Giornate di Algebra Lineare Numerica e Applicazioni 2009

Algoritmi Genetici per la Ricostruzione di Immagini

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Direct problem

$$\mathbf{y} = A\mathbf{x} + \mathbf{n}$$

$\mathbf{x} \in \mathbb{R}^{n^2}$ original image
$\mathbf{y} \in \mathbb{R}^{n^2}$ observed image
$\mathbf{n} \in \mathbb{R}^{n^2}$ white, Gaussian noise with zero mean and known variance σ^2
$A \in \mathbb{R}^{n^2} \times \mathbb{R}^{n^2}$ blur operator

Original image

Observed image

Inverse problem: Image Restoration

The image restoration problem consists of finding an estimation of the original image $\mathbf{x}$, given the blur matrix A, the observed image $\mathbf{y}$ and the variance σ^2 of the noise.

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This problem is ill-posed in the sense of Hadamard.

Edge-preserving regularization technique

The solution of the problem can be defined as the minimum of the following primal energy function:

$$E_{\lambda,\alpha}(\mathbf{x}, \mathbf{b}) = DT(\mathbf{x}, \mathbf{y}) + \sum_{c \in C_k} [\lambda^2 (D_c^k \mathbf{x})^2 (1 - b_c) + \alpha b_c],$$

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α is a cost to pay for adding a discontinuity in the restored image.

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Definition

The dual energy function is defined as follow:

$$\bar{E}_{\lambda,\alpha}(\mathbf{x},\mathbf{b}) = \inf_{\mathbf{b}} E_{\lambda,\alpha}(\mathbf{x},\mathbf{b}).$$

GNC Deterministic Algorithm. – I

A classical deterministic technique for minimizing the dual energy function is the GNC (*Graduated Non-Convexity*) algorithm.

The GNC technique requires to find a finite family of approximating functions

$$\{E_d^{(p_i)}\}_{i\in\{1,\ldots,\bar{\imath}\}},$$

such that the first $E_d^{(p_1)}$ is convex and the last $E_d^{(p_{\bar{\imath}})}$ is the original dual energy function.

GNC Deterministic Algorithm. – II

GNC Algorithm

Instance: $(A, \mathbf{x}^{(0)})$
Output: $\mathbf{x}^{(i-1)}$

$i = 1;$
while $i \neq \bar{\imath}$ do
$\mathbf{x}^{(i+1)}$ is equal to the stationary point among the steepest descent direction of $E_d^{(p_{i+1})}$, starting from $\mathbf{x}^{(i)}$;
$i = i + 1;$

The stationary points, in the while iterations, are determined by a NLSOR (*Non–Linear Successive Over–Relaxation*).

The computational cost of the GNC algorithm is $O(\bar{\imath}\lambda N^2)$, where $O(\lambda N^2)$ is the estimated cost of the NLSOR algorithm.

Blind Image Restoration.

Classical Blind Image Restoration Problem

To find A^* and $\mathbf{x}^*$ such that:

$$(A^*, \mathbf{x}^*) = \arg\min_{(A,\mathbf{x})} E_d(A, \mathbf{x})$$

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Problem

The conjunct minimization of the energy function is not an easy task because it is strongly non linear.

Toward Genetic Blind Image Restoration

Proposed Approach: Nested Minimization Problem

To find A^* such that:

$$A^* = \arg\min_{A} E_d(A, \mathbf{x}(A))$$

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The computation of $\mathbf{x}(A)$ is the not–blind problem associated to A.

Proposed Approach: Nested Minimization Problem

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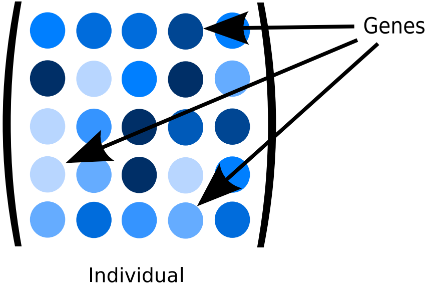
$$\mathbf{x}^* = \mathbf{x}(A^*).$$

The computation of $\mathbf{x}(A)$ is the not–blind problem associated to A.

To compute A^* we used a GA (*Genetic Algorithm*).

Chromosomes and genes representation.

Chromosomes are blur operators, represented by the associated blur matrix $M \in \chi_\mu^{(2z+1)\times(2z+1)}$, with $\chi_\mu = \{0, \ldots, \mu\}$ and μ fixed.



Fitness.

Straightforward Fitness Function.

The fitness function $F(\cdot)$ of an individual A is

$$F(A) = -E_d(A, \mathbf{x}(A)),$$

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Remark

The computational cost of evaluating E_d for large images is very expansive, being proportional to image size $N \times N$.

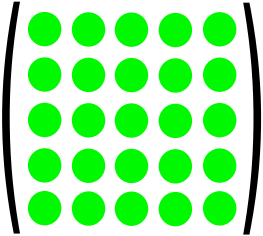
Selection.

The selection is implemented by a tournament method.

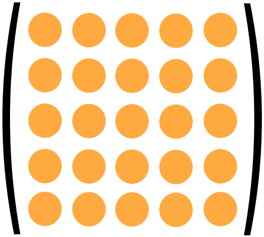
In details, a tournament runs among two individuals randomly chosen from the population, higher is the fitness and higher is the probability that an individual is selected as winner in at least one tournament.

Crossover.

1. Randomly select two Individuals;



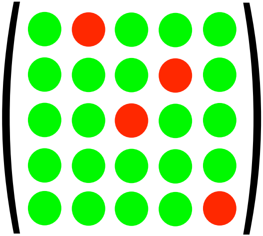
First Blur Mask



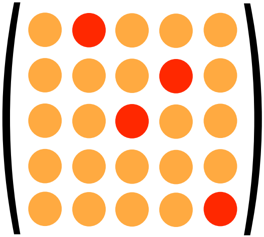
Second Blur Mask

Crossover.

1. Randomly select two Individuals;
2. Randomly select some blur mask entries in $\{1, \ldots, 2z+1\} \times \{1, \ldots, 2z+1\}$;



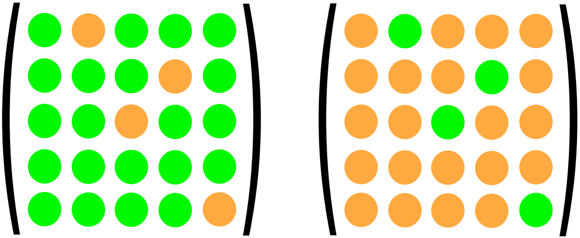
First Blur Mask



Second Blur Mask

Crossover.

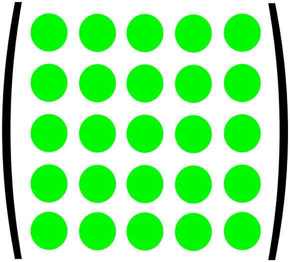
1. Randomly select two Individuals;
2. Randomly select some blur mask entries in $\{1, \ldots, 2z+1\} \times \{1, \ldots, 2z+1\}$;
3. Switch the corresponding elements selected.



First Blur Mask   Second Blur Mask

Mutation.

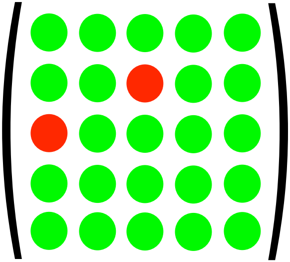
1. Randomly select an Individual;



Selected Blur Mask

Mutation.

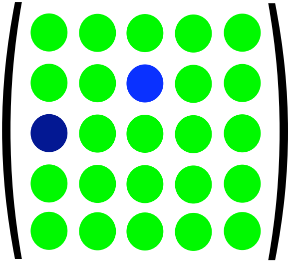
1. Randomly select an Individual;
2. Randomly select some blur mask entries in $\{1, \ldots, 2z + 1\} \times \{1, \ldots, 2z + 1\}$;



Selected Blur Mask

Mutation.

1. Randomly select an Individual;
2. Randomly select some blur mask entries in $\{1, \ldots, 2z+1\} \times \{1, \ldots, 2z+1\}$;
3. Substitute that entries with a random numbers in χ_μ.



Selected Blur Mask

Termination Condition.

The ideal case is that GAs should typically run until they reach a solution which is close to the global optimum.

The stop condition is assumed to be an homogeneity measure of the population. Namely, if the standard deviation of the population is less than a threshold then the algorithm is stopped.

To avoid too long computations the process is usually terminated by the former technique or after a fixed number of generation MG.

The New Fitness: Dynamic Local Evaluation.

To drop down the computational cost and preserving the whole image features:

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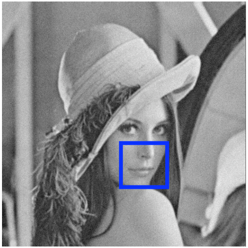
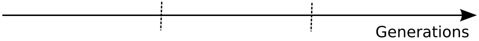
1. Select at random an $\tilde{N} \times \tilde{N}$ subimage;





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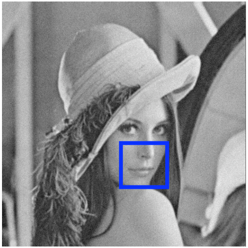
To drop down the computational cost and preserving the whole image features:

1. Select at random an $\tilde{N} \times \tilde{N}$ subimage;
2. Let the GA evolves the population to fit that subimage for *par* generations;



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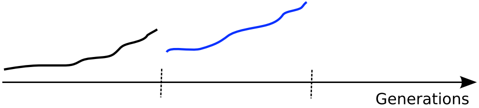

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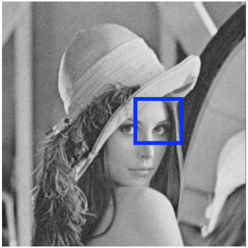

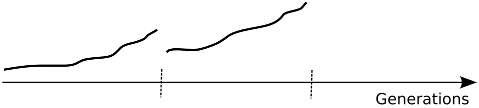

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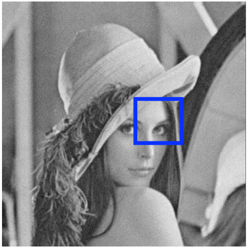
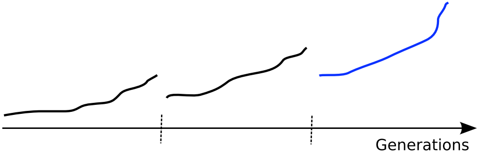




Subimage Significancy.

In order to let the subimage "be significant" providing useful information to the optimization process, it is necessary to put some restrictions on its smoothness.

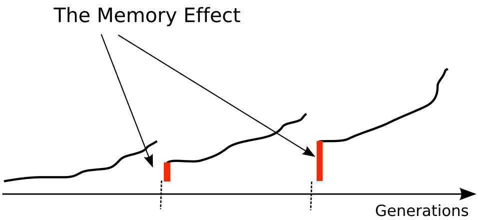
The $\tilde{N} \times \tilde{N}$ subimage $\tilde{\mathbf{y}}$ is selected from the data $\mathbf{y}$ in such a way that its smoothness $S(\tilde{\mathbf{y}})$ lays in the prefixed range $[l, u]$.

If both parameters l, u are correctly chosen then any randomly chosen subimage $\tilde{\mathbf{y}}$ can be representative of the whole data.

DLE Motivation.

Why does it work?

Individuals maintain a memory of their past optimization activity and they convey it when the fitness function changes, i.e. a new subimage is selected.



Speed Up Induced by DLE.

The main advantage of the dynamical local evaluation technique is the dramatic cost reduction.

It is easy to see that because of the form of the cost term

$$O(MG\ MP\,\bar{\imath}\,N^2\,\lambda)$$

the dynamical local evaluation technique will offer a significant computational cost reduction, since it drops down the image dimension N^2 and also $\bar{i}$, the number of iteration needed to restore the subimage.

Genetic Blind Image Restoration With DLE

Instance: the data image $\mathbf{y}$
Output: the reconstructed image $\mathbf{x}$

initialize randomly the population $P^{(0)}$
while $gen \leq MG$ or the population is not homogeneous do
- if $gen \equiv 0 \mod par$ then pick up a subimage $\tilde{\mathbf{y}}$;
- generate a random starting point $\mathbf{x}^{(0)}$;
- evaluate the *fitness* $F(P_{id}^{(gen)})$ of each individual *id* with respect to $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{x}}^{(0)}$;
- *select* the individuals by a tournament to populate the remain $P^{(gen+1)}$;
- compute the *crossover* of some randomly selected individuals in $P^{(gen+1)}$;
- *mutate* some randomly selected individuals in $P^{(gen+1)}$;
- $gen = gen + 1$;

$A = P_0^{(gen+1)}$;

$\mathbf{x} = \tilde{\mathbf{x}}(A, \mathbf{y})$;

Experimental Results.

In each test we have fixed:

- the population size $MP = 100$,
- the maximum generation $MG = 2000$,
- the maximum blur mask entry $\mu = 15$,
- the subimage extraction period $par = 10$.

Test 1: Syntetic Image.

Figure: Data image 128×128 synthetic image

Blur Operator:

$$M_1 = \begin{pmatrix} 10 & 0 & 10 \\ 0 & 1 & 0 \\ 10 & 0 & 10 \end{pmatrix}.$$

Noise: $\sigma^2 = 0$.
Parameters:
$\lambda = 1$, $\alpha = 225$ and $\tilde{N} = 36$

No restriction on the selection of the subarea due to the geometric structure of the image.

Test 1: Results.

(a) (b) (c)

Figure: (a) data image; (b) first order not–blind reconstruction; (c) first order blind reconstruction.

Test 2: Real Image no Noise.

Figure: Data image 512×512 real image

Blur Operator:

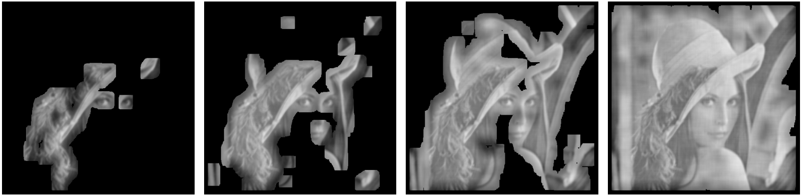
$$M_2 = \begin{pmatrix} 4 & 3 & 3 & 7 & 7 & 2 & 2 \\ 1 & 0 & 0 & 1 & 6 & 2 & 7 \\ 4 & 1 & 7 & 7 & 6 & 7 & 0 \\ 0 & 0 & 7 & 7 & 7 & 2 & 2 \\ 0 & 5 & 0 & 7 & 7 & 4 & 5 \\ 3 & 4 & 7 & 7 & 2 & 4 & 2 \\ 2 & 7 & 7 & 7 & 3 & 7 & 5 \end{pmatrix}.$$

Noise: $\sigma^2 = 0$.

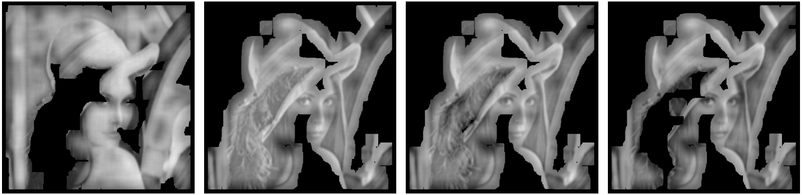
Parameters:

$\lambda = 2$, $\alpha = 4$ and $\tilde{N} = 50$

Test 2: Choosing l and u.



$[6000, +\infty[$ $[5500, +\infty[$ $[5000, +\infty[$ $[4000, +\infty[$



$[4000, 5000]$ $[5000, 7000]$ $[5000, 6000]$ $[5000, 5500]$

Figure: part of the image in *Test 2* covered by the subimage extraction procedure with different lower bounds l and upper bounds u.

Test 2: Data Image.

Test 2: II Order Not–Blind Restoration.

Test 2: II Order Blind Restoration with DLE.

Test 3: Real Image with Noise

Figure: Data image 512×512 real image

Blur Operator:

$$M_3 = \begin{pmatrix} 0 & 1 & 7 & 1 & 0 \\ 1 & 7 & 10 & 7 & 1 \\ 7 & 10 & 15 & 10 & 7 \\ 1 & 7 & 10 & 7 & 1 \\ 0 & 1 & 7 & 1 & 0 \end{pmatrix}.$$

Noise: $\sigma^2 = 100$.

Parameters:

$\lambda = 8$, $\alpha = 640$ and $\tilde{N} = 50$

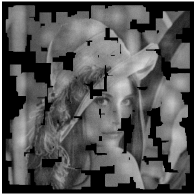
Test 3: Choosing l and u (values $\times 1000$).

$[1065, 1075]$

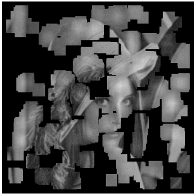
$[1068, 1070]$

$[1072, 1078]$

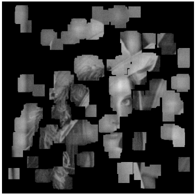
$[1075, 1080]$



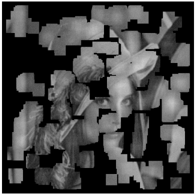
$[1080, 1085]$



$[1082, 1085]$



$[1084, 1086]$



$[1082, 1083]$

Figure: part of the image in *Test 3* covered by the subimage extraction procedure with different lower bounds l and upper bounds u.

Test 3: Data Image.

Test 3: II Order Not–Blind Restoration.

Test 3: II Order Blind Restoration with DLE.

Numerical Comparison of the Result's Quality.

Mean Squared Error between the ideal image and the not–blind (notB) and blind restored image (BPGA).
It has been impossible to compare the results of our algorithm with the final results of a classical GA (BCGA), due to excessive computational cost of the latter one.
In the table is shown the BCGA column is referred to the BCGA stopped after the same amount of time our algorithm takes to reach the termination condition.

Test	σ^2	mask	notB	BPGA	BCGA
1	0	3×3	0.7923	4.2649	6.5237
2	0	7×7	9.3727	8.6938	22.8943
3	100	5×5	7.8614	7.8736	19.6485

Test 4: Real Image – Inverse Gray Level Quantization

Figure: Data image 512×512 real image quantized in 5 leves of gray for each RGB components

A priori Blur Mask:

$$M_4 = \begin{pmatrix} 10 & 0 & 10 \\ 0 & 1 & 0 \\ 10 & 0 & 10 \end{pmatrix}.$$

Noise: $\sigma^2 = 0$.

Parameters:

$\lambda = 3$, $\alpha = 1000$ and $\tilde{N} = 50$

Test 4: Data Image.

Test 4: II Order Not–Blind Restoration.

Test 4: II Order Blind Restoration with DLE.

Test 5: Real Image – Inverse Gray Level Quantization

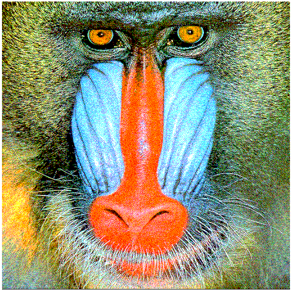


Figure: Data image 512×512 real image quantized in only 8 colors (i.e. black and white RGB components)

A priori Blur Mask:

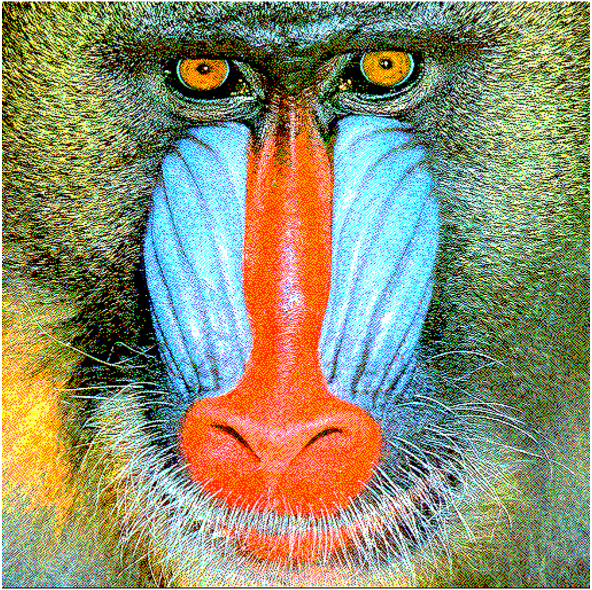
$$M_5 = \begin{pmatrix} 10 & 0 & 10 \\ 0 & 1 & 0 \\ 10 & 0 & 10 \end{pmatrix}.$$

Noise: $\sigma^2 = 0$.

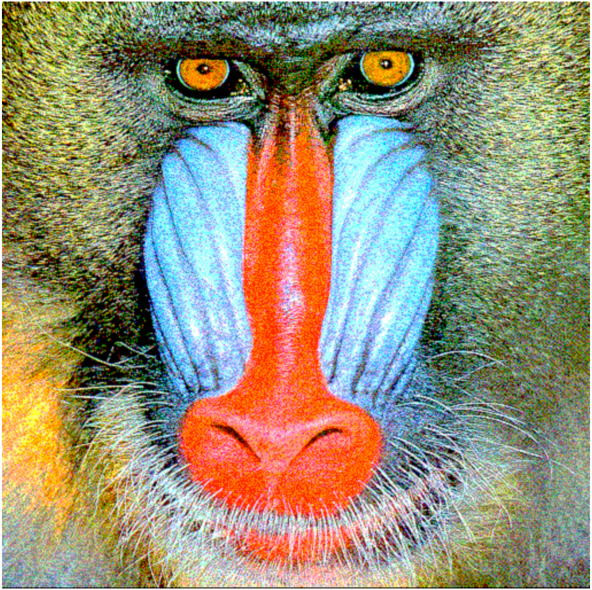
Parameters:

$\lambda = 0.5$, $\alpha = 100$ and $\tilde{N} = 50$

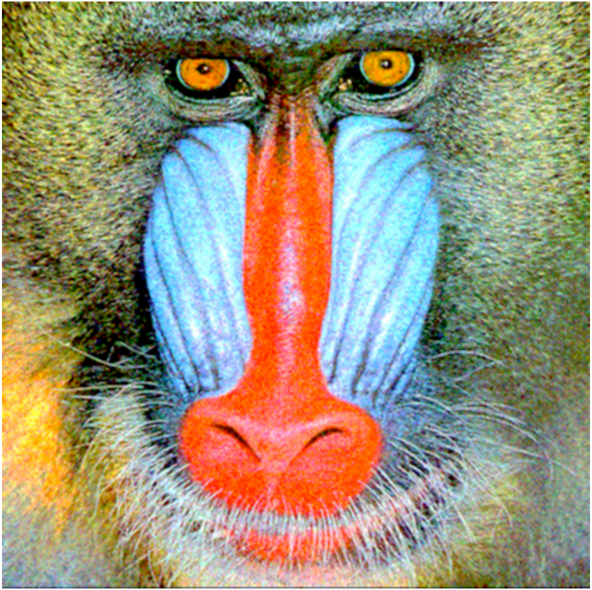
Test 5: Data Image.



Test 5: II Order Not–Blind Restoration.



Test 5: II Order Blind Restoration with DLE.



Conclusion and Future Works.

We have presented an evolutionary algorithm for the blind image restoration problem which uses an innovative technique of dynamical local fitness evaluation.

- The dynamical local evaluation technique offers a drastic computational cost reduction.
- The image quality is not compromised and sometimes it is even improved.
- The technique finds a natural application in contexts where bandwidth, computational and storage capabilities are limited.

Future works will aim to both formalize the local evaluation and extend it in other numerical contexts in which data have a geometrical organization.