(N \geq 2)$, $\Gamma = \partial\Omega$, (Γ_0, Γ_1) is a measurable partition of Γ, Δ_Γ denotes the Laplace–Beltrami operator on Γ, ν is the outward normal to Ω, and the terms P and Q represent nonlinear damping terms, while f and g are nonlinear perturbations.