

Calcoliamo

$$\int x \log(1+x) dx.$$

Integrando per parti si ottiene

$$\begin{aligned} \int x \log(1+x) dx &= \int \left(\frac{x^2}{2}\right)' \log(1+x) dx \\ &= \frac{x^2}{2} \log(1+x) - \int \frac{x^2}{2} \frac{1}{x+1} dx \\ &= \frac{x^2}{2} \log(1+x) - \frac{1}{2} \int \frac{x^2}{x+1} dx \\ &= \frac{x^2}{2} \log(1+x) - \frac{1}{2} \int \left(x - 1 + \frac{1}{x+1}\right) dx \\ &= \frac{x^2}{2} \log(1+x) - \frac{1}{2} \left(\frac{x^2}{2} - x + \log(1+x)\right) \\ &= \frac{x^2}{2} \log(1+x) - \frac{x^2}{4} + \frac{x}{2} - \frac{1}{2} \log(1+x) + C \end{aligned}$$