## Probability and Markov Chains

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## PERSONAL COURSE WEBPAGE:

http://www.math.TUGraz.at/~woess/perugia23

**Programme.** The programme will consist of a part of the following sub-topics. The choices will depend on the preliminary knowledge and preferences of the participants.

- 1. Quick reminder: probability spaces, random variables, expectation, typical probability distributions.
- 2. Convergence of random variables.
- 3. Law of Large Numbers. Variant A: classical (Kolomogorov), Variant B (more advanced): via Birkhoff's ergodic theorem (the latter to be proved).
- 4. Characteristic functions (= Fourier transformation) and Central Limit Theorem.
- 5. Martingales, exchangeability.
- 6. Basics of Markov chain theory.
- 7. Topics from advanced Markov chain theory, links with potential theory.

Knowledge of measure theory is indipsensable. It is suggested that participants who are not familiar with it undertake some preliminary reading. Particularly suitable: the initial parts of the books by Durrett and Klenke, as well as

https://staff.agu.edu.vn/ltduy/files/2013/01/Basic-measure-theory.pdf

## Recommended books:

- 1. Rick Durrett: Probability Theory and Examples. Cambridge University Press, first edition 1990, fifth edition 2019. Freely abvailable at https://services.math.duke.edu/~rtd/PTE5\_011119.pdf
- 2. Achim Klenke: Probability Theory A Comprehensive Course. Springer Universitext, first English edition 2008, third English edition 2022.
- 3. Wolfgang Woess: Denumerable Markov Chains. EMS Textbooks in Mathematics, European Math Soc., 2009.

**Exercises.** Participants should autonomously solve exercises and present them at the blackboard for discussion.

**Final evaluation:** based on presentations of exercises and the final exam. All course material is accessible at the course webpage (see above).

For all questions, you are invited to contact W. Woess at the above email address.

Graz, February 2023 Wolfgang Woess