

10) Calcolare

$$(i - \sqrt{3})^8 + (i + \sqrt{3})^8.$$

Trasformiamo gli addendi della formula in coordinate polari:

$$\begin{aligned}i - \sqrt{3} &= 2 \left(\cos\left(\frac{5}{6}\pi\right) + i \sin\left(\frac{5}{6}\pi\right) \right) \\i + \sqrt{3} &= 2 \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right).\end{aligned}$$

Dunque

$$\begin{aligned}(i - \sqrt{3})^8 + (i + \sqrt{3})^8 &= 2^8 \left(\cos\left(\frac{20}{3}\pi\right) + i \sin\left(\frac{20}{3}\pi\right) \right) + 2^8 \left(\cos\left(\frac{4}{3}\pi\right) + i \sin\left(\frac{4}{3}\pi\right) \right) = \\&= 256 \left(\cos\left(\frac{2}{3}\pi\right) + i \sin\left(\frac{2}{3}\pi\right) + -\cos\left(\frac{\pi}{3}\right) - i \sin\left(\frac{\pi}{3}\right) \right) = -256\end{aligned}$$