Bipolar preference problems

Stefano Bistarelli***, Maria Silvia Pini**** **, Francesca Rossi**** **and K. Brent Venable**** [1]

1 INTRODUCTION

Real-life problems present several kinds of preferences. In this paper we focus on problems with both positive and negative preferences, that we call *bipolar problems*. Although seemingly specular notions, these two kinds of preferences should be dealt differently to obtain the desired natural behaviour. In fact, assume, for example, to have a scenario with two objects A and B. If we like both A and B, i.e., if we give to A and B positive preferences, then the overall scenario should be more preferred than having just A or B alone, and so the combination of such a preferences should give an higher positive preference. Instead, if we dislike both A and B, i.e., if we give to A and B negative preferences, then the overall scenario should be less preferred than having just A or B alone and so the combination of such a negative preferences should give a lower negative preference. When dealing with both kinds of preferences, it is natural to express also indifference, which means that we express neither a positive nor a negative preference over an object. A desired behaviour of indifference is that, when combined with any preference, it should not influence the overall preference.

Finally, besides combining positive preferences among themselves, and also negative preferences among themselves, we also want to be able to combine positive with negative preferences, allowing compensation. For example, if we have a meal with meat (which we like very much) and wine (which we don't like), then what should be the preference of the meal? To know that, we should be able to compensate the positive preference given to meat with the negative preference given to wine.

In this paper we start from the soft constraint formalism [2] based on c-semirings, that models only negative preferences. We then extend it via a new mathematical structure, which allows to handle positive preferences as well and we address the issue of the compensation between positive and negative preferences, studying the properties of this operation. Parts of this paper have appeared in [3].

2 SOFT CONSTRAINT FORMALISM

A soft constraint [2] is a classical constraint [4] where each instantiation of its variables has an associated value from a (totally or partially ordered) set. This set has two operations, which makes it similar to a semiring, and is called a c-semiring. A c-semiring is a tuple $(A, +, \times, \mathbf{0}, \mathbf{1})$ where: A is a set and $\mathbf{0}, \mathbf{1} \in A$; $+$ is commutative, associative, idempotent, $\mathbf{0}$ is its unit element, and $\mathbf{1}$ is its absorbing element; $\times$ is associative, commutative, distributes over $+$, $\mathbf{1}$ is its unit element and $\mathbf{0}$ is its absorbing element. Consider the relation $\leq_S$ over A such that $a \leq_S b$ iff $a + b = b$. Then: $\leq_S$ is a partial order; $+$ and $\times$ are monotone on $\leq_S$; $\mathbf{0}$ is its minimum and $\mathbf{1}$ its maximum; $(A, \leq_S)$ is a lattice and, $\forall\, a, b \in A$, $a + b = lub(a, b)$. Moreover, if $\times$ is idempotent, then $(A, \leq_S)$ is a distributive lattice and $\times$ is its glb. Informally, the relation $\leq_S$ gives us a way to compare (some of the) tuples of values and constraints. In fact, when we have $a \leq_S b$, we will say that *b is better than a*.

Given a c-semiring $S = (A, +, \times, \mathbf{0}, \mathbf{1})$, a finite set D (the domain of the variables), and an ordered set of variables V, a constraint is a pair $\langle def, con\rangle$ where $con \subseteq V$ and $def : D^{|con|} \to A$. Therefore, a constraint specifies a set of variables (the ones in con), and assigns to each tuple of values of D of these variables an element of A. A soft constraint satisfaction problem (SCSP) is just a set of soft constraints over a set of variables. For example, fuzzy CSPs [5] and weighted CSPS [2] are SCSPs that can be modeled by choosing resp. c-semirings $S_{FCSP} = ([0, 1], max, min, 0, 1)$ and $S_{WCSP} = (\Re^+, min, sum, +\infty, 0)$.

3 NEGATIVE PREFERENCES

The structure we use to model negative preferences is exactly a c-semiring [2] as described in the previous section. In fact, in a c-semiring there is an element which acts as indifference, that is $\mathbf{1}$, since $\forall a \in A$, $a \times \mathbf{1} = a$, and the combination between negative preferences goes down in the ordering (in fact, $a \times b \leq a, b$), that is a desired property. This interpretation is very natural when considering, for example, the weighted c-semiring $(R^+, min, +, +\infty, 0)$. In fact, in this case the real numbers are costs and thus negative preferences. The sum of different costs is worse in general w.r.t. the ordering induced by the additive operator (that is, min) of the c-semiring. From now on, we will use a standard c-semiring to model negative preferences, denoted as: $(N, +_n, \times_n, \bot_n, \top_n)$.

4 POSITIVE PREFERENCES

When dealing with positive preferences, we want two main properties to hold: combination should bring to better preferences, and indifference should be lower than all the other positive preferences. A positive preference structure is a tuple $(P, +_p, \times_p, \bot_p, \top_p)$ s. t. P is a set and $\top_p$, $\bot_p \in P$; $+_p$, the additive operator, is commutative, associative, idempotent, with $\bot_p$ as its unit element ($\forall a \in P$, $a +_p \bot_p = a$) and $\top_p$ as its absorbing element ($\forall a \in P$, $a +_p \top_p = \top_p$); $\times_p$, the multiplicative operator, is associative, commutative and distributes over $+_p$ ($a \times_p (b +_p c) = (a \times_p b) +_p (a \times_p c)$), with $\bot_p$ as its unit element and $\top_p$ as its absorbing element[2].

The additive operator of this structure has the same properties as the corresponding one in c-semirings, and thus it induces a partial order over P in the usual way: $a \leq_p b$ iff $a +_p b = b$. This allows to prove that $+_p$ is monotone ($\forall a, b, d \in P$ s. t. $a \leq_p b$, $a \times_p d \leq_p$

[1] $*$: University of Pescara and CNR, Pisa, Italy. E-mail: bista@sci.unich.it and Stefano.Bistarelli@iit.cnr.it. $**$: Department of Pure and Applied Mathematics, University of Padova, Italy. E-mail: {mpini,frossi,kvenable}@math.unipd.it.

[2] In fact, the absorbing nature of $\top_p$ can be derived from the other properties.

$b \times_p d$) and that it is the least upper bound in the lattice $(P, \leq_p)$ ($\forall a, b \in P$, $a \times_p b \geq_p a +_p b \geq_p a, b$).

On the other hand, $\times_p$ has different properties w.r.t. $\times_n$: the best element in the ordering ($\top_p$) is now its absorbing element, while the worst element ($\bot_p$) is its unit element. $\bot_p$ models indifference. These are exactly the desired properties for the combination and for indifference w.r.t. positive preferences. An example of a positive preference structure is ($\Re^+$,max,sum,0,$+\infty$), where preferences are positive real numbers aggregated with sum and compared with max.

5 BIPOLAR PREFERENCE STRUCTURES

For handing both positive and negative preferences we propose to combine the two structures described in sections 4 and 3 in what we call a *bipolar preference structure*. A bipolar preference structure is a tuple $(N, P, +, \times, \bot, \Box, \top)$ where, $(P, +_{|P}, \times_{|P}, \Box, \top)$ is a positive preference structure; $(N, +_{|N}, \times_{|N}, \bot, \Box)$ is a c-semiring; $+ : (N \cup P)^2 \longrightarrow (N \cup P)$ is an operator s. t. $a_n + a_p = a_p$, $\forall a_n \in N$ and $a_p \in P$; it induces a partial ordering on $N \cup P$: $\forall a, b \in P \cup N$, $a \leq b$ iff $a + b = b$; $\times : (N \cup P)^2 \longrightarrow (N \cup P)$ is a commutative and monotone ($\forall a, b, c \in N \cup P$, if $a \leq b$, then $a \times c \leq b \times c$) operator.

Bipolar preference structures generalize both c-semirings and positive structures. In fact, when $\Box = \top$, we have a c-semiring and, when $\Box = \bot$, we have a positive structure. Given the way the ordering is induced by $+$ on $N \cup P$, easily, we have $\bot \leq \Box \leq \top$. Thus, there is a unique maximum element (that is, $\top$), a unique minimum element (that is, $\bot$); the element $\Box$ is smaller than any positive preference and greater than any negative preference, and it is used to model indifference.

A bipolar preference structure allows to have a richer structure for one kind of preference, that is common in real-life problems. In fact, we can have different lattices $(P, \leq_p)$ and $(N, \leq_n)$. In the following, we will write $+_n$ instead of $+_{|N}$ and $+_p$ instead of $+_{|P}$. Similarly for $\times_n$ and $\times_p$. When $\times$ is applied to a pair in $(N \times P)$, we will sometimes write $\times_{np}$ and we will call it compensation operator.

An example of bipolar structure is the tuple (N=$[-1, 0]$, P=$[0, 1]$, $+$=max, $\times$, $\bot$=-1, $\Box$=0, $\top$=1), where $\times$ is such that $\times_p$= max, $\times_n$=min and $\times_{np}$=sum. Negative preferences are between -1 and 0, positive preferences between 0 and 1, compensation is sum, and the order is given by max. In this case $\times$ is not associative.

In general, operator $\times$ may be not associative. For example, if $\top \times \bot = c \in (N \cup P) - \{\top, \bot\}$ or if there are at least two elements $p \in P - \{\top\}, n \in N - \{\bot\}$ s.t. $p \times n = \Box$ and $\times_n$ or $\times_p$ is idempotent, then $\times$ is not associative. Since these conditions often occur in practice, it is not reasonable to require associativity of $\times$.

The combination of a positive and a negative preference is a preference which is higher than, or equal to, the negative one and lower than, or equal to, the positive one.

Possible choices for combining strictly positive with strictly negative preferences are thus the average, the median, the min or the max operator. Moreover, by monotonicity, if $\top \times \bot = \bot$, then $\forall p \in P$, $p \times \bot = \bot$. Similarly, if $\top \times \bot = \top$, then $\forall n \in N$, $n \times \top = \top$.

6 BIPOLAR PREFERENCE PROBLEMS

Once we have defined bipolar preference structures, we can define a notion of bipolar constraint, which is just a constraint where each assignment of values to its variables is associated to one of the elements in a bipolar preference structure. Given a bipolar preference structure $(N, P, +, \times, \bot, \Box, \top)$ a finite set D (the domain of the variables), and an ordered set of variables V, a constraint is a pair $\langle def, con \rangle$ where $con \subseteq V$ and $def : D^{|con|} \rightarrow (N \cup P)$.

A bipolar CSP (V, C) is then just a set of variables V and a set of bipolar constraints C over V.

A solution of a bipolar CSP (V, C) is a complete assignment to all variables in V, say s, with an associated preference $pref(s) = (p_1 \times_p \ldots \times_p p_k) \times (n_1 \times_n \ldots \times_n n_l)$, where, for $i := 1, \ldots, k$ $p_i \in P$, for $j := 1, \ldots, l$ $n_j \in N$, and $\exists \langle def, con \rangle \in C$ such that $p_i = def(s \downarrow_{con})$ or $n_j = def(s \downarrow_{con})$. A solution s is optimal if there is no other solution s' with $pref(s') > pref(s)$. In this definition, the preference of a solution s is obtained by combining all the positive preferences associated to its projections over the constraints, combining all the negative preferences associated to its projections over the constraints, and then, combining the two preferences obtained so far. If $\times$ is associative, then other definitions of solution preference could be used while giving the same result.

7 RELATED AND FUTURE WORK

Bipolar reasoning and preferences have recently attracted some interest in the AI community. In [1], a bipolar preference model based on a fuzzy-possibilistic approach is described, but positive and negative preferences are kept separate and no compensation is allowed. In [6] only totally ordered unipolar and bipolar preference scales are used, while we have presented a way to deal with partially ordered ones. On totally ordered scales the t-norm and t-conorm of [6] correspond to $\times_n$ and $\times_p$, while the uninorm of [6], similar to $\times$, is less general than $\times$, since it is associative.

We plan to adapt constraint propagation and branch and bound techniques to deal with bipolar problems and we intend to develop a solver for bipolar CSPs, which should be flexible enough to accommodate for both associative and non-associative compensation operators. We also intend to consider the presence of uncertainty in bipolar problems, possibly using possibility theory and to develop solving techniques for such scenarios. We also want to generalize other preference formalisms, such as multicriteria methods and CP-nets.

8 ACKNOWLEDGEMENTS

This work has been supported by Italian MIUR PRIN project “Constraints and Preferences” (n. 2005015491).

REFERENCES

[1] S. Benferhat, D. Dubois, S. Kaci, and H. Prade, ‘Bipolar representation and fusion of preferences in the possibilistic logic framework’, in *KR-02*. Morgan Kaufmann, (2002).

[2] S. Bistarelli, U. Montanari, and F. Rossi, ‘Semiring-based constraint solving and optimization’, *Journal of the ACM*, **44**(2), 201–236, (1997).

[3] S. Bistarelli, M. S. Pini, F. Rossi, and K. B. Venable, ‘Positive and negative preferences’, in *CP 2005 workshop on preferences and soft constraints*, (2005).

[4] R. Dechter, *Constraint processing*, Morgan Kaufmann, 2003.

[5] H. Fargier, T. Schiex, and G. Verfaille, ‘Valued Constraint Satisfaction Problems: Hard and Easy Problems’, in *IJCAI-95*, pp. 631–637. Morgan Kaufmann, (1995).

[6] M. Grabisch, B. de Baets, and J. Fodor, ‘The quest for rings on bipolar scales’, *Int. Journ. of Uncertainty, Fuzziness and Knowledge-Based Systems*, (2003).