Computing Coalition of Arguments with CSPs*

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1 Extending Dung Argumentation to Coalitions

An Argumentation Framework (AF) is a pair $\langle \mathcal{A}_{rgs}, R\rangle$ of a set $\mathcal{A}_{rgs}$ of arguments and a binary relation R on $\mathcal{A}_{rgs}$ called the attack relation [1]. The justified arguments under different extensional semantics (e.g. conflict-free ones) are then evaluated, and the claims of these arguments define the inferences of the underlying theory.

Our aim is to partition a set of arguments into coalitions of arguments [2–4], merging the two worlds of coalition formation and argumentation. A classical scenario could be represented by the need to aggregate a set of distinct arguments into different lines of thought. The basic idea is to start from a single set of arguments and partition them to several agents, with the condition that each subset (i.e. coalition) has to show the same properties defined by Dung, e.g. admissibility or stability [1]. In order to model and solve the proposed extended problems we use *Constraint Programming* [5]: the solution of the obtained *Constraint Satisfaction Problem* [5] (*CSP*) represents a partition of the arguments.

According to our extension, the classical Dung AF is a particular case of our extended framework and it corresponds to a partition with a (unique) maximal subset (the extension as described in Sec. 1) plus different singletons (the arguments which are not included in the extension). An example representing the original framework and our extension is illustrated in Fig. 1, in which case (A) represent the unique subset obtained

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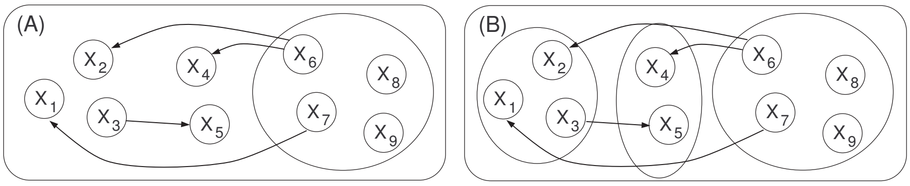

Fig. 1: Differences between classical Dung AF (A) and the extended partitioned framework (B) presented in this paper.

in the classical framework and case (B) shows the results obtained with our extension. Fig. 1 (A) represents a conflict-free extension as described in [1], while Fig 1 (B) represents a conflict-free partition of coalition, since each coalition is conflict-free (see Def. 1).

In the following, we extend the definitions given in [1] in order to consider coalitions instead of singletons.

Definition 1. *A partition of coalitions* $\mathcal{G} = \{\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_n\}$ *is* ***conflict free*** *iff for each* $\mathcal{B}_i \in \mathcal{G}$, $\mathcal{B}_i$ *is conflict free, i.e. no attacks inside the same coalition.*

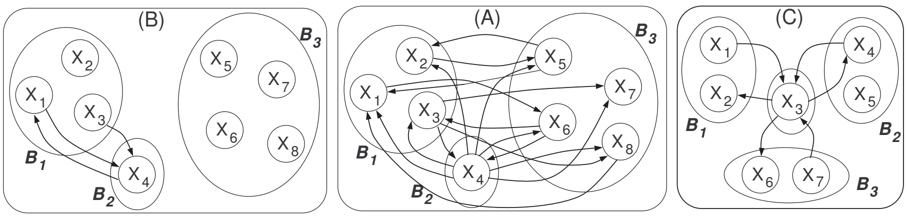

Fig. 2: A stable (A), admissible and complete (B) and an admissible but not complete (C) partition of coalitions.

Notice that singletons are always conflict-free; for example, coalition C_1 in Fig.2 (A) is conflict-free. Now we revise the concept of attacks among coalitions and the notion of stable partition of coalitions:

Definition 2. *A coalition* $\mathcal{B}_i$ ***attacks*** *another coalition* $\mathcal{B}_j$ *if any of its elements attacks at least one element in* $\mathcal{B}_j$*, i.e.* $\exists a \in \mathcal{B}_i, b \in \mathcal{B}_j$ *s.t.* $a\,R\,b$. A conflict free *partition* $\mathcal{G} = \{\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_n\}$ *is* ***stable*** *iff for each coalition* $\mathcal{B}_i \in \mathcal{G}$*, all its elements* $a_k \in \mathcal{B}_i$ *are attacked by all the other coalitions* $\mathcal{B}_i$ *s.t.* $i \neq j$*, i.e.* $\forall a_i \in \mathcal{B}_k, \exists b_z \in \mathcal{B}_j . b_z\,R\,a_i$*.* $(\forall k \neq j)$*.*

Fig. 2 (A) represents a stable partition: each argument in $\mathcal{B}_1$ attacks at least one argument in $\mathcal{B}_2$ and one argument in $\mathcal{B}_3$ (and the same is true for $\mathcal{B}_2$ and $\mathcal{B}_3$). To have a stable partition means that each of the arguments cannot be moved from one coalition to another without inducing a conflict in the new coalition.

In the next two definitions we separately extend the concept of admissible and complete extensions respectively. Fig. 2 (B) represents an admissible and complete partition of coalitions, while Fig. 2 (C) represents a not stable partition.

Definition 3. *A* conflict free *partition $\mathcal{G} = \{\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_n\}$ of coalitions is **admissible** iff for each argument $a \in \mathcal{B}_i$ attacked by $b \in \mathcal{B}_a$, then $\exists c \in \mathcal{B}_i$ attacks $a \in \mathcal{B}_a$ (i.e. $c\,R\,a$), that is each $\mathcal{B}_i$ defends itself on each of its attacked arguments.*

In a complete partition, each rational agent is able to defend its line of thought because it counter-attacks all its attacking lines.

Definition 4. *An admissible partition $\mathcal{G} = \{\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_n\}$ is a **complete** partition of coalitions iff each argument a which is defended by $\mathcal{B}_i$ is in $\mathcal{B}_i$ (i.e. $a \in \mathcal{B}_i$).*

Fig. 2 (B) is an admissible partition because is conflict-free and both $\mathcal{B}_1$ and $\mathcal{B}_2$ defend themselves: x_3 attacks x_4 but x_4 attacks x_1. Fig. 2 (C) represents an admissible partition but it is not stable because x_6 is defended also by coalitions $\mathcal{B}_1$ (via x_1) and $\mathcal{B}_2$ (via x_4) but belongs to $\mathcal{B}_3$.

2 Mapping Partition Problems to CSPs

A CSP P is a triple $P = \langle X, D, C\rangle$ where X is an n-tuple of variables $X = \langle x_1, x_2, \ldots, x_n\rangle$, D is a corresponding n-tuple of domains $D = \langle D_1, D_2, \ldots, D_n\rangle$ such that $x_i \in D_i$, C is a t-tuple of constraints $C = \langle C_1, C_2, \ldots, C_t\rangle$, where a constraint is a relation on the X variables [5].

In this section we show a mapping from the AF extended to coalitions (see Sec. 1) to CSPs, i.e. $\mathcal{M} : AF \rightarrow CSP$. $\mathcal{M}$ is described as follows: we define a variable for each argument $a_i \in \mathcal{A}_{rgs}$, i.e. $V = \{a_1, a_2, \ldots, a_n\}$ and each of these argument can be taken or not, i.e. the domain of each variable is $D = \{1, n\}$. The value of a variable represents the coalition to which argument a_i belongs; for example if $a_1 = 2$ it means that the first argument belongs to the second coalition. We can have a maximum of n coalitions, that is all singletons. Notice that b attacks a meas that b is a father of a in the interaction graph, and c attacks b attacks a means that c is a grandfather of a.

Conflict-free constraints: since we want to find the conflict-free sets, if $a_i \, R \, a_j)$ is in the graph we need to prevent the same coalition to include both arguments a_i and a_j: $c_{a_i,a_j}(a_i = k, a_j = k)$ is not allowed. The other possible assignment of the variables,i .e. if $a_i \neq a_j$ are permitted: in these cases we are choosing only one argument between the two (or none of the two) and thus, we have no conflict inside the same coalition. **Admissible constraints:** for the admissibility of a partition, if a_i has several grandfathers $a_{g1}, a_{g2}, \ldots, a_{gk}$ and only one father a_f, we need to add a $k+1$-ary constraint $c_{a_i=h,a_{g1},\ldots,a_{gk}}(a_i = h, a_{g1} = j_1, \ldots, a_{gk} = j_k)$ is not allowed if $\forall j_i . j_i \neq h$. The explanation is that at least a grandfather must be taken in the the same coalition, in order to defend a_i from one of his fathers a_f. Notice that, if a node is not attacked (i.e. he has no fathers), he can be taken or not in the admissible set. **Complete constraints:** if we have a son node a_i with multiple grandsons $a_{s1}, a_{s2}, \ldots, a_{sk}$, we need to add that the constraint $c_{a_i,a_{s1},\ldots,a_{sk}}(a_i = j, a_{f1} = j, \ldots, a_{sk} = j)$ is allowed, and the other assignment for $a_i, a_{s1}, a_{s2}, \ldots, a_{sk}$ are prohibited. In words, if a node is not taken in a coalition (i.e. $a_i = j$), all of its grandsons must be included in the same coalition. **Stable constraints:** this kind of constraints can be represented with a global constraint such that for each couple of arguments a_i, a_j in the problem belonging to two different coalitions, respectively k and z, at least one of the attacks to a_j has to come from a node in coalition k: if $b_1, b_2, \ldots b_n$ are all the arguments that attacks a_j, $c_{a_i=k,a_j \neq k,b_1,b_2,\ldots,b_n}((b_1 = k) \vee (b_2 = k) \vee \cdots \vee (b_n = k))$.

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