

## ADDENDUM TO: COMPARISON BETWEEN DIFFERENT TYPES OF ABSTRACT INTEGRALS IN RIESZ SPACES

ANTONIO BOCCUTO - ANNA RITA SAMBUCINI

In [3] we did not give explicitly the definition of measurability for real-valued functions, with respect to finitely additive measures with values in a Dedekind complete Riesz space. We note that, in [3], all involved functions are intended to be measurable.

We now report the definition of measurability, which we gave in [2] (Definition 3.2).

**DEFINITION 0.1.** *Let  $X$  be any nonempty set,  $R$  be a Dedekind complete Riesz space,  $\mathcal{A} \subset \mathcal{P}(X)$  be an algebra. Given a non-negative mapping  $f : X \rightarrow \mathbb{R} \cup \{+\infty\}$  and a finitely additive set function  $\mu : \mathcal{A} \rightarrow R$ , we say that  $f$  is measurable with respect to  $\mu$  (or  $\mu$ -measurable) if for all  $t \geq 0$  the set  $\{x \in X : f(x) > t\}$  belongs to  $\mathcal{A}$ .*

*If  $f : X \rightarrow \mathbb{R}$  is a (non necessarily positive) map, we say that  $f$  is measurable if both its positive part  $f^+$  and its negative part  $f^-$  are measurable.*

In [3], all involved simple functions are intended to be measurable.

In Definition 4.1., we have to introduce an *outer measure*, because otherwise this definition could not make sense. More precisely, proceeding analogously as in [1], Definition 1.3., p. 86, we introduce an outer measure as follows:

---

Lavoro svolto nell' ambito dello G.N.A.F.A. del C.N.R..

A.M.S. Classification: 28A70.

DEFINITION 0.2 Let  $X$  be a nonempty set,  $\mathcal{A} \subset \mathcal{P}(X)$  be an algebra,  $R$  be a Dedekind complete Riesz space,  $\mu : \mathcal{A} \rightarrow R$  be a finitely additive set function. We call outer measure associated with  $\mu$  the  $R$ -valued set function  $\tilde{\mu}$ , defined on the whole of  $\mathcal{P}(X)$  by setting:

$$(1). \quad \tilde{\mu}(A) \equiv \inf\{\mu(B) : A \subset B, B \in \mathcal{A}\} \quad \forall A \in \mathcal{P}(X)$$

So, the definition of  $(o)$ -convergence in measure should be formulated as follows:

DEFINITION 0.3 We say that a sequence  $(f_n)_n$  of extended real-valued functions, defined on  $X$ ,  $(o)$ -converges in measure to  $f$  if

$$(o) - \lim_n \tilde{\mu}(\{x \in X : |f_n(x) - f(x)| > \varepsilon\}) = 0 \quad \forall \varepsilon > 0,$$

and analogously should be formulated the definition of  $(B)$ -convergence in measure.

In [3], on page 264, on the lines 5, 10, 13 and on the last line the positive finitely additive set function  $\mu$  should be replaced by the outer measure  $\tilde{\mu}$  defined in (1).

Thus, when we study the "level sets" of a function, the set function  $\mu$  have to be replaced by  $\tilde{\mu}$ , except in the case in which the involved function are simple: in this case we will take *a priori* simple measurable maps.

In the bibliography of this addendum, in [2] we indicate the exact reference: we are sorry for incorrectness of citation [2] which appeared in the bibliography of [3].

We thank Prof. Z. Lipecki for pointing us these gaps about the paper [3].

#### REFERENCES

- [1] Bhaskara Rao K. P. S., Bhaskara Rao M., *Theory of Charges*, Academic Press Inc., London, (1983).
- [2] Boccuto A., Sambucini A. R., *On the De Giorgi-Letta integral with respect to means with values in Riesz spaces*, Real Analysis Exchange, **21** (1995/96), pp. 793-810.
- [3] Boccuto A., Sambucini A. R., *Comparison between different types of abstract integral in Riesz spaces*, Rend. Circ. Mat. Palermo, Serie II, **46** (1997), pp. 255-278.